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Original Article

Optimization and Mathematical Modulation of the NRF Number in the NRF-LEACH Approach

Yassine Kharchachi[1*], Khalid Housni[1]

[1]*Department of Computer Science, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco.*

[*]*Corresponding Author : yassine.kharchachi@uit.ac.ma*

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Abstract - *Among them, LEACH is one of the most established approaches in reducing energy consumption in wireless sensor networks, especially by restricting long-range transmissions towards the base station, while its fixed-round extension, NRF-LEACH, prolongs network lifetime by keeping the same cluster structure for a number of rounds. However, its performance depends heavily on finding the optimal number of fixed rounds, and there is no clear analytical formulation for the NRF value, which is a significant gap in the literature that affects energy efficiency, network stability, and the lifetime of the sensor network. In this work, we introduce a hybrid method combining multi-objective optimization and statistical modeling to find the optimal NRF value for different packet sizes and sensor densities, and then use the optimized results to construct an analytical model of the NRF parameter using statistical fitting, which can predict the NRF value directly from network characteristics. The experimental results obtained using NS-3 simulations demonstrate that the optimal NRF value changes significantly with packet size and node density, and among the models tested, power regression is the most consistent with the simulation data, with the correlation coefficient nearly 0.94 and the coefficient of determination nearly 0.89, which proves the validity of the proposed modeling strategy and also indicates the necessity to select an appropriate NRF value for the application of NRF-LEACH in practical scenarios.*

Keywords - *WSN, CH, LEACH, NRF-LEACH, NSGA-II, NRF.*

1. Introduction

Wireless Sensor Networks (WSNs) are effective in a variety of applications, including environmental monitoring, healthcare, industrial automation, and military systems [1-3]. Due to the limited energy resources of sensor nodes, which are typically non-replaceable, energy conservation is crucial for any communication or routing mechanism [1-3]. Hierarchical routing protocols, which are known to decrease communication overhead and prolong network lifetime by grouping nodes into clusters and fusing data locally before forwarding it to the base station, have gained a lot of attention in this area [1, 4-6]. LEACH is among the most influential of these protocols because of its distributed operation and randomized cluster formation strategy [1]. Although it is still relevant, LEACH repeats a setup phase at each round where cluster heads are elected, and clusters are reconfigured, which consumes a lot of energy and reduces the lifetime of the network, particularly in dense deployments or high-communication scenarios [7-9]. To overcome this limitation, the NRF-LEACH variant proposes keeping the same cluster structure for multiple consecutive rounds, avoiding repeated setup phases, and resulting in significant energy savings and increased network lifetime, with improvements reported to be larger than those of the original protocol. However, the effectiveness of this variant depends heavily on the number of rounds during which the cluster structure is preserved. The selection of this parameter, called NRF, has a significant impact on the protocol's performance. However, to the best of our knowledge, the NRF parameter has not been formally characterized in the literature, and existing works introduce enhanced clustering strategies or combine LEACH with optimization techniques but they rely on fixed or empirically selected NRF values and lack a principled way to determine the optimal NRF in various network conditions [7-14], which is a clear research gap. With growing scale and heterogeneity of WSN deployments, a clear understanding of the evolution of the optimal NRF as a function of packet size, node density, and network behavior is necessary to ensure consistent performance across diverse scenarios. In order to fill this gap, this paper proposes a hybrid methodology for estimating the optimal NRF based on multiobjective optimization and statistical modeling of the optimized results to determine the optimal NRF value that optimizes multiple performance metrics [15–18]. The Pareto-optimal solutions for each network configuration for three key metrics (the number of rounds, the cumulative number of dead nodes, and the total number of bits delivered to the base station) are calculated using the NSGA-II algorithm [15–18]. Then, the Pareto-optimal solutions are statistically fitted into an analytical expression of the NRF parameter, and the optimal NRF can be estimated directly from packet size and sensor density, providing a generalized and practical formula that can be embedded into the NRF-LEACH protocol or other clustering-based routing mechanisms.

The study is guided by the following research questions:

1. What are the effects of packet size, network density, and overall energy behavior on the optimal NRF value?
2. Are multi-objective optimization methods, such as NSGA-II, able to find the best NRF values reliably across many different scenarios?
3. Is it possible to create a mathematical model that can derive the accurate NRF value from observable network parameters?

This research not only offers a systematic, optimization-based analysis of the NRF parameter but also an analytical model capable of predicting its value under realistic conditions. These contributions make the NRF-LEACH protocol more comprehensible and, at the same time, assist in the selection of optimal configuration parameters in real-world wireless sensor networks by providing practical insights.

2. Related Works

Energy-efficient routing in wireless sensor networks has encouraged the development of a vast number of clustering methods to reduce communication overhead and prolong network lifespan. The original LEACH protocol has been enhanced by changing the way cluster-heads are selected or by defining the clustering process differently. A case in point is the use of energy-aware or distance-dependent criteria in numerous works, which has resulted in the non-selection of nodes with very little residual energy (the references to those studies are [12–14]), leading to a more even spread of energy consumption across the entire network. These methods usually outperform the original LEACH, yet they still require frequent reconfiguration stages, which reduces the network's overall efficiency in the long run. Other researchers have proposed different approaches and considered hybrid or multi-stage clustering techniques, such as the combination of redundant and reserve cluster heads to keep the cluster alive during node failure and to strengthen communication between the base station and the clusters by increasing the number of packets delivered [8]; or hybrid optimization models that use techniques like Particle Swarm Optimization (PSO), K-means clustering, or fuzzy logic not only to select the cluster heads but also to minimize the risk of early node death [10]. All these approaches indeed have the potential to yield considerable performance gains; however, they often come with the cost of additional computational steps or the need for centralized decision-making, which may not be suitable for large-scale or resource-constrained networks. Moreover, different types of LEACH have been put forward, which also share a common goal of achieving more uniform energy consumption, like the ones that are based on residual energy [10], multi-hop routing [11], and hierarchical decision processes [13]. In particular, BCE-LEACH relies on the combination of both energy and distance metrics, which not only balance cluster-head selection but also lead to the introduction of relay nodes for multi-hop communication, resulting in longer network lifetime as compared to the traditional LEACH [10-11]. The latest developments include, among others, shortest-path algorithms, data fusion techniques, and cluster-head coordination mechanisms aimed at improving the efficiency of the system by increasing its throughput and reducing the redundancy in the collected data [13]. The works reviewed so far indicate that there is still a strong interest in improving clustering methods and data delivery performance in Wireless Sensor Networks (WSNs).

Other authors have used machine learning techniques to aid in the clustering process, such as using LEACH in combination with learning-based algorithms, such as a variation of K-means or a weighted decision scheme to choose cluster heads based on parameters related to energy, distance, or node density [19-21]. These methods provide more flexibility, but generally involve extra computation or need an external training stage, which is not suitable for highly constrained sensor deployments. Lastly, competition-based or location-aware clustering strategies have been proposed to minimize overlap between clusters and minimize intra-cluster transmission distances [14], which often use spatial metrics, residual energy, or local density to select appropriate cluster-head candidates and can lead to more balanced energy distribution in dense scenarios. However, none of these studies tackle the problem of how many rounds to run in order to maintain a cluster structure over time. From the existing literature, it has been observed that no prior work has provided a formal framework to identify or model the optimal number of fixed rounds in NRF-LEACH, and most studies focus either on improving the cluster-head selection or proposing new clustering strategies, but the NRF parameter that is at the core of the performance of NRF-LEACH is empirically selected, without a systematic evaluation of NRF across different packet sizes, node densities, and network conditions, and no analytical model exists for predicting its optimal value, which motivate the present study, which combines multi-objective optimization with statistical modeling to establish a robust and generalizable method for estimating the optimal NRF value under different network scenarios.

3. NRF-LEACH Protocol

The NRF-LEACH protocol enhances the traditional LEACH algorithm by reducing the number of cluster reconfigurations, since each round starts with a setup phase when cluster heads are elected, and nodes join the nearest cluster head, which is a major energy consumer in dense or long-duration deployments.

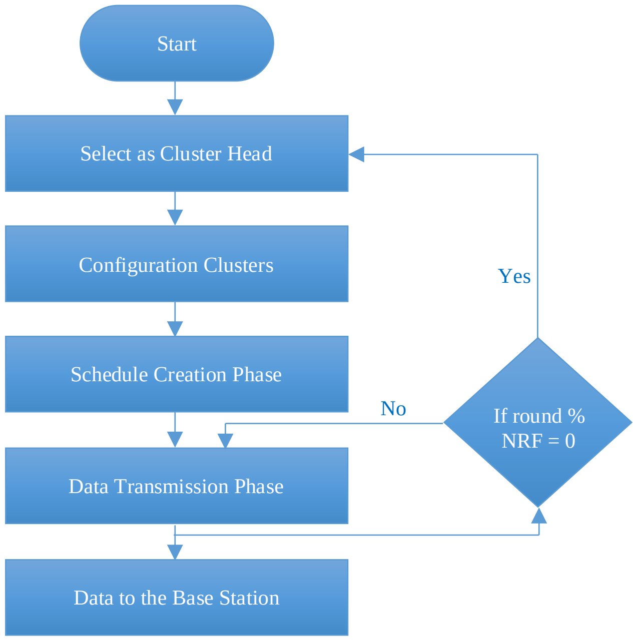

Fig. 1 NRF-LEACH protocol flowchart

NRF-LEACH overcomes this limitation by maintaining the same cluster structure for a fixed number of rounds, called NRF, after the initial setup phase; in the following NRF–1 rounds, the cluster configuration is preserved and the network avoids repeated advertising and cluster-formation procedures, which considerably reduces the energy overhead for cluster reinitialization and prolongs the network lifetime when the NRF value is appropriate for the network characteristics.

There are two primary stages in the protocol: the configuration phase and the transmission phase. Cluster heads broadcast an advertisement message during the configuration phase, and every node that is not a cluster head chooses the nearest cluster head and sends a membership message. These communications incur both transmission and reception costs. The total energy consumed during this phase corresponds to the sum of (eq. 1) the energy spent by cluster heads to broadcast advertisements, (eq. 2) the energy needed by non-cluster-head nodes to receive these messages, and (eq. 3) the energy required for nodes to transmit membership requests. In LEACH, this cost is incurred every round, whereas in NRF-LEACH it is only incurred once for a sequence of NRF rounds. As a result, the overall energy required to configure the network decreases proportionally with increasing NRF, provided that the cluster structure remains stable. During the transmission phase, nodes periodically send sensed data to their cluster head, which aggregates and forwards the information to the base station. This phase is identical in both LEACH and NRF-LEACH, with the difference that NRF-LEACH benefits from reduced control-message overhead. However, preserving the same cluster structure for several rounds may also introduce imbalances when residual energy levels diverge among nodes or when the distribution of nodes becomes unfavorable after several rounds. Because of this, it is important to choose the NRF parameter carefully. If it is too small, the benefit of avoiding setup phases is limited, and if it is too large, outdated cluster configurations may cause performance to deteriorate.

The commonly used energy-related equations to model NRF-LEACH capture the cost of transmitting and receiving advertisement messages, membership messages, and aggregated data, which includes the contribution of the electronic circuitry, free-space or multipath propagation, packet size, and node-to-cluster-head distances, which allows the amount of energy saved by avoiding repeated configuration phases to be quantified and highlights the direct influence of the NRF value on the protocol efficiency, which is particularly important for small packet sizes or networks with moderate density, where transmission costs represent a larger proportion of the total energy budget. In general, NRF-LEACH provides a straightforward mechanism to minimize the long-term cost of cluster formation. However, its performance is highly dependent on the selection of the NRF parameter and how it affects the energy consumption, stability, and data delivery, which motivates the need to develop a method to determine the optimal NRF value for different network conditions. This problem is addressed in the following sections via multi-objective optimization and statistical modeling.

4. Explicit Contributions

While several variants of LEACH have been proposed to enhance energy efficiency and prolong the lifetime of WSNs, existing studies do not offer a principled approach to determine the optimal number of fixed rounds in NRF-LEACH. This study fills the gap by proposing a hybrid optimization and modeling framework. The study's main contributions are summarized below:

1. An optimized NRF parameter through a multi-objective systematic method. This research employs the NSGA-II algorithm that helps to determine the best NRF values by considering the large number of network configurations along with three performance indicators: the number of rounds, the cumulative number of dead nodes, and the total number of bits sent to the base station. To this date, there has been no other study that applies the method of multi-objective optimization for the setting of the NRF parameter.
2. A simulation dataset of enormous size that simulates a variety of network situations. The NS-3 simulation yielded a total of over 480,000 simulations through the manipulation of packet sizes, node densities, and NRF values, which in turn not only represented a thorough understanding of the behavior of NRF-LEACH over a wide range of conditions but also profoundly solidified the basis for making generalizable conclusions.

3. A statistical model-based method that leads to an analytical formula for the NRF value; the optimum NRF values are derived from the NSGA-II and employed for the model adaptation, while multiple fitting strategies are explored.
4. Research comparing approach that reveals the importance of the proposed model. The comparison quantitatively evaluates logarithmic, polynomial, and power-law regression models using mean squared error, correlation coefficient, and coefficient of determination measures, and finds that the power-law model is the best for the NRF, packet size, and sensor density connection, so the derived analytical formula allows direct NRF value estimation.
5. The configuration has been identified in the results to be the one that maximizes the advantages of the NRF-LEACH, especially in the case of small and medium packet sizes. This analytical model gives practitioners an easy way to choose an appropriate NRF value to enhance network efficiency without having to reconfigure or trial-and-error evaluate.

These contributions offer the first systematic approach to estimating the optimal number of fixed rounds in NRF-LEACH, which integrates multiobjective optimization, extensive simulations, and statistical modeling, thereby making the protocol more practical and laying the groundwork for future enhancements to clustering-based routing protocols.

5. Energy Consumption Model

NRF-LEACH uses the classical radio energy model that assumes that the cost of transmitting and receiving data is different and considers the propagation effects that depend on the distance between the transmitter and the receiver.

During the configuration phase, two types of messages are exchanged: advertising messages sent by cluster heads and membership messages issued by non-cluster-head nodes. The energy required for these operations is summarized by the following equations. During the advertising step, each cluster head broadcasts an advertisement of size *k* bits. If N_{ch} denotes the number of elected cluster heads, the energy spent by all cluster heads to transmit these advertisements is expressed as in [22-24] (eq. 1).

$$E_{adv\_tx} = N_{ch} * E_{elect} * k + N_{ch} * \varepsilon_{fs} * k * d^2 \qquad (1)$$

where E_{elec} represents the electronic circuitry cost per bit, ε_{fs} is the free-space amplification factor, and *d* denotes the average transmission distance. In the cluster-formation step, each non-cluster-head node transmits a membership request to the closest cluster head. The transmission energy for these requests is given as in [22-24] by (eq. 2):

$$E_{adv\_rx} = N_{ch} * (\mathrm{N} - N_{ch}) * E_{elec} * \mathrm{k} \qquad (2)$$

where *N* is the total number of nodes.

In the cluster-formation step, each non-cluster-head node transmits a membership request to the closest cluster head. The transmission energy for these requests is:

$$E_{join\_tx} = (N - N_{ch}) * \left(E_{elect} * \mathrm{k} + \varepsilon_{fs} * k * d^2\right) \ (3)$$

Each cluster head receives, on average, N/N_{ch} membership requests. The energy consumed in receiving these messages is given by (eq. 4) :

$$E_{join\_rx} = N_{ch} * \frac{N}{N_{ch}} * E_{elect} * \mathrm{k} = \mathrm{N} * E_{elect} * \mathrm{k} \quad (4)$$

The total energy required by the setup phase in the standard LEACH protocol is given by (eq. 5)

$$E_{setup}^{LEACH} = E_{adv\_tx} + E_{adv\_rx} + E_{join\_tx} + E_{join\_rx} (5)$$

In NRF-LEACH, the configuration phase is carried out once every *NRF* rounds. For the remaining *(NRF − 1)* rounds, no reconfiguration is performed. The total energy consumed for the configuration phase in NRF-LEACH becomes (eq. 6).

$$E_{setup}^{NRF-LEACH} = \frac{1}{NRF} * E_{setup}^{LEACH} \qquad (6)$$

The corresponding energy saved by avoiding repeated setup phases is given by (eq. 7).

$$E_{saved} = E_{setup}^{LEACH} - E_{setup}^{NRF-LEACH} \qquad (7)$$

These equations highlight the direct influence of the NRF parameter on network performance. Increasing the number of rounds during which the same cluster structure is maintained reduces configuration overhead proportionally.

However, an excessively large NRF value may result in poorly balanced energy consumption among nodes as cluster-head roles remain unchanged for too long. This trade-off explains why the determination of an optimal NRF value is essential to the effective operation of NRF-LEACH.

6. Methodology

The approach we are suggesting employs large-scale simulations, multi-objective optimization, and statistical modeling to tackle the problem and determine the optimal number of fixed rounds in the NRF-LEACH protocol.

The process consists of three major steps: (1) creation of data and formulation of the optimization problem, (2) using the NSGA-II algorithm, and (3) building a mathematical model for the NRF parameter. The various steps of our method are thoroughly explained below and presented in Figure 2.

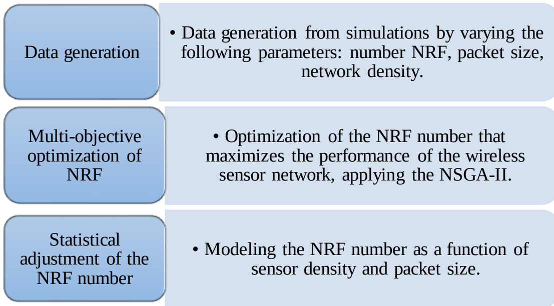

Fig. 2 Flowchart of statistical adjustment methodology

6.1. Experimental Environment

Simulations were performed using the NS-3 network simulator, which offers a detailed and modular platform for simulating hierarchical routing protocols. Three network configurations were considered to reflect varying levels of sensor density: sensor nodes were randomly deployed in square areas of 100×100 m², 200×200 m², and 300×300 m², corresponding to networks of 100, 200, and 300 nodes, respectively; the base station was placed at the center of each deployment area; each sensor node was initialized with an energy of 0.5 J; and radio communication followed the standard energy dissipation model with free-space and multipath propagation parameters. Packet sizes ranged from 8 to 512 bits to evaluate NRF-LEACH with small, medium, and large data payloads; for each packet size and for each node density, the protocol was run with NRF values ranging from 1 to 500, resulting in over 480,000 independent simulations.

6.2. Data Generation

Three performance indicators were recorded for each simulation:

- The number of rounds completed before the network became non-operational,
- The total number of dead nodes at the end of the simulation,
- The total amount of bits sent to the base station.

These performance indicators were selected because they reflect different dimensions of network performance: lifetime, power used by the nodes, and net throughput. The resulting dataset is a triad of performance figures for every tested NRF value, which constitutes the training basis for determining the NRF value that ensures the optimal compromise between energy efficiency and data delivery performance. The conducted tests are listed in the table below.

Table 1. Simulations carried out

Density	Packet Size	NRF
100 Nodes / 10000 m^2	16-bit, 24-bit, 32-bit....512-bit	From 1 to 500
200 Nodes / 40000 m^2	16-bit, 24-bit, 32-bit....512-bit	From 1 to 500
300 Nodes / 90000 m^2	16-bit, 24-bit, 32-bit....512-bit	From 1 to 500

6.3. Problem Formulation

Finding the optimal NRF was treated as a multi-objective optimization problem, with the optimal NRF taken as an objective-available solution.

- Maximize the number of rounds (network lifetime),
- Minimize the number of dead nodes,
- Maximize the number of bits received at the base station.

These are conflicting objectives. For example, maximizing network lifetime may lead to cluster heads depleting their energy sooner, increasing the number of dead nodes, while minimizing node mortality may reduce throughput or shorten the operational duration. Since the problem is multi-objective, it seeks a set of trade-off solutions, and hence Pareto-based optimization is more appropriate.

6.4. NSGA-II Optimization Setup

The NSGA-II algorithm was chosen due to its ability to handle conflicting objectives, its ability to preserve solution diversity, and its ability to approximate the Pareto front with less computational effort. The collected simulation data is processed by the NSGA-II algorithm by treating each NRF value as a candidate solution. The NSGA-II parameters utilized in this study are as follows:

- Population size: 200
- Number of generations: 300
- Crossover probability: 0.9
- Mutation probability: 0.1
- Selection method: binary tournament
- Crossover operator: simulated binary crossover
- Mutation operator: polynomial mutation
- Ranking method: non-dominated sorting
- Diversity preservation: crowding distance metric

The algorithm generates a Pareto front for each packet size and each density configuration, which consists of a set of non-dominated NRF values that are used to select the most appropriate NRF for each scenario.

A single proposed NRF value is produced by the integration of the three performance indicators in a weighted way. The weights determined through experiments ($\alpha = 0.2$, $\beta = 0.6$, and $\gamma = 0.2$) reflect the importance of minimizing node death while keeping the throughput and life expectancy of the device at acceptable levels.

6.5. Statistical Modeling of the NRF Parameter

Using the NRF values found by NSGA-II for each simulation scenario, the model fits the time series simultaneously. The dependent variable is the optimized NRF value, and the independent variables are packet size and node density. The following three regression models were assessed:

- Logarithmic fitting (eq. 11),
- Polynomial fitting of third order (eq. 12),
- Power-law fitting (eq. 13)

Mean squared error (eq. 9), correlation coefficient (eq. 8), and coefficient of determination (R^2) (eq. 10) were used to assess each model, with the power-law model consistently achieving the best performance in all cases, with a correlation coefficient near 0.94 and R^2 approaching 0.89. The final analytical expression can be used to derive a predictive formula for estimating the optimal NRF value based on network parameters directly.

$$r = \frac{\sum_{i=1}^{n}\sqrt{(X_i-\overline{X})(Y_i-\overline{Y})}}{\sqrt{\sum_{i=1}^{n}(X_i-\overline{X})^2\sum_{i=1}^{n}(Y_i-\overline{Y})^2}} \tag{8}$$

$\overline{X}$ and $\overline{Y}$ are the means of the variables X and Y.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{9}$$

$$R^2 = \frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}(y_i-\overline{y})^2} \tag{10}$$

y_i is the real value, $\hat{y}_i$ is the predicted or adjusted value and $\overline{y}$ are the means of the variables y.

$$y = a + b\log(x) + c\log(z) \tag{11}$$

$$y = a_0 + a_1x + a_2z + a_3x^2 + a_4x^1z^1 + a_5z^2 + a_6x^3 + a_7x^2z^1 + a_8x^1z^2 + a_9z^3 \tag{12}$$

$$y = a\,x^b\,z^c \tag{13}$$

By turning the optimization process's output into a useful tool, this modeling stage allows users to configure NRF-LEACH without the need for lengthy simulations.

6.6. Workflow Summary

The entire methodological process can be outlined in the following steps:

1. Data generation through simulation of NRF-LEACH with various settings of NRF, packet size, and node density,
2. Metrics of performance will be computed (rounds, dead nodes, delivered bits),
3. NSGA-II is used to get Pareto-optimal NRF values,
4. Weights are applied to the aggregation process to determine one representative NRF for each configuration,
5. Statistical models are created with packet size and density as factors,
6. The best regression model is chosen, and a final analytical expression is obtained.
7. Validation of the model through independent simulation data.

The method adopting this strategy has the merit of being both strong and applicable in different scenarios and grants NRF parameter determination in NRF-LEACH through a systematic and reproducible approach.

7. Results and Discussion

The large-scale simulation outcomes, along with the multi-objective optimization process that followed, are discussed in this section under three major components: the raw simulation data, the behavior of the NSGA-II optimization, and the statistical modeling of the NRF parameter, stressing the effects of packet size and sensor density on the optimal number of fixed rounds.

7.1. Analysis of Simulation Results

In Figures 3, 4, and 5, respectively, the performance of NRF-LEACH in the three distinct deployment scenarios is illustrated, where the networks consist of 100, 200, and 300 nodes, the total number of rounds, the base station's packets received, and the cumulative number of dead nodes were all registered for the range of NRF values from 1 to 500, in terms of different packet sizes.

7.1.1. Scenario 1: 100 Nodes in 100×100 m²

The results for packet sizes from 8 to 512 bits are reported in Figure 3. For small packet sizes (8, 16, and 32 bits), the number of rounds (sub-figures a1, a2, and a3) and the number of packets received (sub-figures b1, b2, and b3) improve significantly as NRF increases from 1 to about 60. After that, the gains are small, indicating that after NRF ≈ 60, the number of repeated configuration phases is no longer a significant contributor to energy consumption. As can be seen in sub-figures a4 and a5, the number of rounds improves, but more modestly for larger packet sizes (64 and 128 bits). As can be seen in sub-figures b4 and b5, throughput starts to decline for NRF values greater than 60 since data transmission causes some cluster heads to deplete their energy prematurely.

7.1.2. Scenario 2: 200 Nodes in 200×200 m²

As shown in Figure 4, these general trends are still valid with an increased network density. The area where NRF brings the most significant gains is still centered between 2 and 60, with diminishing returns beyond this range, but the magnitude of these improvements is slightly reduced, especially for the number of rounds, due to higher data traffic and more frequent intra-cluster interactions that cause the energy depletion to be faster in denser environments.

7.1.3. Scenario 3: 300 Nodes in 300×300 m²

Figure 5 confirms that the impact of NRF weakens as network density increases. Although NRF values between 2 and 60 continue to offer noticeable gains in certain configurations, the benefits become less pronounced and more dependent on packet size. Larger networks tend to experience higher communication overhead, which mitigates the savings obtained by avoiding repeated setup phases.

7.2. Interpretation of Key Trends

Across all scenarios, the most consistent observation is that NRF-LEACH performs best when NRF lies within the interval [2]. In this region, avoiding the overhead of repeated setup phases leads to measurable energy savings. For larger NRF values, however:

- Residual energy becomes unevenly distributed,
- Cluster heads remain fixed for too long,
- The transmission cost becomes dominant (especially for larger packets),
- Some nodes deplete their energy before the cluster structure is refreshed.

This behavior highlights the importance of selecting an NRF value that balances configuration overhead and cluster-head stability.

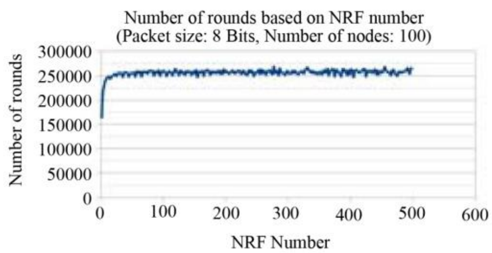

Fig al: Number of rounds based on NRF number

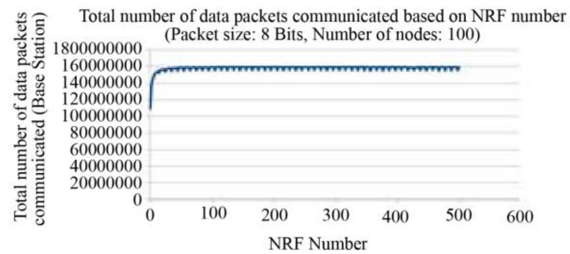

Fig bl: Number of rounds based on NRF number

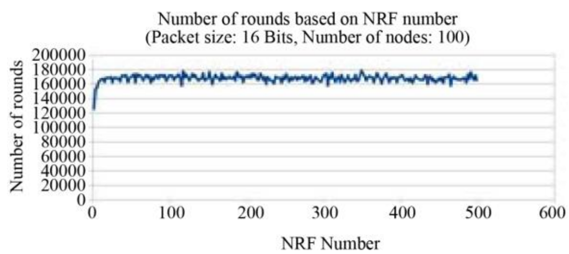

Fig a2: Number of rounds based on NRF number

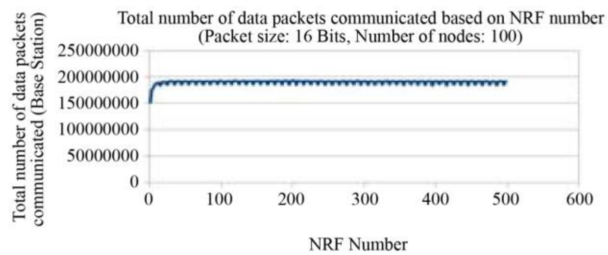

Fig b2: Number of rounds based on NRF number

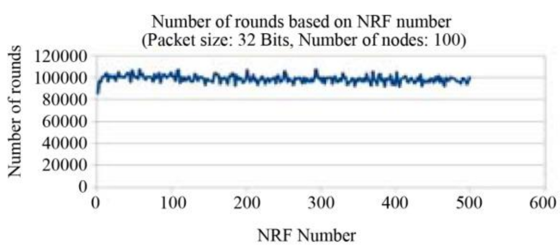

Fig a3: Number of rounds based on NRF number

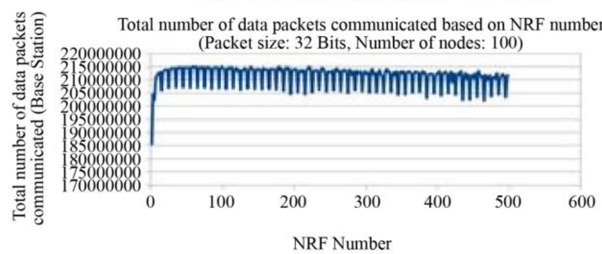

Fig b3: Number of rounds based on NRF number

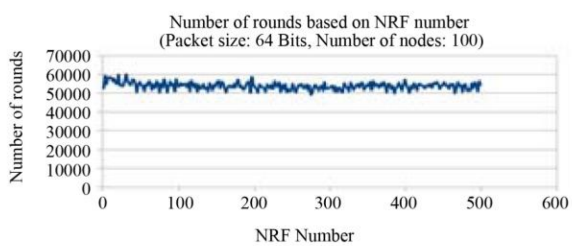

Fig a4: Number of rounds based on NRF number

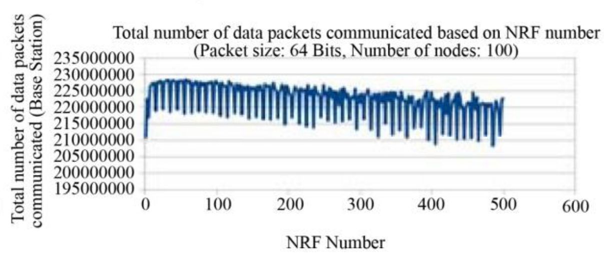

Fig b4: Number of rounds based on NRF number

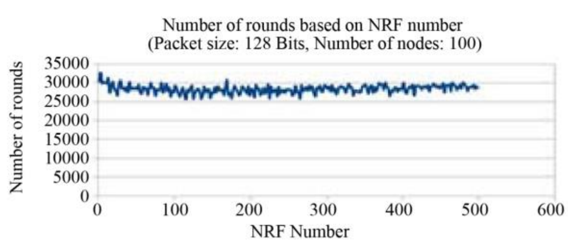

Fig a5: Number of rounds based on NRF number

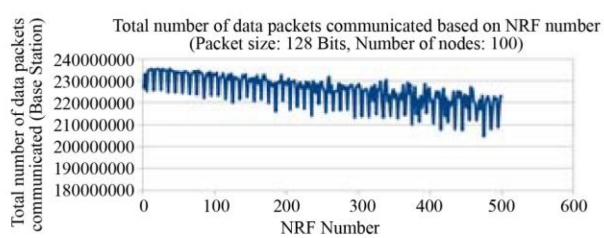

Fig b5: Number of rounds based on NRF number

Fig. 3 Scenario 1 (100 Nœuds 100 * 100 m^2)

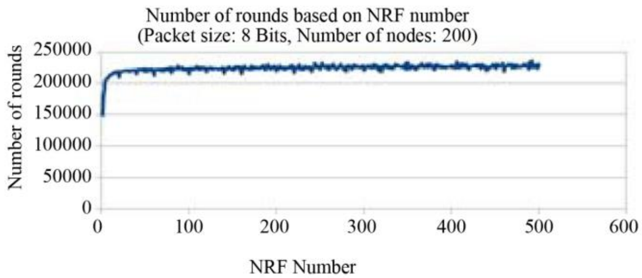

Fig al: Number of rounds based on NRF number

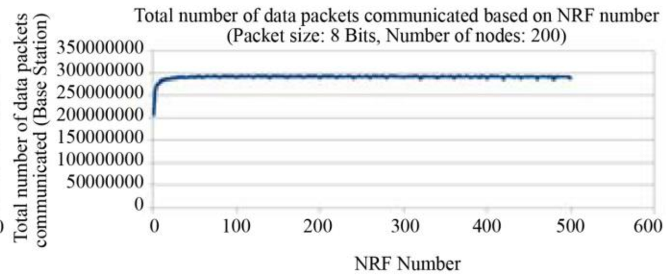

Fig bl: Number of rounds based on NRF number

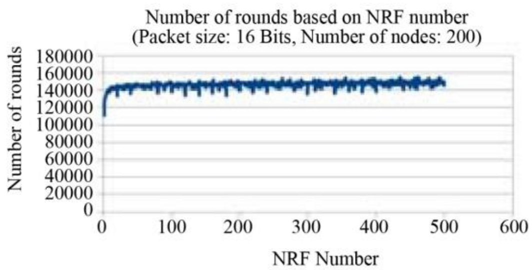

Fig a2: Number of rounds based on NRF number

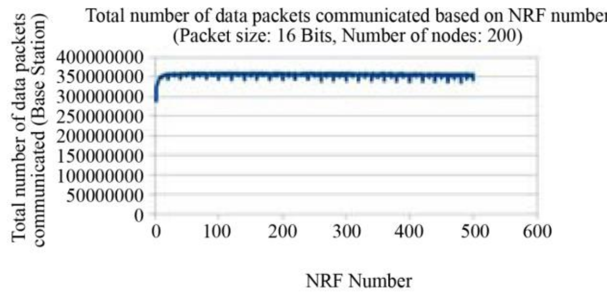

Fig b2: Number of rounds based on NRF number

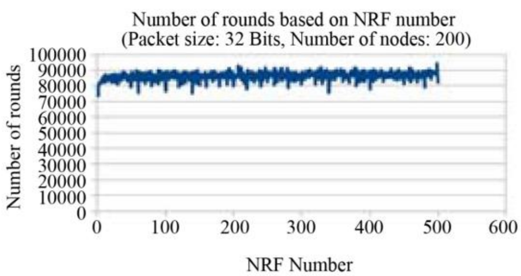

Fig a3: Number of rounds based on NRF number

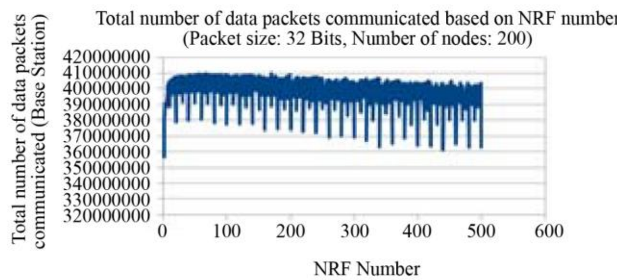

Fig b3: Number of rounds based on NRF number

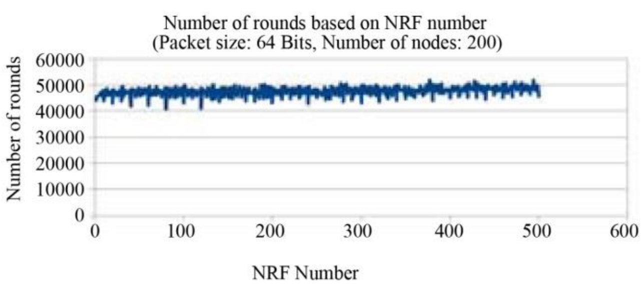

Fig a4: Number of rounds based on NRF number

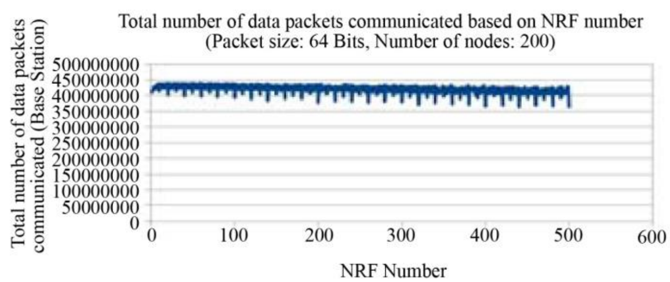

Fig b4: Number of rounds based on NRF number

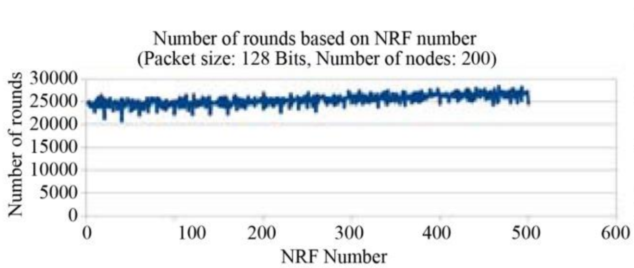

Fig a5: Number of rounds based on NRF number

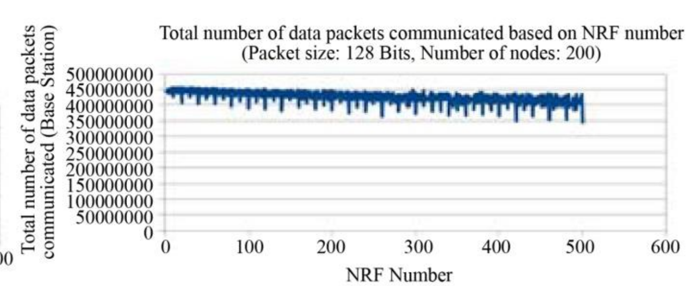

Fig b5: Number of rounds based on NRF number

Fig. 4 Scenario 2 (200 Nœuds 200 * 200 m^2)

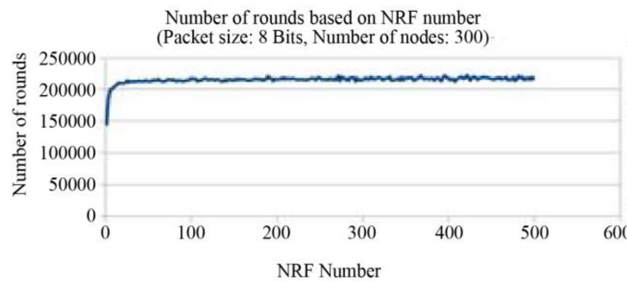

Fig al: Number of rounds based on NRF number

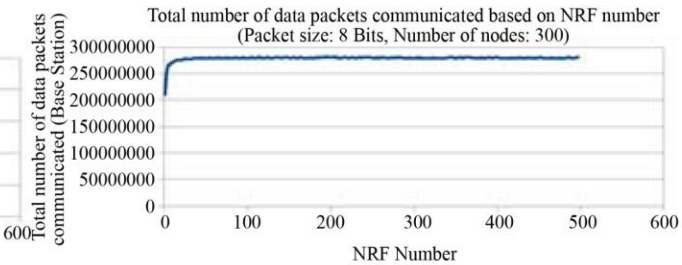

Fig bl: Number of rounds based on NRF number

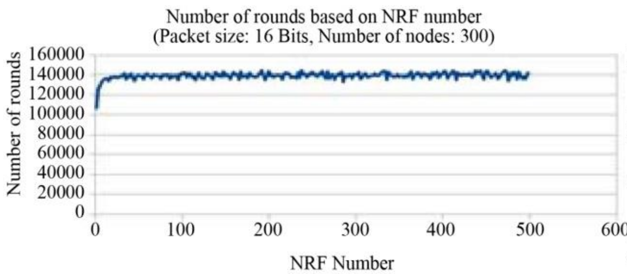

Fig a2: Number of rounds based on NRF number

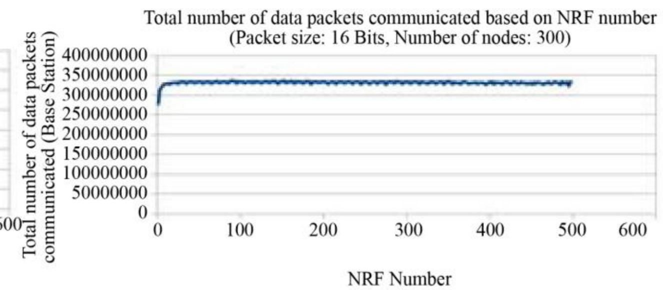

Fig b2: Number of rounds based on NRF number

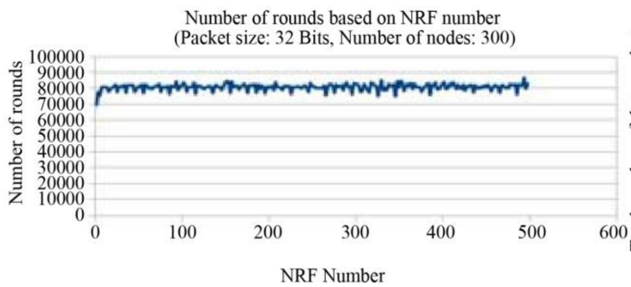

Fig a3: Number of rounds based on NRF number

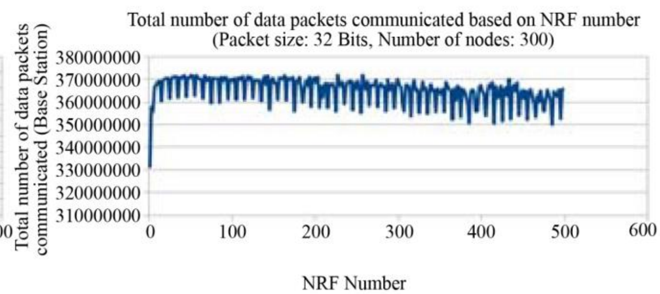

Fig b3: Number of rounds based on NRF number

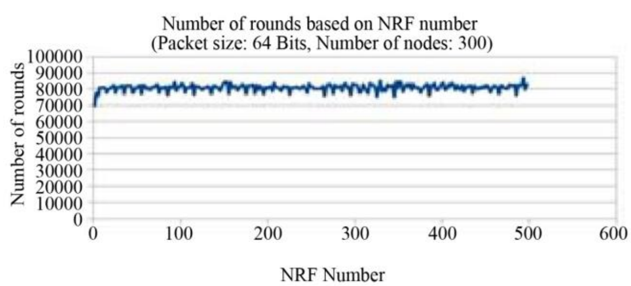

Fig a4: Number of rounds based on NRF number

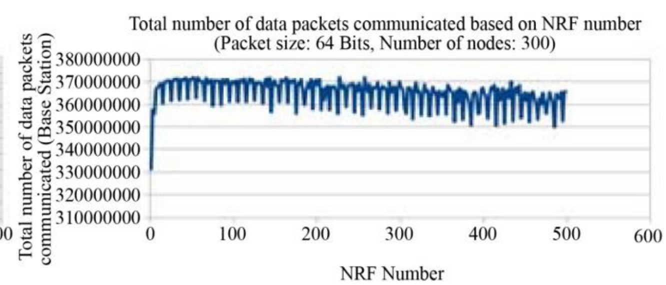

Fig b4: Number of rounds based on NRF number

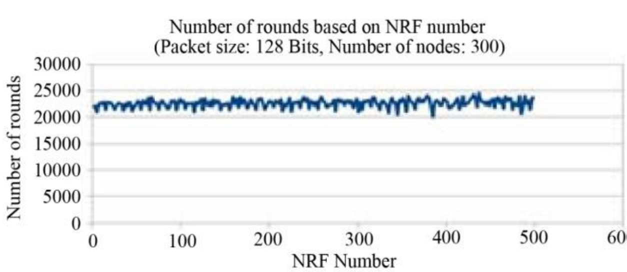

Fig a5: Number of rounds based on NRF number

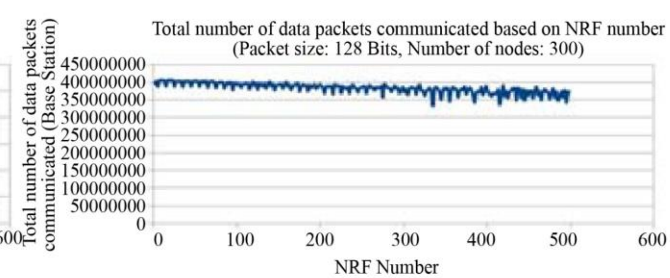

Fig b5: Number of rounds based on NRF number

Fig. 5 Scenario 3 (300 Nœuds 300 * 300 m^2)

7.3. NSGA-II Optimization Results

NSGA-II was applied to the simulation dataset to identify Pareto-optimal solutions for each combination of packet size and sensor density. Trade-offs between the three performance metrics are evident in the resulting Pareto fronts.

A Pareto front for a typical set of 7 configuration runs shows that maximizing the number of rounds often comes at the cost of maximizing throughput, and that minimizing the number of dead nodes adds to the trade-off. The diversity of solutions and the smooth distribution along the front confirm that NSGA-II applies here.

The weighted selection method (α=0.2, β=0.6, γ=0.2) that gives more weight to solutions with lower node mortality, but still produces reasonable operational duration and throughput, yields a single representative NRF value for each configuration and provides the statistical modeling that underpins the results.

7.4. Comparative Summary of Optimized NRF Values

Table 2 summarizes representative results taken from the NSGA-II outputs to show how packet size and density affect the optimal NRF.

Table 2. Representative optimized NRF values

Packet Size (bits)	Density: 100 nodes	Density: 200 nodes	Density: 300 nodes
8	50–60	45–55	40–50
32	35–50	30–45	25–40
128	10–30	5–25	5–20
512	<10	<10	<10

These results clearly show that:

- Larger NRF values (better stability) are possible with smaller packets,
- Larger packets require smaller NRF values (to preserve energy balance),
- Higher network density lowers the optimal NRF range.

7.5. Statistical Fitting and Model Performance

Three regression models were tested: logarithmic, third-order polynomial, and power-law regression. Table 3 summarizes their performance metrics.The power-law model provides the best agreement with the optimized *NRF* values. Its high correlation and R^2 values indicate that packet size and density have a nonlinear influence that is well captured by this model. This model, therefore, serves as the final analytical expression for estimating the *NRF* value.

Table 3. Regression performance metrics

Model Type	MSE	Correlation	R^2	Estimated parameters
Logarithmic regression	903.97	0.92	0.72	a= 21770.49 , b= -0.52, c=-0.71
Polynomial regression (order 3)	826.47	0.86	0.74	[341.17, -3.35, -2.45 e-5, 9.54 e-3, 5.86 e-3, -4.50 e-3, -8.36 e-6, -8.87 e-6, 3.57 e-7, 6.75 e-6]
Power-law	351.37	0.94	0.89	a = 89082.97, b = -0.71, c=-0.80

7.6. Structural Fitting

Table 4 summarizes the simulation parameters used throughout the study. These parameters were applied consistently across all network densities and packet-size configurations in order to generate a homogeneous dataset suitable for comparative analysis. The values of ε_{fs}, ε_{amp}, and E_{elec} follow the standard radio model typically used in LEACH-based studies, while the probability of selecting cluster heads was fixed at *0.05.*

Packet sizes ranged from *8* to *512 bits*, and the network density was varied over *100, 200,* and *300* nodes. The three-dimensional adjustment surfaces from the logarithmic, polynomial, and power-law regression models are shown in Figures 6, 7, and 8, respectively, with the scatter points representing the outputs from the NSGA-II optimization to give a qualitative assessment of how well the regression models capture the distribution of optimized NRF values as a function of packet size and sensor density.

Table 4. Simulation Parameters

Parameters	Values
Size of network	100m/100m, 200/200, 300/300
Base Station (BS)	(50m,50m), (100m,100m), (150m,150 m),
ε_{efs}	8 pj/bit/m2
ε_{amp}	0.0013pj/bit/m4
E_{elec}	50 nj/bit
d_0	87,7 m
Initial Energy	0.5 j
Cluster Head probability	0.05
Total nodes	100, 200, 300
SCHEDULE_MESSAGE	16 bits
MESSAGE_LENGTH	8, 16, 24, 32, 40, 48 ……..512bits

As we can see in Figure 6, the logarithmic model approximates the overall shape of the data, but it does not capture the variation in the data for larger packet sizes, where the surface flattens out. The polynomial model in Figure 7, being a higher-order structure, has more flexibility, but shows localized oscillations that do not follow the actual distribution of the data, particularly in regions where packet size increases most rapidly.

On the other hand, as Figure 8 shows, the power-law model generates a continuous surface that tracks the cloud of experimental points in all three cases. The quantitative results in Table 3 are consistent with the qualitative observation that the power-law regression has the lowest mean squared error (351.37), the highest correlation coefficient (0.94), and the highest coefficient of determination (0.89), which collectively demonstrate that the power-law function can more accurately model the nonlinear behavior of the NRF parameter and its sensitivity to packet size and density than the logarithmic and polynomial models.

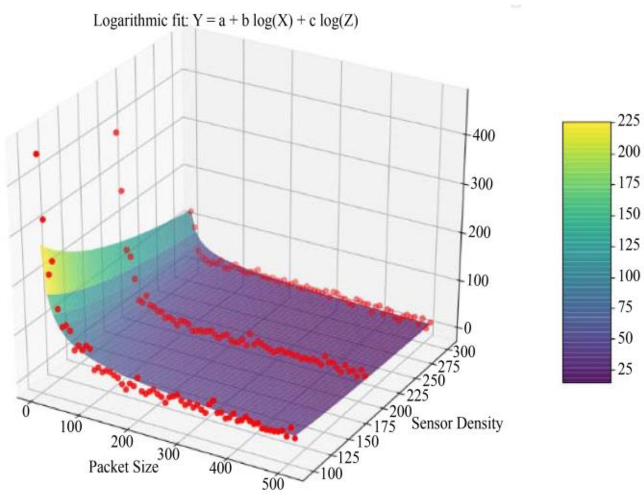

Fig. 6 Statistical adjustment of the NRF number (Logarithmic fit)

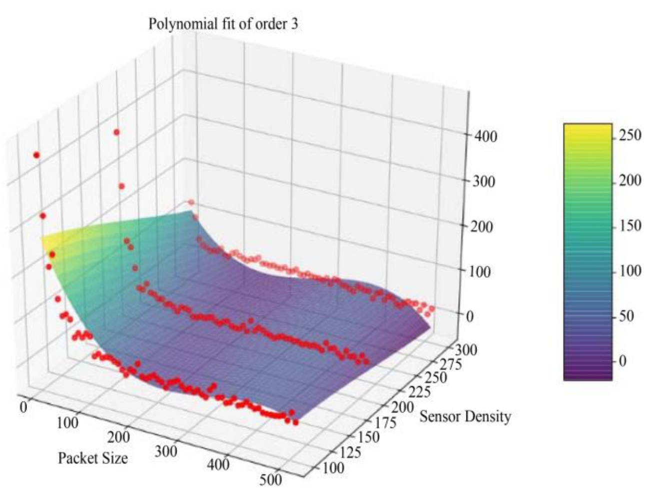

Fig. 7 Statistical adjustment of the NRF number (Polynomial fit of order 3)

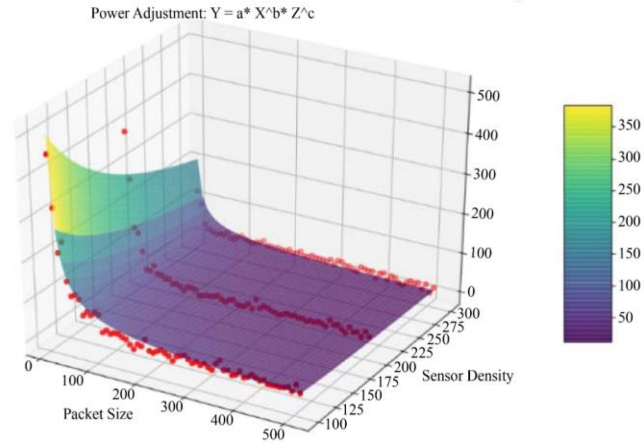

Fig. 8 Statistical adjustment of the NRF number (Power Adjustment)

Taken together, the numerical indicators and visual inspection lead to the conclusion that the power-law model provides the most reliable analytical representation of the relationship between the NRF parameter, packet size, and sensor density. This model is therefore selected as the final predictive equation for estimating the optimal NRF value in the NRF-LEACH protocol.

7.7. Discussion

The results confirm that NRF-LEACH can significantly extend network lifetime when the NRF parameter is chosen appropriately. The study shows that:

- The NRF parameter is very sensitive to packet size,
- Values in the range of 2 to 60 serve up the most consistent and universal advantages.
- Too large an NRF value causes an energy imbalance among nodes,
- The correlation between NRF and network parameters is of a nonlinear nature.

The new analytical model that has been derived here is a very effective tool for configuring NRF-LEACH without the need for extensive simulations. Besides, it gives the network operators the capability to calculate the NRF value from the size of packets and the density of sensors, which will help them to be more effective in the deployment of hierarchical routing strategies.

8. Ethical Considerations

The simulation data generated using the NS-3 network simulator is the only data on which this study is based. There were no real-world measurements, no personal data, and no sensitive information that was collected or processed. The work did not involve human or animal subjects or identifiable personal information, so ethical approval and informed consent are not required, and all the procedures are in accordance with the standard research practices in computer science and wireless networking.

9. Conclusion

The research considered the effect on the performance of the NRF-LEACH protocol caused by the parameter NRF and put forward a methodical approach for finding the best value of the NRF for various network setups. This was done through the combination of large-scale simulations, multi-objective optimization, and statistical modeling methods, which not only give a detailed characterization of the protocol behavior but also a practical analytical model. The results indicate that the NRF values that provide the greatest benefits for the NRF-LEACH depend heavily on selecting an NRF value that provides a trade-off between configuration overhead and cluster-head stability, and that values in the lower and medium range offer consistent gains, with excessively large values resulting in unbalanced energy consumption and premature node failures.

The NSGA-II algorithm was used to determine Pareto-optimal NRF values that represent reasonable trade-offs between network lifetime, data delivery, and node survivability. Based on the optimized NRF values, a statistical model was established to capture the nonlinear relationship between NRF, packet size, and sensor density, where the power-law model showed the highest correlation and determination coefficients among the tested approaches, and its analytical formulation can be used as a simple and efficient method to estimate the optimal NRF for new deployments without any further simulations. Despite its strengths, the study has certain limitations. The simulations assume homogeneous sensor nodes with identical energy resources, whereas real deployments may include heterogeneous devices with different power profiles. The analysis also considers static networks and does not account for node mobility, environmental interference, or dynamic variations in packet generation rates. Furthermore, the optimization and modeling processes rely on the assumptions of the NS-3 energy model, which may differ from the characteristics of specific hardware platforms. There are multiple avenues for future work that can be explored to extend further the current approach: extending the method to heterogeneous or mobile sensor networks, incorporating additional parameters such as link quality, interference patterns, or variable data-generation rates, incorporating machine learning techniques to predict NRF values under uncertain or time-varying conditions, and applying the methodology to other clustering-based protocols to derive generalized models for energy-efficient routing in wireless sensor networks. Overall, the study contributes a structured framework for analyzing and configuring NRF-LEACH, offering both theoretical insights and practical guidelines for improving energy efficiency and extending the lifetime of wireless sensor networks.

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