New Heuristic for Constrained Non- Renewable Resource Allocation in Stochastic Metagraphs with Discrete Random Times

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Abstract

In recent years, some researchers have attempted to show projects using metagraphs. In real word projects, activity durations are non-deterministic. Non-deterministic quantities can be shown with random variables. Here, it is assumed that the activity duration is a discrete random variable with a known probability function. It is supposed that a certain type ofnon-renewable (consumable) resource is necessary to execute the activities. However, the amount of the available resource is constrained and known. In addition, probability function of activity time depends on the resource allocated to it. In this research, a new heuristic algorithm has been developed to allocate the constrained consumable resource to the edges (activities) of the metagraph such that, the delay in project completion time is decreased. In other words, the probability of project completion before the due date is maximized. Some examples have been solved using the new and previous algorithms. Then, results have been compared.

This probability has been used for time-cost trade off in the project, in a way that the total cost of project is minimized. Therefore, the optimum value of the resource is defined.

Keywords — *Time-cost trade off; Consumable resource allocation; Stochastic metagraph; Project completion time.*

I. INTRODUCTION

In many projects, the time cost trade-off is very important because the objective of this analysis is to complete the original project with the lowest cost possible.

We usually show the projects using various kinds of networks however, in recent decades a new tool has been developed for this purpose. This new tool is called metageraph [1]. Metagraphs and specifications have been introduced in [2]. Metageraphs are used in decision support systems [3]. Metagraphs provide a powerful and flexible tool for analysing and scheduling workflows [4], [5], [6]. We can study metagraphs and their application in [7]. Constrained resource allocation in fuzzy metagraphs has been examined in [8]. Time cost trade-off in fuzzy metagraphs with trapezoidal fuzzy numbers for edges has been studied in [9].A method has been proposed for constrained consumable resource allocation in stochastic metagraphs [10].

This paper studies the time cost trade-off in stochastic metagraphs when the edges time are discrete random variables. It is assumed that a certain kind of consumable resource is necessary to execute the project activities. Here, anew heuristic algorithm is developed to solve the above-mentioned problem. This new heuristic algorithm is better than the method introduced in [10]. The advantage of the new algorithm has been shown in some examples.

II. THE PROBLEM

In this research, to complete the project with the lowest possible cost, a certain method is developed to define the amount of resource allocated to each activity.

Here, two algorithms have been proposed to solve the problem. The first algorithm allocates constrained resources and the second creates time cost trade-off.

A. Assumptions

- The metagraphs of this project starts with an invertex and terminates in an outvertex.
- The completion time for each edge is a discrete random variable, which has a given probability function.
- Only a certain type of consumable (non-renewable) resource is needed for implementation of activity.
- Probability function of activity duration depends on the value of resource allocated to that activity.
- The due date of the project is known as a constant value.
- The value of allocated resource to each activity is limited.

III. NEW ALGORITHM FOR RESOURCE ALLOCATION

This algorithm comprises two sub-algorithms for constrained consumable resource allocation in stochastic metagraphs.

A. Notations of Sub-Algorithm 1

i No. of iteration

e_j jth activity(edge) $j=1,2,...,n$

Rs the available amount of limited resource

$Rs^{(i)}$ Available limited resource in iteration i

$R^{(i)}$ N-tuple orderedof allocated resource to activities 1 to n in ith iteration of algorithm $R^{(i)} = (S_{L_1}^{(i)},...,S_{L_n}^{(i)})$.

R_{max} Maximum amount of allocated resource which can be allocated to the project

S_{L_j} the amount of resource allocated to j th activity, $L_j = 1,2,...,k_j$

B. Sub-Algorithm 1

Step 1. Set $i = 0$, $R^{(i)} = (S_{L_1}^{(i)},S_{L_2}^{(i)},...,S_{L_n}^{(i)})$ and $Rs^{(i)} = R_{max}$ as the initial assignment.

Step 2. Compute the mean time of the project paths. Choose a path with minimum average.

Step 3. Find the critical indexes of activities that are on the selected path using computer simulations.

Step 4. Choose an activity that has the least critical index.

If two or more activitiesexist with the same critical index exist, then choose one of them according to the following priority:

- ***A.*** Select an activity that lies on the minimum number of paths.
- ***B.*** Select an activity completion time of which has the maximum variance.

Step 5. For the selected activities in step 4. Set $S_{L_j} = S_{L_j} - 1$ and $i = i + 1$, $Rs^{(i+1)} = Rs^{(i)} - 1$, if $Rs^{(i)} = Rs$, stop and determine the allocated resource to each activity, otherwise go back to step 2.

C. Notations of Sub-Algorithm 2

i No. of iteration

e_j jth activity(edge) j=1,2,...,n

Rs The available amount of limited resource

$Rs^{(i)}$ Available limited resource in iteration i

$R^{(i)}$ N-tuple orderedof allocated resource to activities 1 to n in ith iteration of algorithm$R^{(i)} = (S_{L_1}^{(i)},...,S_{L_n}^{(i)})$.

Rn n-tuple ordered for allocated resource to activities 1 to n after changing the allocated resource of pair activities

S_{L_j} The amount of resource allocated to j th activity, $L_j = 1,2,...,k_j$

I No. of simulation iteration

N Number of simulation

t Due date ofthe project

T Completion time random variable of project

k_1 First random integer number from 1 to n

k_2 Second random integer number from 1 to n

P The probability of project completion before the due date

Pn The new value of P

D. Sub-Algorithm 2

Step 1. Set $I = 1,N$.

Step 2. Using the assignment of $Rs^{(i)}$, which is obtained in the sub-algorithm 1, simulate the metagraph and obtain $P = P(T \leq t|Rs)$.

Step 3. Create two random integers k_1 and $k_2$from 1 to$n$.

Step 4. Recognize the allocated resource to activitiesk_1 and k_2 in the $R^{(i)} = (S_{L_1}^{(i)},S_{L_2}^{(i)},...,S_{L_n}^{(i)})$.

Step 5. Check the$S_{L_{k_1}}$ and $S_{L_{k_2}}$, if one of them can be increased and another can be reduced.Then increase one of them and decrease the other oneand go to step 6 otherwise set $I = I + 1$ and go to step 10.

Step6. DefineRn.

Step7. Based on Rnobtained, compute $Pn = P(T \leq t)$ using the simulation.

Step 8. If$Pn > P$, go to step 9, otherwise, put $I = I + 1$ and go to step 10.

Step 9. Set$R^{(i)} = Rn$, $P = Pn$ and $I = I + 1$.

Step 10. If$I = N$ stop, otherwise go to step 3.

IV. NEW ALGORITHM FOR TIME COST TRADE-OFF

The costs of project can be classified as direct costs and indirect costs.

A. Direct cost

The costs of providing resources, materials, workers, energy and the cost of equipment and similar costs used for implementing activities, are considered direct costs of the project. It is supposed that C is the cost of one unit of the resources and Rs is the allocated resource to the metagraph. If the direct cost has linear relation with Rs then the direct cost (f_1) can be shown as follows:

$$f_1 = C \times Rs$$

B. Indirect costs

Indirect cost of the project depends on the completion time of the project. By increasing the completion time of the project, the probability of penalty payment will be increased and vice versa. Suppose that $p(T \leq t)$ is the probability of project completion before the due date and Q is the penalty of delay for one unit of time. Indirect cost (f_2) can be shown as follow:

$$f_2 = [1 - p(T \leq t)] \times Q$$

C. Total Costs

Total cost is the sum of direct costs and indirect costs as shown below:

$$C(Rs) = f_1 + f_2$$

The aim of developing the algorithm is to minimize the total cost. After minimizing the optimum value of the total cost, Rs will be defined. Also using the algorithm, cumulative distribution function of completion time of the project can be computed for various value of Rs.

1) Notations of algorithm for time cost trade-off

R n-tuple ordered of allocated resource for activities

Rs_{max} Maximum value of resource that can be allocated to project

Rs_{min} Minimum value of resource that can be allocated to project

2) Steps of the algorithm

Step 1. Set $C_{min} = \infty$

Step 2. Set $Rs = Rs_{min}$

Step 3. Compute$C(Rs) = f_1 + f_2$

Step 4. If $C(Rs) < C_{min}$, set $C_{min} = C(Rs)$, $Rs^* = Rs$ andgo to step5, otherwise go to step5.

Step 5. If $Rs = Rs_{max}$ go to step 6, otherwise, set $Rs = Rs + 1$ and go to Step 3.

Step 6.C_{min} is the optimum value of total cost and $Rs^* = Rs$. In this case, we should consume Rs^*.

In step 3 of the algorithm above, $P(T \leq t)$ is estimated using the simulation which was described previously in algorithm 1.

V. EXAMPLE 1

Suppose that the metagraph of Figure 1 shows a project. Probability function of activity completion time random variables has been given in table Ⅰ. These functions depend on amount of allocated resource. In this project$Rs = 18, t = 11$, $C = 12$ and $Q = 420$.

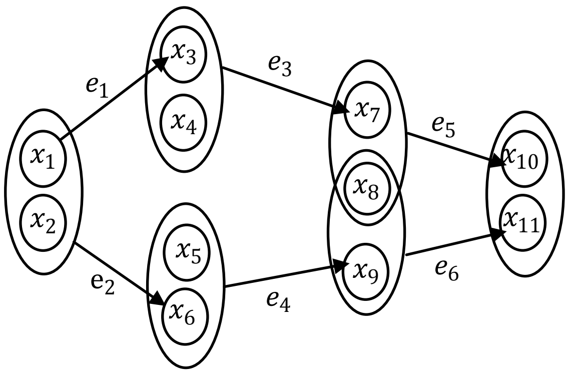

Fig 1: Metagraph of example 1

Table I. Probability Function of Activity Durations in Example 1

L_1	S_{L_1}	$P_1(S_{L_1},D_1)$	L_2	S_{L_2}	$P_2(S_{L_2},D_2)$
1	2	$=\frac{1}{2}$ $D_1 = 2$ $=\frac{1}{2}$ $D_1 = 3$	1	3	$=\frac{1}{2}$ $D_2 = 4$ $=\frac{1}{2}$ $D_2 = 5$
2	3	$=\frac{1}{3}$ $D_1 = 1$ $=\frac{2}{3}$ $D_1 = 2$	2	4	$=\frac{3}{5}$ $D_2 = 3$ $=\frac{2}{5}$ $D_2 = 4$
L_3	S_{L_3}	$P_3(S_{L_3},D_3)$	L_4	S_{L_4}	$P_4(S_{L_4},D_4)$
1	4	$=\frac{3}{7}$ $D_3 = 5$ $=\frac{4}{7}$ $D_3 = 6$	1	2	$=\frac{2}{5}$ $D_4 = 2$ $=\frac{3}{5}$ $D_4 = 3$
2	5	$=\frac{4}{7}$ $D_3 = 4$ $=\frac{3}{7}$ $D_3 = 5$	2	3	$=\frac{1}{3}$ $D_4 = 1$ $=\frac{2}{3}$ $D_4 = 2$
L_5	S_{L_5}	$P_5(S_{L_5},D_5)$	L_6	S_{L_6}	$P_6(S_{L_6},D_6)$
1	2	$=\frac{1}{3}$ $D_5 = 2$ $=\frac{2}{3}$ $D_5 = 3$	1	3	$=\frac{1}{4}$ $D_6 = 4$ $=\frac{3}{4}$ $D_6 = 5$
2	3	$=\frac{1}{4}$ $D_5 = 1$ $=\frac{3}{4}$ $D_5 = 2$	2	4	$=\frac{6}{7}$ $D_6 = 3$ $=\frac{1}{7}$ $D_6 = 4$

A. Steps of Sub-Algorithm 1 for Solving the Example are as Follows:

Step 1. Set $i = 0$, $R^{(0)}$=*(3,4,5,3,3,4)* and $Rs^{(0)} = R_{max} = 22$as the initial assignment.

Step 2. The mean time of path $e_1 - e_3 - e_5$is 7.83 and the mean time of path $e_2 - e_4 - e_6$ is 8.2.Then, $e_1 - e_3 - e_5$will be selected.

Step 3. The critical indexes of the selected path activities have been estimated by simulation and shown in table Ⅱ.

Table II . Critical Indexes for the selected Path Activities

path	e_1	e_3	e_5
$e_1 - e_3 - e_5$	0.5594	0.5594	0.5594

Step 4. Choose $e_3$since it has the maximum variance**.**

Step 5. For the activity selected in step 4, set $S_{L_3} = S_{L_3} - 1$ and $i = i + 1 = 1$, $Rs^{(1)} = Rs^{(0)} - 1 = 21$since,$Rs^{(1)} \neq Rs$ then go to step 2.

Step 2. The mean time of path$e_1 - e_3 - e_5$ is 8.97 and the mean time of path $e_2 - e_4 - e_6$ is 8.2. Then,$e_1 - e_3 - e_5$will be selected.

Step 3. The critical indexes of the selected path activities have been estimated by simulation and shown in table.

Table III . Critical indexes for the Selected Path Activities

path	e_2	e_4	e_6
$e_2 - e_4 - e_6$	0.3873	0.3873	0.3873

Step 4. Choose e_2 since it has the maximum variance.

Step 5. For the activity selected in step 4, set $S_{L_2} = S_{L_2} - 1$ and $i = i + 1 = 2$, $Rs^{(2)} = Rs^{(1)} - 1 = 20$ since, $Rs^{(2)} \neq Rs$ then go to step 2.

Step 2. The mean time of path $e_1 - e_3 - e_5$ is 8.97 and the mean time of path $e_2 - e_4 - e_6$ is 9.3. Then, $e_1 - e_3 - e_5$ will be selected.

Step 3. The critical indexes of selected path activities have been estimated by simulation and shown in table IV.

Table IV. Critical indexes for the Selected Path Activities

path	e_1	e_5
$e_1 - e_3 - e_5$	0.5769	0.5769

Step 4. Choose e_5 because it has maximum variance.

Step 5. For selected activity in step 4, set $S_{L_5} = S_{L_5} - 1$ and $i = i + 1 = 3$, $Rs^{(3)} = Rs^{(2)} - 1 = 19$ since, $Rs^{(3)} \neq Rs$ then go to step 2.

Step 2. The mean time of path $e_1 - e_3 - e_5$ is 9.88 and the mean time of path $e_2 - e_4 - e_6$ is 9.3. Then, $e_2 - e_4 - e_6$ will be selected.

Step 3. The critical indexes of the selected path activities have been estimated by simulation and shown in table V.

Step 4. Choose e_6 since it has the maximum variance.

Table V. Critical indexes for the Selected Path Activities

path	e_4	e_6
$e_2 - e_4 - e_6$	0.4562	0.4562

Step 5. For the activity selected in step 4, set $S_{L_6} = S_{L_6} - 1$ and $i = i + 1 = 4$, $Rs^{(4)} = Rs^{(3)} - 1 = 18$ since, $Rs^{(4)} = Rs$ stop and go to solve the sub-algorithm 2.

B. Steps of sub-algorithm 2

Step 1. Set $I = 1$ and $N = 1000$.

Step 2. Suppose that $Rs^{(4)} = (3,3,4,3,2,3)$. Using the simulation, $P = P(T \leq 11|Rs = 18) = 0.7485$ is obtained.

Step 3. k_1 and k_2 are created randomly. Results are $k_1 = 1, k_2 = 4$.

Step 4. The allocated resource to activities $k_1 = 1$ and $k_2 = 4$ in the $Rs^{(4)} = (3,3,4,3,2,3)$ are $S_{L_{k_1}} = 3$ and $S_{L_{k_2}} = 3$.

Step 5. Check the $S_{L_{k_1}}, S_{L_{k_2}}$, one of them cannot be increased and the other cannot be reduced so set $I = I + 1$ and go to step 10.

Step 10. since $I \neq N$ go back to step 3.

Step 3. Create two random integers k_1 and k_2 from 1 to 6. If $k_1 = 1$ and $k_2 = 2$.

Step 4. The allocated resource to activities $k_1 = 1$ and $k_2 = 2$ in the $Rs^{(4)} = (3,3,4,3,2,3)$ are $S_{L_{k_1}} = 3$ and $S_{L_{k_2}} = 3$.

Step 5. Check the $S_{L_{k_1}}, S_{L_{k_2}}$, one of them can be increased and the other can be reduced. Increase one of them and decrease the other and go to step 6.

Step 6. $Rn=(2,4,4,3,2,3)$.

Step 7. Based on the obtained Rn, $Pn = P(T \leq t) = 0.8183$ is computed.

Step 8. Since $Pn > P$, go to step 9.

Step 9. $R^{(i)} = Rn$, $P=Pn$ and $I=I+1$.

Step 10. If $I \neq N$ stop, then go to step 3.

The steps of sub-algorithm 2 were repeated 1000 times using the computer simulation. The Final allocation is Rn=(3,3,4,4,3,3,4,5) and $Pn = 0.9831$.

C. Steps of algorithm 2 with $C = 12$ and $Q = 420$ are as follows:

Step 1. Set $C_{min} = \infty$.

Step 2. Set $Rs = 16$.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 192 + 309.8 = 501.8$$

Step 4. $501.8 < C_{min}$, $C_{min} = 501.8$, $Rs^* = 16$.

Step 5. $Rs_{max} \neq Rs$ so $Rs = Rs + 1 = 17$, and go to step 3.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 204 + 96.2 = 300.2$$

Step 4. $300.2<501.8, C_{min}=300.2$, $Rs^* = 17$.

Step 5. $Rs_{max} \neq Rs$ so $Rs = Rs + 1 = 18$, and go to step 3.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 216 + 15.8 = 231.8$$

Step 4. $231.8<300.2, C_{min}=231.8$, $Rs^* = 18$.

Step 5. $Rs_{max} \neq Rs$ so $Rs = Rs + 1 = 19$, and go to step 3.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 228 + 0 = 228$$

Step 4. $228 < 231.8$, $C_{min} = 228$, $Rs^* = 19$.

Step 5. $Rs_{max} \neq Rs$ so $Rs = Rs + 1 = 20$, and go to step 3.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 240 + 0 = 240$$

Step 4. If $240 > 228$, $C_{min} = 228$, $Rs^* = 21$.

Step 5. $Rs_{max} \neq Rs$ so $Rs = Rs + 1 = 21$, and go to step 3.

Step 3. Calculate $C(Rs)$.

$$C(Rs) = 252 + 0 = 252$$

Step 4. $252 > 228, C_{min} = 228$, $Rs^* = 22$.

Step 5. $Rs_{max} = 22$ so go to step 6.

Step 6. $C_{min} = 228$ for R=(3,3,4,3,2,4) and $Rs = 19$ is obtained.

Value of the total cost of the project for various amounts of available resources have been computed and shown in table VI.

Table VI. Cost Information and the Probability of Completing the Project for Various Amounts Of Resources

Rs	R	$P(T \leq t \mid Rs)$	$C(Rs)$
16	(2,3,4,2,2,3)	0.2623	501.8
17	(2,3,4,3,2,3)	0.7709	300.2
18	(3,3,4,3,2,3)	0.9623	231.8
19	(3,3,4,3,2,4)	1	228
20	(3,3,4,3,3,4)	1	240
21	(3,4,4,3,3,4)	1	252
22	(3,4,5,3,3,4)	1	264

Minimum value of total cost is 228. In this case, probability of project complation befor thedue date is 1.

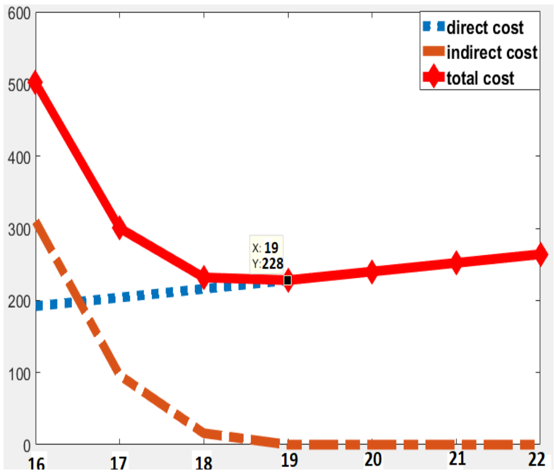

Fig 2:Direct Cost, Indirect Cost and Total Cost Curves for Example 1

VI. EXAMPLE 2

Suppose that the metagraph of Figure 3shows a project. Probability function of completion time random variables have been given in table VII. These functions depend on theamount of allocated resource. In this project$t = 14$, $C = 14$ and$Q = 180$.

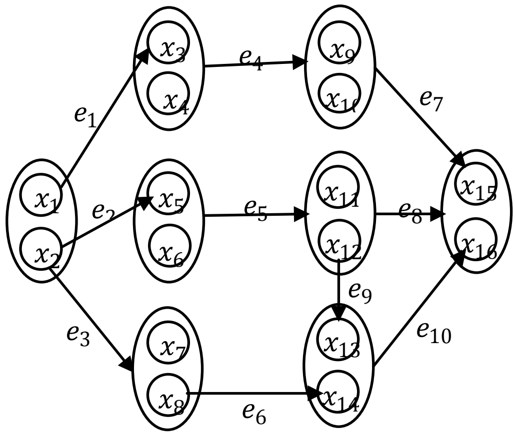

Fig 3: Metagraph of a Project with 10 Activities

Table VI. Probability Function of Activity Durations in Example 2

L_1	S_{L_1}	$P_1(S_{L_1},D_1)$	L_2	S_{L_2}	$P_2(S_{L_2},D_2)$
1	5	$=\frac{1}{3}$ $D_1=4$	1	3	$=\frac{1}{2}$ $D_2=5$
		$=\frac{2}{3}$ $D_1=5$			$=\frac{1}{2}$ $D_2=6$
2	6	$=\frac{1}{5}$ $D_1=3$	2	4	$=\frac{1}{3}$ $D_2=4$
		$=\frac{4}{5}$ $D_1=4$			$=\frac{2}{3}$ $D_2=5$
L_3	S_{L_3}	$P_3(S_{L_3},D_3)$	L_4	S_{L_4}	$P_4(S_{L_4},D_4)$
1	2	$=\frac{1}{2}$ $D_3=7$	1	8	$=\frac{1}{4}$ $D_4=3$
		$=\frac{1}{2}$ $D_3=8$			$=\frac{3}{4}$ $D_4=4$
2	3	$=\frac{3}{4}$ $D_3=6$	2	9	$=\frac{6}{7}$ $D_4=2$
		$=\frac{1}{4}$ $D_3=7$			$=\frac{1}{7}$ $D_4=3$
L_5	S_{L_5}	$P_5(S_{L_5},D_5)$	L_6	S_{L_6}	$P_6(S_{L_6},D_6)$
1	8	$=\frac{1}{2}$ $D_5=6$	1	6	$=\frac{1}{2}$ $D_6=5$
		$=\frac{1}{2}$ $D_5=7$			$=\frac{1}{2}$ $D_6=6$
2	9	$=\frac{2}{3}$ $D_5=6$	2	7	$=\frac{1}{3}$ $D_6=4$
		$=\frac{1}{3}$ $D_5=7$			$=\frac{2}{3}$ $D_6=5$
L_7	S_{L_7}	$P_7(S_{L_7},D_7)$	L_8	S_{L_8}	$P_8(S_{L_8},D_8)$
1	1	$=\frac{1}{2}$ $D_7=6$	1	5	$=\frac{1}{2}$ $D_8=3$
		$=\frac{1}{2}$ $D_7=7$			$=\frac{1}{2}$ $D_8=4$
2	2	$=\frac{3}{4}$ $D_7=5$	2	6	$=\frac{3}{4}$ $D_8=2$
		$=\frac{1}{4}$ $D_7=6$			$=\frac{1}{4}$ $D_8=3$
L_9	S_{L_9}	$P_9(S_{L_9},D_9)$	L_{10}	$S_{L_{10}}$	$P_{10}(S_{L_{10}},D_{10})$
1	2	$=\frac{1}{3}$ $D_9=1$	1	4	$=\frac{1}{4}$ $D_{10}=2$

2	3	$=\frac{2}{3}$ $D_9=2$ $=\frac{2}{3}$ $D_9=1$ $=\frac{1}{3}$ $D_9=2$	2	5	$=\frac{3}{4}$ $D_{10}=3$ $=\frac{6}{7}$ $D_{10}=1$ $=\frac{1}{7}$ $D_{10}=2$

In this example, first $P(T \leq t|Rs)$ has been computed for various amounts of available resources using algorithm 1. Then, the values of the total costs of the project have been computed and shown in table Ⅷ

Table VII. Cost Information and the Probability of Completing the Project for Various Amounts of Resources

Rs	R	$P(T \leq t\|Rs)$	$C(Rs)$
44	(5,3,2,8,8,6,1,5,2,4)	0	796
45	(5,3,2,8,8,6,1,5,2,5)	0.0251	805.5
46	(5,4,2,8,8,6,1,5,2,5)	0.0849	808.7
47	(5,4,2,9,8,6,1,5,2,5)	0.2599	791.2
48	(5,4,2,9,8,6,1,6,2,5)	0.4667	768
49	(5,4,3,9,8,6,1,6,2,5)	0.6567	747.8
50	(5,4,3,9,8,6,1,6,3,5)	0.7450	745.9
51	(5,4,3,9,9,6,1,6,3,5)	0.7999	750
52	(6,4,3,8,9,6,2,6,3,5)	0.8444	756
53	(6,4,3,9,9,7,1,6,3,5)	0.8465	769.6
54	(6,4,3,9,9,7,2,6,3,5)	0.8467	783.6

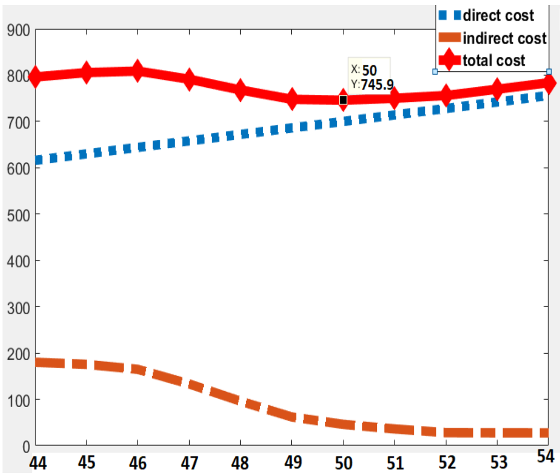

Fig 4: Direct cost, Indirect Cost and total Cost Curves for example 2

The minimum total cost of the project is 745.9 for Rs=50 and constrained consumable resource allocated to activitieshas been shown by R=(5,4,3,9,8,6,1,6,3,5). Also, probability of the project completion before thedue date is 0.7450.

VII. COMPARISON OF METHODS BY OTHER EXAMPLE

To define the minimum total cost in the project, the constrained resource should be allocated to activities in a way that, the probability of completion of stochastic metagraph before the due date of the projectis maximized. In order to achieve this, in this research, the algorithm 1 has been developed for resource allocationin stochastic metagraphs with discrete random times.Results of this method is better than the results of the reference [10]. The method introduced in [10], will be called algorithm A.H.

In this part, some examples have been solved with both methods results of which have beenshown in table ⅨXand table Ⅹ.

Table Ⅸ compares $P(T \leq t|Rs)$ for 18 examples using algorithm 1 and algorithm A.H.

Table Ⅹ compares the results of allocated resource to activities for 18 examples of reference [10] using algorithm 1 and algorithm A.H.

Table Ⅺ shows the results of allocated resource and time cost trad-off for 18 examples using algorithm 1 and algorithm 2.

Table VIII. Comparison of Results and $P(T \leq t|Rs)$using Algorithm1 and Algorithm A.H

Example number	Rs	t	$P(T \leq t\|Rs)$related to obtained resource allocation by algorithm A.H	$P(T \leq t\|Rs)$related to obtained resource allocation by algorithm 1	Better algorithm
1	18	6	0.7986	0.8040	New algorithm

2	17	7	0.9429	0.9427	New algorithm & Algorithm A.H
3	16	8	0.9001	0.9091	New algorithm & Algorithm A.H
4	16	7	0.6376	0.6370	New algorithm & Algorithm A.H
5	18	11	0.9571	0.9623	New algorithm
6	20	7	0.8573	0.8570	New algorithm & Algorithm A.H
7	23	8	0.5446	0.5898	New algorithm
8	23	10	0.6809	0.6742	New algorithm & Algorithm A.H
9	30	17	0.9587	0.9629	New algorithm
10	28	13	0.9484	0.9480	New algorithm & Algorithm A.H
11	38	15	0.5312	0.5550	New algorithm
12	29	15	0.9795	0.9831	New algorithm
13	51	14	0.7636	0.7999	New algorithm
14	38	15	0.7021	0.7009	New algorithm & Algorithm A.H
15	40	14	0.8812	0.8856	New algorithm & Algorithm A.H
16	40	15	0.9861	0.9880	New algorithm & Algorithm A.H
17	42	14	0.9685	0.9685	New algorithm & Algorithm A.H
18	43	13	0.7713	0.7864	New algorithm
19	40	14	0.9176	0.9171	New algorithm & Algorithm A.H

Table IX. Comparison of allocated Resource to Activities using Algorithm 1 and Algorithm A.H

Example number	Rs	t	Obtained resource allocation by algorithm A.H	Obtained resource allocation by algorithm 1
1	18	6	(3,3,3,4,5)	(4,3,3,3,5)
2	17	7	(3,4,3,3,4)	(3,3,3,3,5)
3	16	8	(4,3,2,4,3)	(4,3,2,4,3)
4	16	7	(3,3,2,5,3)	(3,3,2,5,3)
5	18	11	(2,3,5,2,2,4)	(3,3,4,3,2,3)
6	20	7	(3,3,3,3,5,3)	(3,3,3,3,5,3)
7	23	8	(4,2,5,3,5,4)	(4,2,5,2,6,4)
8	23	10	(3,5,3,4,4,4)	(3,5,3,4,4,4)
9	30	17	(6,2,1,3,5,6,3,4)	(6,3,1,2,5,6,3,4)
10	28	13	(2,4,3,4,2,6,3,4)	(2,4,3,4,2,6,3,4)
11	38	15	(2,5,3,2,6,8,5,7)	(2,5,3,2,6,8,5,7)
12	29	15	(2,4,4,5,3,2,4,5)	(3,3,4,4,3,3,4,5)
13	51	14	(5,4,3,9,9,6,2,6,2,5)	(5,4,3,9,9,6,1,6,3,5)
14	38	15	(8,6,2,4,4,2,1,4,4,3)	(8,6,2,4,4,2,1,4,4,3)
15	40	14	(6,6,6,2,2,3,3,7,3,2)	(6,6,6,2,2,3,3,7,3,2)
16	40	15	(3,2,4,4,3,4,1,5,3,3,2,6)	(3,2,4,4,3,4,1,5,3,3,2,6)
17	42	14	(3,3,2,4,5,5,3,4,2,3,6,2)	(3,3,2,4,6,5,3,4,2,3,6,1)
18	43	13	(2,2,3,4,5,5,1,7,4,4,3,3)	(2,2,3,5,5,4,1,7,4,4,3,3)
19	40	14	(4,3,4,5,5,2,4,3,2,3,2,3)	(4,3,4,5,5,2,4,3,2,3,2,3)

Table X. The Results of Algorithm 1 and Algorithm 2to Compute the Total Cost

Example number	$Rs_{min} \le Rs \le Rs_{max}$	t	C	$Min\ C(Rs)$	Q	Rs	$P(T \le t \mid Rs)$
1	14≤ Rs ≤21	6	2	44.5	50	16	0.7500
2	14≤ Rs ≤19	7	6	111.16	160	17	0.9427
3	13≤ Rs ≤18	8	25	408.88	140	15	0.7580
4	14≤ Rs ≤19	7	25	425	450	17	1
5	16≤ Rs ≤22	11	12	228	420	19	1
6	16≤ Rs ≤22	7	9	202.3	188	21	0.9291
7	19≤ Rs ≤25	8	45	1326	950	24	0.7411
8	20≤ Rs ≤26	10	30	721	130	22	0.5308
9	25≤ Rs ≤33	17	35	1052	230	30	0.9629
10	21≤ Rs ≤29	13	25	716.6	320	28	0.9480
11	35≤ Rs ≤43	15	30	1170	350	39	1
12	24≤ Rs ≤32	15	6	171.83	50	28	0.9234
13	44≤ Rs ≤54	14	14	745.9	180	50	0.7450
14	34≤ Rs ≤44	15	20	817	340	40	0.9499
15	36≤ Rs ≤46	14	17	701.8	230	41	0.9790
16	32≤ Rs ≤44	15	35	1366	300	37	0.7646
17	34≤ Rs ≤46	14	75	3178	900	42	0.9685
18	36≤ Rs ≤48	13	28	1267	412	44	0.9157
19	32≤ Rs ≤44	14	16	655.43	220	39	0.8571

VIII. CONCLUSION AND FUTURE RESEARCH

This paper has developed a new algorithms for constrained non-renewable(consumable) resource allocation in stochastic metagraphs with discrete random variable for activity times. This algorithm can allocate the limited resource better than that of algorithm A.H.. Besides, this paper has developed a new heuristic algorithm for time cost trade-off, which can define the minimum total cost.

Future researchers can study the time cost trade-off problems in stochastic metagraphs when the activity times are continuous random variables. Also, Similar research can be conducted in projects which require more than one kind of constraint consumable resource or in the ones which need two or more different kinds of resources.

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