A Pedestrian View on Random Walk

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Abstract*—In this paper the essential features of a random walk are described. Random walk is correlated to other physically observed motions. Methods to simulate the random walk is briefly discussed.*

Keywords —*Random walk, Brownian motion.*

I. Introduction

(Size 10 & Normal)Statistical mechanics is almost invariably introduced through the concept of steps taken by a drunken walker/random walker. The term Random Walk was coined by Karl Pearson[1] and they are a very popular model of stochastic processes with a rich history [2,3]. Various physical parameters such as mean values, dispersion etc which could be related to some physical events such as Brownian motion. This problem of studying the random steps taken in one dimension has been in turn extended to other forms of random walks (shown in the Fig.1below) which in turn is related to some physical events or processes.

Mathematically, a random walk is a stochastic sequence $\{S_n\}$, with $S_0=0$ defined by

$$S_n = \sum_{k=1}^{n} X_k \qquad (1)$$

Where $\{X_k\}$ are independent and identically distributed random variables.

II. RandomWalk

The theory of random walk is restated in terms of a particle picture and the walks are replaced by the successive steps the particle would take in a random environment.

In this introductory article we mainly consider Random walks on a lattice which is a special case of the full class of random walks. A random walk consists of a connected path formed by randomly adding new bonds to the end of the existing walk, subject to any restrictions which distinguish one kind of random walk from another. The mean-square end-to-end distance $\langle R^2 \rangle$ of a walk with N steps may diverge as N goes to infinity as [4]

$$\langle R^2(N)\rangle = aN^{2\nu}\left(1 + bN^{-\Delta} + ...\right) \qquad (2)$$

In the above equation ν is a 'critical exponent' that determines the universality class. Here *a* and*b* are some 'non universal' constants which depend on the model and lattice structure chosen and Δ is a 'correction to scaling' exponent. In such cases there is a strong analogy to critical behaviour in percolation or in temperature driven transitions in systems of interacting particles. The equivalent of the partition function for a system undergoing a temperature driven transition is given by the quantity Z_N which simply counts the number of distinct random walks on the lattice and which behaves as

$$Z_N \propto N^{\gamma-1} q_{eff}^N \qquad (3)$$

As $N \to \infty$, γ is another critical exponent and q_{eff} is an effective coordination number which is related to the exchange constant in a simple magnetic model. The formalism for describing this geometric phenomenon is thus the same as for temperature driven transitions, even including corrections to scaling in the expression for the mean-square end-to-end distance as represented by the term in $N^{-\Delta}$ in equation (2). The determination of ν and γ for different kinds of walks is essential to the classification of these models into different universality classes. We now know that the lattice dimensionality as well as the rules for the generation of walks affect the critical exponents and thus the universality class [5]. Examples of several kinds of walks are shown in Fig 1.

In the case of a simple random walks defined by $X_k = \pm 1$, with $P(X_k = 1) = p$ and $P(X_k = -1) = 1 - p = q$ the walker may cross the walk an infinite number of times with no cost. In d dimensions the end-to-end distance diverges with the number of steps N according to

$$\sqrt{\langle R^2(N)\rangle} \propto N^{\frac{1}{2}} \qquad (4)$$

III. Simulation & Result

A simulation of the simple random walk can be carried out by picking a starting point and generating a random number[6] to determine the direction of each subsequent, additional step. After each step the end-to-end distance can be calculated (Fig 2). Errors may be estimated by carrying out a series of independent random walks and performing a statistical analysis of the resultant distribution. Thus, the simple random walk has a trivial result $\nu = \frac{1}{2}$.

At this point we briefly mention a simple variant of the random walk for which the choice of the (n+1) step from the nth step of a return to the point reached at the (n-1) step, i.e. an 'immediate reversal', is forbidden. Although for this so-called 'non-reversal random walk' (NRRW) the exponents remain unchanged, i.e. $\nu = \frac{1}{2}, \gamma = 1$ as for the ordinary random walk, prefactors change. This means that in eqn.(2) $q_{eff} = (q-1)$ for the NRRW whereas $q_{eff} = q$ for the ordinary random walk, etc.

This NRRW model represents, in fact, a rather useful approach for the modelling of polymer configurations in dense melts, and since one merely has to keep track of the previous step and then choose one of the remaining $q-1$ possibilities, it is straightforward to implement. Furthermore, this NRRW model is also a good starting point for the simulation of 'self-avoiding walk' [7,8].

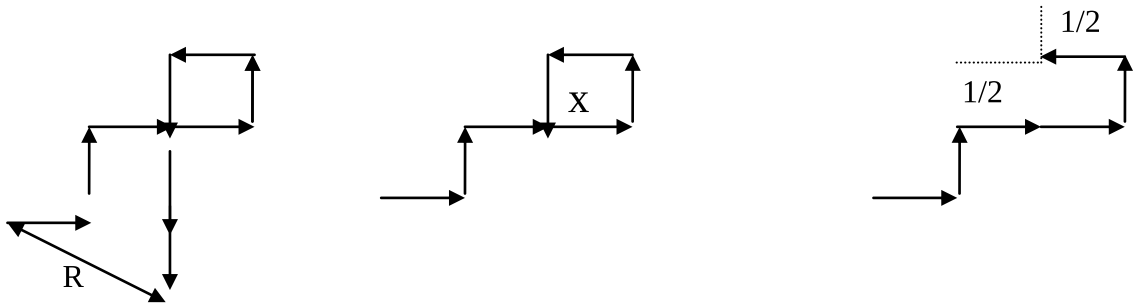

Fig 1: Examples of different kinds of random walks on a square lattice. For the RW every possible new step has the same probability. For the SAW the walk dies if it touches itself. The GSAW walker recognizes the danger and takes either of the two steps shown with equal probability

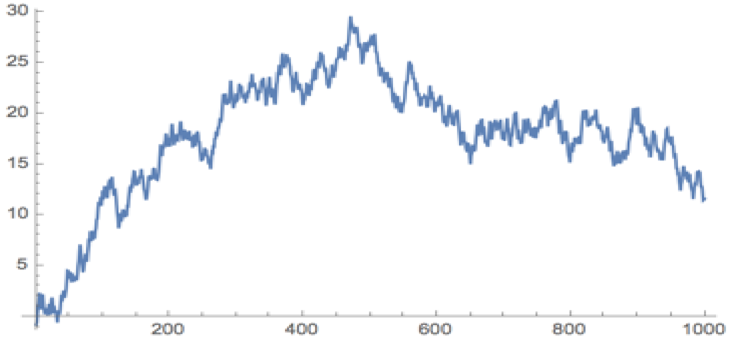

Fig 2: Simulation of a random walk

IV. Conclusion

We in this article have only tried to scratch the surface of a widely studied subject. I have not included some of the more elaborate articles and review papers so that to the uninitiated the task doesn't become too daunting. If one may wish to look further they can do so by referring to papers mentioned in the references here.

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