Damping Force Effect with Respect to Time and Displacement of Two Identical Spheres using Discrete Element Method

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Abstract
The objective of this contribution is to present a numerical simulation method to model the collision of particles in a plane using object-oriented techniques. The method chosen is the Lagrangian time-driven method and it uses the position, and the velocity of particles as independent variables. These are obtained by time integration of the two-dimensional dynamics equations which were derived from the classical Newtonian mechanics approach based on the second law of Newton for the translational motion of each particle in the granular material. This includes keeping track of all forces acting on each particle at every time-step. Contact forces depend on the overlap geometry, material properties and dynamics of particles and include normal components of repulsion force. The back-ground version of DEM and time integration algorithm are developed and implemented into C++ code. The implementation of time-integration algorithm is verified by simple test concerning particle-particle interaction for which analytical expression exist. In this paper Damping force due to particle – particle contact at different time step and displacement are investigated.

Keywords — *DEM simulation, Granular materials, Elastic effect*

I. INTRODUCTION

A granular material is a conglomeration of many discrete solid particles which is characterized by a loss of energy due to dissipative collisions whenever the particles interact. They can be considered as the fourth state of matter which is very different from solids, liquids or gas. Granular materials are very simple. If they are non-cohesive, then the forces between them are essentially only repulsive so that the shape of the material is determined by external boundaries and gravity. Practically many solid particles which we make use of in the kitchen are granular particles like sugar, rice, coffee, cereals etc. Walking outside we step on the soil which is again a particulate and hence falls under the category of granular matter. The unusual and unique character exhibited by granular material systems have led to a resurgence of interest within several scientific and engineering disciplines ranging from physics, soil mechanics and chemical engineering (Jaeger and Nagel [1992]; Behringer [1993, 1995]; Bideau and Hansen [1993]; Jaeger et al. [1994, 1996a, 1996b]; Mehta [1994]; Hayakawa et al. [1995]). The science of granular media has a long history. Much of the engineering literature has been devoted to understanding how to deal with these materials. Notable contributions in the literature include Coulomb [1773], who proposed the ideas of static friction; Faraday [1831], who discovered the convective instability in a vibrated container filled with powder, and Reynolds [1885], who introduced the motion of dilatancy, which implies that a compacted granular material must expand in order for it to undergo shear. Processes involving particulate or granular flows are prevalent throughout the pharmaceutical, chemical, energy, food handling, mineral processing, powder metallurgy, and mining industries. In addition, numerous phenomena found in nature involve such material flows.

The discrete element method (DEM), originally developed by Cundall and Strack [1971,1979], has been used successfully to simulate chute flow (Dippel *et.al* 1996), heap formation (Luding, 1997), hopper discharge (Thompson and Grest, 1991; Ristow and Herrmann, 1994), blender segregation (Wightman et al., 1998; Shinbrot et al., 1999; Moakher et al., 2000) and flows in rotating drums (Ristow, 1996; Wightman et al., 1998). The DEM allows for the simulation of particle motion and interaction between the particles, considering not only the obvious geometric and material effects such as particle shape, material non-linearity, viscosity, friction, etc., but also the effect of various physical fields of surrounding media, even of chemical reactions (Kantor *et al.* 2000). One of the most promising area of future applications of discrete element method seems to be geotechnical engineering. The discrete approach assumes the soil is an assembly of granules or discrete particles where micromechanical behavior of soil is pre-defined by micromechanical inter granular properties.

II. DISCRETE STATE FORMULATION

The granular media present a space filled by the particle termed here as discrete elements. The media are assumed to be composed of spherical particles with same radii R_i. The particles are assumed to be deformable bodies, deforming each other by normal and shear force. The composition of media is time-dependent because distinct particle change their position by free rigid body motion or by contacting with neighbor particles or walls. Each particle may be in contact with other particles. The boundary conditions of media are determined by planes and treated as particles with an infinite radius and mass.

The dynamic behaviour of media is considered as the dynamics of each particle. Consequently, the overall response of media is predicted by the behaviour of individual particles, the dynamics of which is evaluated by applying the second Newton's law. One of the most important issues considered by a discrete approach is the detection of interaction force between contacting particles. The interaction forces of each contacting particle are locally resolved on the basis of actual geometry of kinematic contact between two spherical particles, inter-particle contact forces and boundary conditions.

III. GEOMETRY OF KINEMATIC CONTACT OF SPHERICAL PARTICLES

Let any two particles i and j be in contact with position vectors x_i and x_j with centre of gravity lying at O_i and O_j having linear velocities v_i and v_j respectively in figure 1.

The contact point c_{ij} is defined to be at the centre of the overlap area position vector x_{cij} .The vector x_{ij} of the relative position point from the centre to gravity of particle i to that of particle j is defined as $x_{ij} = x_i - x_j$. The depth of overlap is h_{ij} . Unit vector in the normal direction of the contact surface through the centre of the overlap area is denoted by n_{ij} . It extends from the contact point to the inside of the particle i as $n_{ij} = -n_{ji}$

Since the particle shape is assumed to be spherical, for sphere of any dimension the contact parameters can be written as follows:

$$h_{ij} = \begin{cases} R_i + R_j - |x_{ij}|, |x_{ij}| < R_i + R_j \\ 0, |x_{ij}| \geq R_i + R_j \end{cases}$$

Where R is the radius of the particle. The relative velocity of the contact point is defined as

$$v_{ij} = v_{cij} - v_{cji}$$

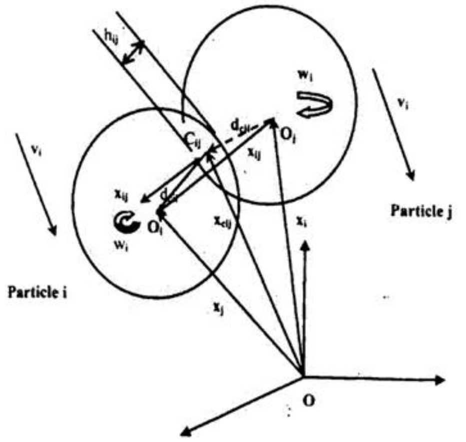

Figure 1.Contact between two particles i and j.

IV. INTER PARTICLE CONTACT FORCE

The contact force can be expressed as the sum of normal and tangential components;

$$F_{ij} = F_{n,ij} + F_{t,ij}$$

Contact force between the spherical particles are modeled as spring, dash-pots and a friction slider. The contact forces between them depend on the overlap geometry, the properties of the material and the relative velocity between the particles in the contact area. Hence in the perfect contact model, it is required to describe the effects of elasticity, energy loss through internal friction and surface friction and attraction on the contact surface for describing the contact force calculations. The normal component of contact force between particles can be expressed as the sum of elastic repulsion, internal friction and the surface attraction forces.

$$F_{n,ij} = F_{n,ij,elastic} + F_{n,ij,viscous}$$

Normal elastic repulsive force is based on the linear Hooke's law of a spring with a spring stiffness constant $k_{n.ij}$ and is given by the expression,

$$F_{n,ij,elastic} = K_{n,ij} h_{ij} n_{ij}$$

Normal energy dissipative force is dissipated during real collisions between particles and, in general, it depends on the history of impact. A very simple and popular model is based on the linear dependency of force on the relative velocity of the particles at the contact point with a constant normal dissipation coefficient γ_n and is expressed as

$$F_{n,ij,viscous} = -\gamma_n m_{ij} v_{n,ij}$$

Governing equation for the motion of granular material in a plane,

$$m_i \frac{d^2 x_i}{dt^2} = m_i a_i$$

$$= F_i$$

$$v_i = \frac{dx_i}{dt}$$

Force acting on i^{th} particle F_i is,

i.e., sum of gravitational force and contact force

$$F_i = m_i g + \sum_{\substack{j=i \\ j \neq i}}^{N} F_{n,ij} + \sum_{\substack{j=i \\ j \neq i}}^{N} F_{t,ij}$$

The contact forces between them depend on the overlap geometry, the material of the particles, and the relative velocity of the particles in the contact area.

V. COMPUTER IMPLEMENTATION

The major computational tasks of DEM at each time step can be summarized as follows*:*

- Detection of contacts between a particle *i* and *j*.
- Computation of contact forces from relative displacement between particles
- Summation of contact forces to determine the total unbalanced force
- Computation of acceleration from force
- Velocity and displacement by integrating the acceleration
- Updating the position of particles

VI. RESULT AND DISCUSSION

A. Elastic-Damping impact of two identical spheres

This simulates the collision of particle with initial velocity and tests the particle-particle contact force. For this test, parameters corresponding to the tangential force are set zero and the effect of normal damping and elastic force is studied. This test also confirms the particle-particle interaction. The diameter of the particle is 0.005 m and the mass of the particle is 10 kg. The value of the normal stiffness parameter is 3×10^5 units.

1. Effect of change of Elastic Damping Force with effect of Displacement

Considers the simplest case of an elastic damping impact of two identical spheres with the same magnitude of velocity. The incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5m.

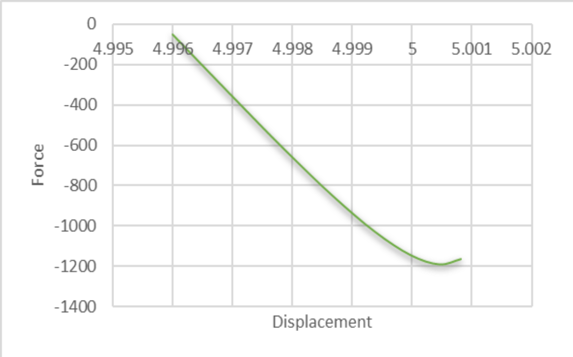

Figure 2. Elastic normal impact of first particle with change in Displacement

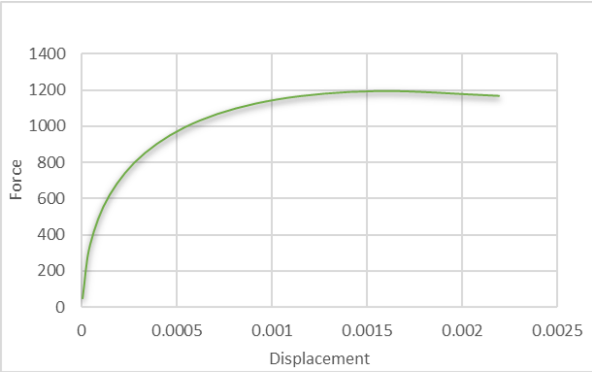

Figure 3. Elastic normal impact of second particle with change in Displacement

The first particle moving with the same velocity and collide the second particle, and the particle rebounds to the original position and the normal damping force reaches a peak during contact as shown in Figures 2. Figure shows the results for a particle with displacement in the x-direction and elastic damping force in the y-directions. The result shows that when displacement increases there is a small increase in damping force compared with elastic force. But the two particles attain the same force but there is change in direction. The velocity magnitude of second particle was set to be zero and the initial position is 10 cm. The first particle is moving with the same velocity and collide the second particle, and the first particle rebounds to the original position. The second particle rebound to the direction of movement of the first particle. The normal damping force reaches a peak during contact as shown in Figures 3. Figure shows the results for a particle with displacement in the x-direction and the elastic damping force in the y-direction.

2. Effect of change of force with respect to time

Considers the simplest case of an elastic damping impact of two identical spheres with the same magnitude of velocity but in opposite directions. The incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5 cm. The particle is moving with the same velocity and collide the second particle. When time step increases then the collision also increases, and the damping force reaches a peak then collision decreases then damping force decreases as shown in Figures 4. Figure shows the results for a particle with change in time in the x-direction and elastic damping force in the y-directions. The result shows that when time increases there is a small increase in damping force compared with elastic force. But the two particles attain the same force but there is change in direction.

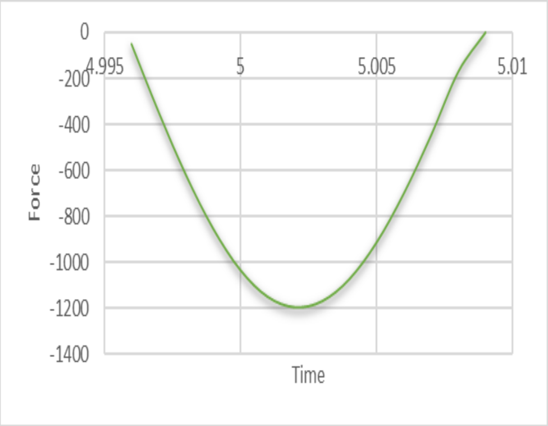

Figure 4. Elastic damping impact of first particle with change in time

The velocity magnitude of second particle was set to be zero and the initial position is 10 m. The first particle is moving with the same velocity and collide the second particle, and the first particle rebounds to the original position and the second particle rebound to the direction of movement of the first particle. The normal damping force reaches a peak during contact as shown in Figures 5. Figure shows the results for a particle with time in the x-direction and elastic damping force in the y-directions.

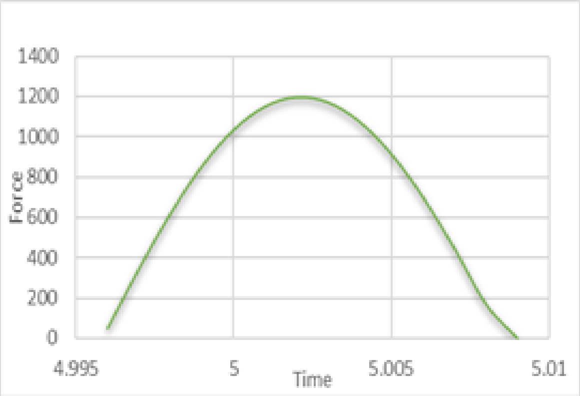

Figure 5. Elastic damping impact of second particle with change in time.

VII. CONCLUSION

The result obtained in the present investigation may be generally described as follows:

1. The described discrete element model composed of visco-elastic spherical particles is implemented into the developed C++ code. This code open for new elements and interaction models may be considered as the first step in the development of an advanced simulation tool for granular and other inhomogeneous materials and is intended for modelling more complex geotechnical problems.
2. The analytical solutions for the impact of two spheres have been examined and derived. In particle-particle collision the particle rebounds to the original position. The normal damping force reaches a peak during contact.
3. The time step increases the damping force reaches a peak during contact and the decrease in time step damping force also decreases.

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