Antiwindup Design for Fuzzy PID Controlled DC Motor under Nonlinearities and Load Variations

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Abstract

A new approach is adopted to calculate antiwindup for speed control of DC motor which is nonlinear parameter varying system. To calculate antiwindup gain a linear matrix inequality based condition by applying lyapunov theory, local sector condition , an upper bound on the nonlinearity and parametric bound is used in that approach to ensure asymptotic and $\mathcal{L}_2$ *stability. Here AWC is designed for fuzzy PID controller which will give better performance in terms of overshoot, rise time and better control of DC motor than conventional PI controller. Fuzzy PID controlled DC motor with antiwindup compensator is modelled and simulated under armature nonlinearity, control input saturation, and load variations in MATLAB and simulation results are provided.*

Keywords — *Antiwindup, nonlinear parameter varying system, linear matrix inequality condition, antiwindup gain, fuzzy PID, load variations.*

I. INTRODUCTION

High performance motor drives plays very important role in industrial and in other applications like rolling mills, traction system and robotics. DC drives are high performance drives and superior to AC drives in terms of complexity and speed torque characteristics. As many applications are using DC drives, speed control of DC motor should be precisely controlled in order to get required performance. The DC motor model used here is nonlinear model and parameter varying system. Nonlinearity DC motor modelling is due to armature reactance and varying parameter considered here is inertia. For controlling DC motor fuzzy PID controller is used and is advantageous than conventional controllers in terms of cost, operating conditions [2], and are superior in rise time and percent overshoot compared to conventional controllers [3].

A closed loop control system consists of controller, actuator, plant and feedback elements. When a large change in set point or reference point occurs, the controller gives large control signal to the actuator. As the actuator is always subjects to limits, saturation of actuator happens which is undesirable consequence. Because of this actuator saturation the integral term in the controller adds up the large error causing large overshoot in the closed loop response and this phenomenon is called windup phenomenon. Because of this overshoot the performance of system may deteriorates and may cause instability also. So we design an antiwindup compensator in order to get back the performance of closed loop system in absence of actuator saturation as much as possible.

The adopted methodology is to calculate antiwindup gain considers variations in parameter of system, simple in computation and reduce design conservatism in the design and has many advantages than existing techniques.[1]

Notations: Notations used throughout this paper are $\bar{u} > 0$ corresponds to saturation limit on the control input *u*. $A_{(i)}$ refers to the i^{th} row of a matrix A. $||x||$ is nucledian norm of a vector x. For a vector x, $||x||_2 = (\int_0^\infty ||x||^2\, dt)^{1/2}$ is standard form $\mathcal{L}_2$ norm and diag(.....) refers the block diagonal matrix whose arguments are diagonal blocks.

II. METHODOLOGY

A. DC Motor Modelling:

Generally DC motor model is considered to be linear without armature reaction or with assumption that effect is removed by compensating windings. So by considering the armature nonlinearity, the system becomes nonlinear system and it is modelled by the following equations [4].

$$\dot{\omega} = -\frac{F}{J}\omega + \frac{a}{J}i + \frac{b}{J}i^2 - \frac{1}{J}T_l$$
$$\frac{di}{dt} = -\frac{a}{L}\omega - \frac{b}{L}\omega i - \frac{R}{L}i + \frac{\lambda}{L}u_{sat} \qquad (1)$$

Where

- u_{sat} is saturated control input
- L is armature inductance
- i is armature current
- a is no load machine constant
- b is a small negative number
- R is armature resistance
- λ is circuitry gain
- J is motor inertia
- ω is rotor rotation speed(rad/sec)
- F is motor viscous friction constant

T_l is torque applied to the rotor by external load and the angular speed of the motor is y(rpm) and is determined by $y = \frac{60}{2\pi}\omega$

B. *Fuzzy PID Controller:*

PID controller is Proportional-Integral-Derivative controller and also called three term controller. PID controller takes the error value as input (error value is the difference of desired set point and measured process variable) and gives the output based on proportional, integral, and derivative terms as a correction to the actuator.

PID controller is mathematically represented as

$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{de(t)}{dt} \qquad (2)$$

where

e(t) is error signal

k_p is proportional gain

k_i is integral gain

k_d is derivative gain

$u(t)$ is control input to actuator

For tuning the values of $k_p, k_i, \& k_d$, we use fuzzy logic. Fuzzy systems are rule base systems which uses If-Then rules. In fuzzy logic input variables will be qualified membership functions and output variables also characterised by membership functions. Rules will be written using If-Then statements which describes the decision to be taken based on the combination of control variables. Implementing fuzzy logic involves three steps.

1) *Fuzzification:*

Fuzzification is converting input crisp data in to fuzzy data or membership functions. To design fuzzy logic controller two variables error and derivative error are taken as inputs and chosen membership functions for input variables are triangular shape. Each universe of discourse is having seven overlapping fuzzy sets, they are NL(negative large), NS(Negative Small), ZE(Zero), PS(Positive Small),PM(Positive Medium) and PL(Positive Large). Each fuzzy variable is one of the member of fuzzy sets and having the degree of membership values from 0 to 1. The membership function of inputs e and de are as shown in fig.1.and fig.2.respectively.

2) *Fuzzy Inference Process*:

Inference process has two concepts , one is rule base and second one is inference engine. Rule base is the set of rules which are in the form of IF-THEN statements and describes the relation between input and output variables in terms of membership function. Inference engine deduce the outputs using the input membership functions and rules. The following tables and figures shows rule base and membership

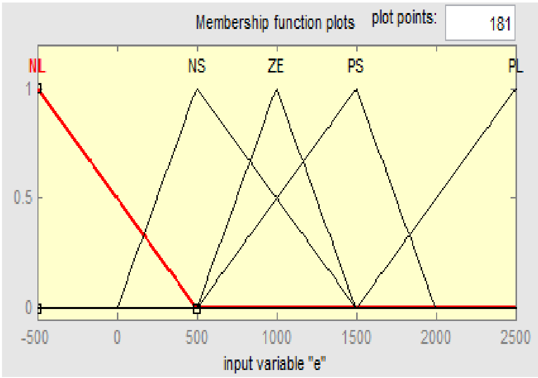

Fig.1. Membership Function for Input Varable "e"

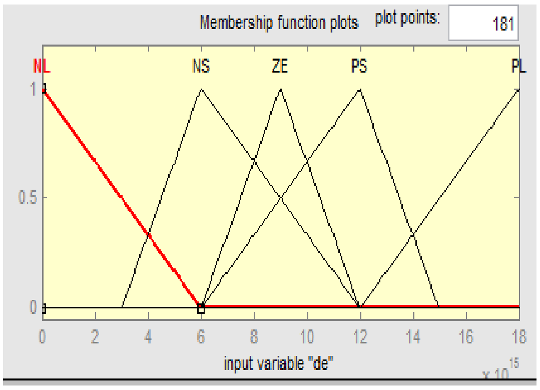

Fig.2. Membership Function for Input Variable "de"

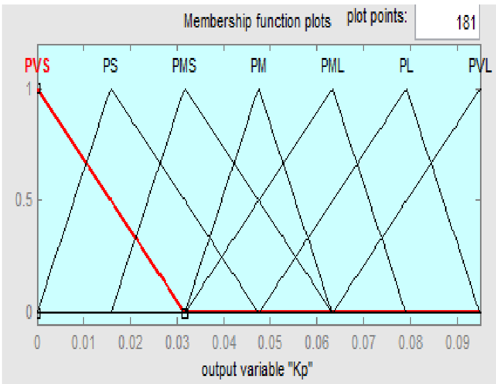

Fig.3. Membership Function for Output Variable k_p

Table I. Rule Base for Parameter k_p

de e	NL	NS	ZE	PS	PL
NL	PVL	PVL	PVL	PVL	PVL
NS	PML	PML	PML	PMS	PMS
ZE	PVS	PVS	PVS	PMS	PMS
PS	PML	PML	PML	PL	PVL
PL	PVL	PVL	PVL	PVL	PVL

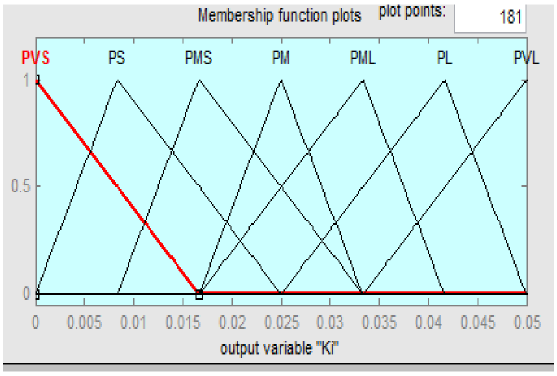

Fig.4. Membership function for Output Variable k_i

Table II. Rule Base for Parameter k_i

de e	NL	NS	ZE	PS	PL
NL	PM	PM	PM	PM	PM
NS	PMS	PMS	PMS	PMS	PMS
ZE	PS	PS	PVS	PS	PS
PS	PMS	PMS	PMS	PMS	PMS
PL	PM	PM	PM	PM	PM

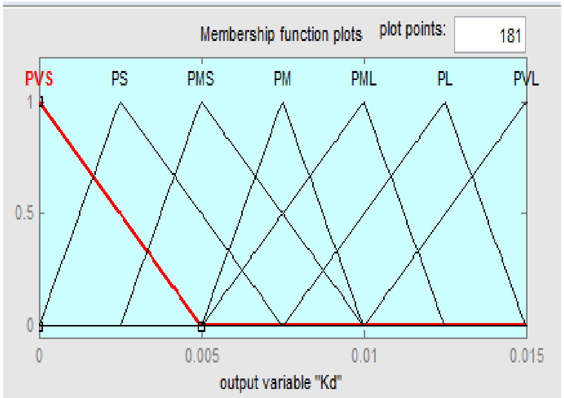

Fig.5. Membership Function for Output Variable k_d

Table III. Rule Base for Parameter k_d

de e	NL	NS	ZE	PS	PL
NL	PVS	PMS	PM	PL	PVL
NS	PMS	PML	PL	PVL	PVL
ZE	PM	PL	PL	PVL	PVL
PS	PML	PVL	PVL	PVL	PVL
PL	PVL	PVL	PVL	PVL	PVL

3) *Defuzzification*:

Defuzzification is process of converting the fuzzy values got from inference process to crisp values. We have different defuzzification methods includes centre of gravity, centre of area, mean of maxima, centroid method etc. Here in this paper the used defuzzfication method in centroid method.

The following fig.6. represents the block diagram of fuzzy PID controller.

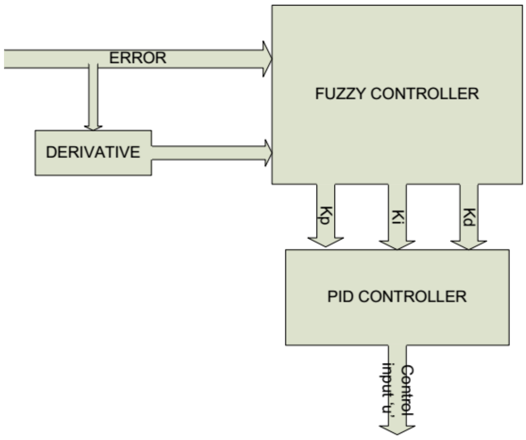

Fig. 6. Block Diagram of Fuzzy PID Controller

C. Antiwindup Compensator :

The adopted methodology is illustrated as follows.

A linear time varying system in state space representation is written as follows.

$$\dot{x} = A(\vartheta(t))x + B(\vartheta(t))u_{sat} + B_w(\vartheta(t))w + \varphi(t, x, \vartheta(t))$$

$$z = C_z(\vartheta(t))x + D_z(\vartheta(t))w$$

$$y_o = C_y(\vartheta(t))x + D_y(\vartheta(t))w \qquad (3)$$

Here

x is state vector

u_{sat} is standard saturated input to the plant

w is external signal which may include reference signal, disturbances and noise

z is exogenous output

$\varphi(t, x, \vartheta(t))$ is nonlinearity and is function time varying parameter $\vartheta(t)$

$y_o \in x^q$ is measured output which is used as feedback to the controller

$A(\vartheta(t)), B(\vartheta(t)), B_w(\vartheta(t))$, $C_z(\vartheta(t)), D_z(\vartheta(t)$, $C_y(\vartheta(t))$ and $D_y(\vartheta(t))$ are linear parameter varying matrices and varying parameter is $\vartheta(t)$ and satisfies the condition :

$$z_v = \{\theta \in R^s; \theta_h \epsilon [\underline{\vartheta}_{(h)}, \overline{\vartheta}_{(h)}]\} \forall h = 1, \dots \dots .. s \qquad (4)$$

Where

$\underline{\vartheta}_{(h)}$ is lower bound of ϑ_h

$\overline{\vartheta}_{(h)}$ is upper bound of ϑ_h

and parameter vector θ belongs to convex set $z$and includes all possible values of parameter $\vartheta(t)$

By considering the conventional Lipschitz condition to the nonlinear system (1), $\varphi(t, x, \vartheta(t))$ can be written as $\varphi(t, x, \vartheta(t)) = H(\theta) f(t, x)$

where $H(\theta)$ is linear parameter varying matrix and nonlinearity is $f(t, x) \in R^n$ which satisfies the condition $||f(t, x) - f(t, \overline{x})|| \leq ||L(x - \overline{x}||$ and *f(t,0)=0,* $\forall$ *t≥0,* where L is matrix , then system (1) becomes

$$\dot{x} = A(\theta)x + B(\theta)u_{sat} + B_w(\theta)w + H(\theta)f(t, x)$$

$$z = C_z(\theta)x + D_z(\theta)w$$

$$y = C_y(\theta)x + D_y(\theta)w \quad \forall \theta \in z \qquad (5)$$

For the nonlinear system(5), feedback controller along with AWC is given by equations

$$\dot{x}_c = A_c x_c + B_c y + E_c(u_{sat} - u)$$
$$u = C_c x_c + D_c y \qquad (6)$$

Where x_c is state of controller

A_c, B_c, C_c and D_c are constant and known matrices E_c is antiwindup gain.

The overall closed loop system formed by combining nonlinear system (5) and controller (6) has the form

$$\frac{d\xi}{dt} = \mathbf{A}(\theta)\xi + \mathbf{T}(\theta)w - (\mathbf{B}(\theta) + \mathbf{R}E_c)\psi + \mathbf{F}(\theta)f(x)$$
$$z = \mathbf{J}(\theta)\xi + D_z(\theta)w$$
$$y = \mathbf{K}(\theta)\xi + D_c D_y(\theta)w\,,\ \ \forall\theta \in z \qquad (7)$$

where

$$\frac{d\xi}{dt} = \begin{bmatrix} x \\ x_c \end{bmatrix},$$
$$\mathbf{A}(\theta) = \begin{bmatrix} A(\theta) + B(\theta)D_c C_y(\theta) & B(\theta)C_c \\ B_c C_y(\theta) & A_c \end{bmatrix}$$
$$\mathbf{B}(\theta) = \begin{bmatrix} B(\theta) \\ 0 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 0 \\ I_{cXc} \end{bmatrix}, \mathbf{F}(\theta) = \begin{bmatrix} H(\theta) \\ 0 \end{bmatrix}$$
$$\mathbf{T}(\theta) = \begin{bmatrix} B_w(\theta) + B(\theta)D_c D_y(\theta) \\ B_c D_y(\theta) \end{bmatrix},$$
$$\mathbf{K} = [D_c C_y(\theta)\ C_c]\,, \mathbf{J}(\theta) = [C_z(\theta)\ 0],$$
$$\psi = u_{sat} - u.$$

Now the method is adopted for computation of AWC gain E_c so that the closed loop system with nonlinearity is $\mathcal{L}_2$ stable is presented below.

Consider the system (1) and a controller of form(4) such that Lipschitz condition is satisfied. If there exists a symmetric matrix > 0 , a diagonal matrix $U > 0$, matrices V and $\mathbf{H}(\theta)$ of appropriate dimensions, and scalars k and μ such that the set of LMI's given by

$$k>0, 0<\mu<1 \qquad (8)$$
$$\begin{bmatrix} Q & \mathbf{M}^T_{(i)}(\theta) - \mathbf{H}^T_{(i)}(\theta) \\ * & \mu\overline{u}^2 \end{bmatrix} \geq 0\ \forall\theta \in z \qquad (9)$$
$$\begin{bmatrix} \Gamma_1(\theta) & \Gamma_2(\theta) & \Gamma_3(\theta) & F(\theta) & 0 & QL^T & QJ^T \\ * & -kI & D_y^T(\theta)D_c^T & 0 & D_z^T(\theta) & 0 & 0 \\ * & * & -2U & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0\ \forall\theta \in z \qquad (10)$$
$$\Gamma_1(\theta) = Q\mathbf{A}^{\mathrm{T}}(\theta) + \mathbf{A}(\theta)$$
$$\Gamma_2(\theta) = \mathbf{T}(\theta) + Q\mathbf{J}^T(\theta)D_z(\theta)$$
$$\Gamma_3(\theta) = \mathbf{H}^T(\theta) - \mathbf{B}(\theta)U - \mathbf{R}V$$
$$\mathbf{M}^T_{(i)}(\theta) = \mathbf{K}_i(\theta)Q$$

are satisfied then

- the closed loop system is locally asymptotically stable for all initial conditions ξ^T(0)Pξ(0)≤1, if w=0;
- the $\mathcal{L}_2$ gain from w to z is bounded by γ, if $w \in \mathcal{L}_2$ for all signals validating $||w||_2^2 \leq \delta^{-1}$
- the state of the closed loop system remains bounded in ξ^T(t)Pξ(t)≤1 for all signals satisfying $\left||w|\right|_2^2 \leq \delta^{-1}$

Then the AWC gain matrix can be calculated as $E_c = VU^{-1}$ and $\mathcal{L}_2$ gain bound is given by $\gamma = \sqrt{k}$

In order to get antiwindup gain E_c we consider an algorithm for solving the set of LMI's that are mentionrd in the method.

The algorithm is as follows:

1. Initialize the i and j values with 1.
2. Construct the LMI's of the mentioned in method by assigning θ=θ_{ij}
3. Increment j, if j≤q, calculate $\theta_{ij} = \theta_{i(j-1)} + \varepsilon_i$ and go to step2, otherwise set j value with 1.
4. Increment i, If i≤s, go to step 2.
5. Solve the set of LMI's in order to get the variables Q, U, V, **H**(θ), k and μ which are obtained in step 2.
6. Calculate E_c by solving$E_c = VU^{-1}$.

III. SIMULATION AND RESULTS

A. DC Motor Modelling

The nonlinear model of DC motor simulated in MATLAB using simulink blocks. The parameters of DC motor model are given by R=3.2Ω,α=0.06 V/rad, L=0.0086H, Gear ratio=30:1, F=0.01Nm s/rad, b=-0.01Ω/rad, λ=17.5 and the motor inertia J has the variation in the range J∈[0.012,0.055].

B. Calculation of AWC Gain

Consider $[x_1\ x_2]^T = [\omega\ i]^T$ then the system described in(1) is converted to state space model representation (3) by using $\vartheta(t) = J^{-1}$, then the region (4) can be written as

$$z_v = \{\theta \in R; \theta \in [18.18, 83.33]\}$$

and the system matrices are as follows.

$$A(\theta) = \begin{bmatrix} -0.01\theta & 0.006\theta \\ -6.9767 & -372.093 \end{bmatrix}, B(\theta) = \begin{bmatrix} 0 \\ 6104.65 \end{bmatrix},$$
$$B_w = \begin{bmatrix} 0 & -\theta \\ 0 & 0 \end{bmatrix}, f(t,\omega,i) = \begin{bmatrix} -0.01\theta i^2 \\ 1.66\omega i \end{bmatrix},$$
$$C_z(\theta) = C_y(\theta) = [-9.54929\ 0],$$
$$D_z(\theta) = D_y(\theta) = [1\ \ 0].$$

A fuzzy PID controller is designed in the simulink using fuzzy logic controller having the inputs error and derivative of error.

The controller without saturation is given by the equations

$$x_c^{\cdot} = y,$$

$$u = 0.025x_c + 0.0829y + 0.00165\dot{y} \quad (11)$$

From the equations we have $A_c = 0$, $B_c = 1$, $C_c = 0.025, D_c = 0.0829$

From the system matrices and controller matrices, the closed loop system matrices can be calculated and are given as

$$\mathbf{A}(\theta) = \begin{bmatrix} -0.01\theta & 0.006\theta & 0 \\ -1617.8567 & -372.093 & 50.872 \\ -9.54929 & 0 & 0 \end{bmatrix},$$

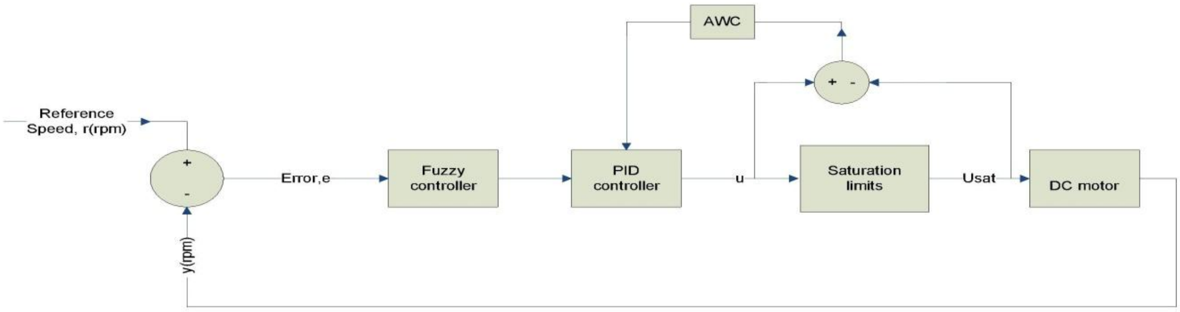

Fig.7.Block Diagram of Fuzzy PID Controlled DC Motor with Antiwindup Compensator

$$\mathbf{B} = \begin{bmatrix} 0 \\ 2034.88 \\ 0 \end{bmatrix}, \mathbf{T}(\theta) = \begin{bmatrix} 0 & -\theta \\ 168.69 & 0 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{F}(\theta) = \begin{bmatrix} -0.01\theta & 0 \\ 0 & -1.16 \\ 0 & 0 \end{bmatrix},$$

K(θ)=[-0.7916 0 0.025],

$\mathbf{J}(\theta) = [-9.54929\ 0\ 0]$.

By using the above matrices a program is written for algorithm mentioned earlier in MATLAB using the commands of robust control tool box, LMI's (8),(9) and (10) were constructed and solved for variable matrices Q, U, V $\mathbf{H}(\theta)$, k and μ. From these matrices antiwindup gain is calculated and is given by $E_c = 2034.8$The block diagram represented closed loop system with fuzzy PID controller and with AWC is shown in fig.7.

The simulation results are provided below. We applied a step signal of 2500 for 10 sec and a step of 4500 for next 10sec. On seeing the fig.(8) we can conclude that Fuzzy PID controller is better than in terms of rise time and percentage overshoot.

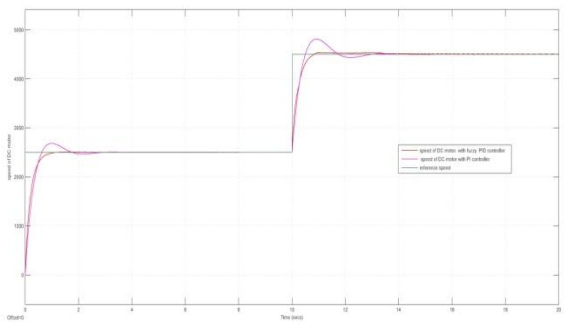

Fig.8. DC Motor Speed with PI and Fuzzy PID Controller

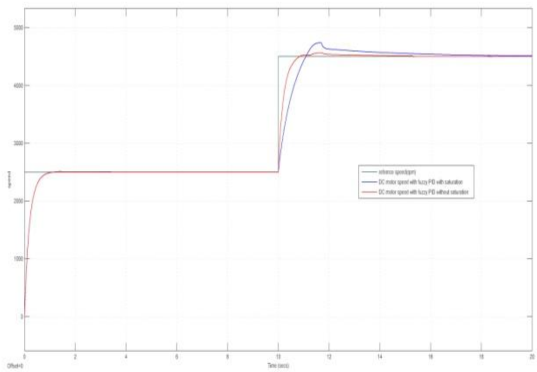

Fig.9 DC Motor Speed Control with Saturation and without Saturation Effect

The conclusions drawn from above result are as follows

- When the reference speed is less, control signal is less so actuator saturation is not happening
- When the reference signal is more, control signal is more so actuator saturation happens which causing large overshoot and more settling time than the system without considering the saturation effect.

So an antiwindup compensator is designed which reduces overshoot and settling time.

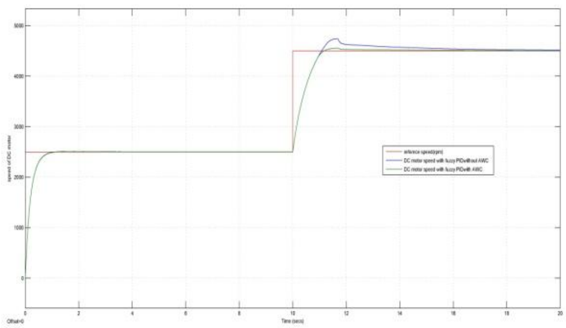

Fig.10. DC Motor Speed without AWC and with AWC

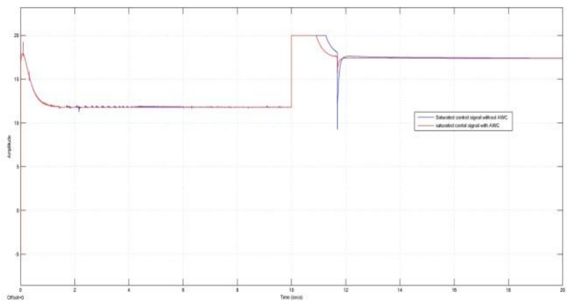

Fig.11 Saturated Control Signal with AWC and Without AWC

When the antiwindup compensator is designed the overshoot in closed loop system response is reduced and settling time also reduced.

Now we will apply a constant reference speed signal and we will apply a change in load torque. First we will apply a large load for some time then we will remove the load on the motor. Then the observed results are like as follows.

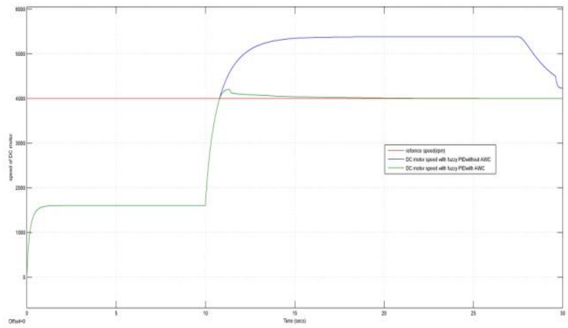

Fig.11 DC Motor Speed under Load Variations with AWC and without AWC

When a large load is applied on the motor the speed of the motor is reduced in both the cases i.e. with AWC and without AWC. But when the load is removed the system response came to normal response with antiwindup compensator. But in case of without compensator the motor speed is increased and doesn't settle to reference speed that means system may become unstable.

The following figure shows the saturated control signal under load variations with and without AWC. When large load is removed the control signal is more which saturates the actuator more and more which is undesirable. But in case of with antiwindup compensator the control signal is reduced and decreases the overshoot which is improving the closed loop response.

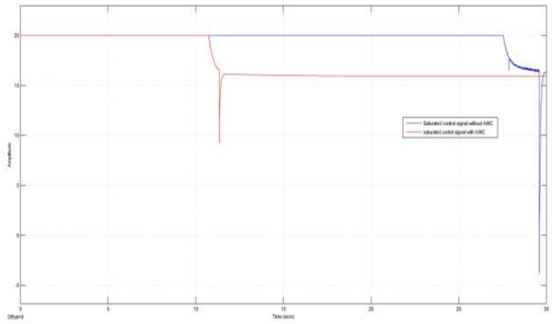

Fig.12. Saturated Control Signal Under Load Variations with AWC and without AWC

CONCLUSIONS

A DC motor is modelled by considering the nonlinearity in the mathematical equations, fuzzy PID controller is designed for speed control of DC motor and a new method is adopted for designing AWC for the system. Compared with the PI controller fuzzy PID controller is better and it is proven by the provided result. By applying step wave as reference signal the system was simulated for large reference signal and load variations and the adopted methodology is found to be effective in improving the closed loop system performance and control signal against windup consequences.

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