Mathematical modelling of traditional stoves using the Thermal Network Approach

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Abstract

This work intended to build a mathematical model for traditional stoves commonly used in rural areas by means of the thermal network approach. The studied stoves had different shapes as around us, but the pots had the same shape and dimension. The three stoves were made of the same type of material (Aluminium) and operated with firewood. Each system (heating device and the pot) was subdivided into $\overline{8}$ isothermal subvolumes, interconnected to one another by a thermal resistance; each resistance corresponding to a particular type of heat transfer. A differential system of 8 equations governing the heat transfer in the ovens' and configurations was obtained. the instantaneous thermal efficiency of each stove was estimated. Numerical simulation studies were carried out in order to appreciate the thermal behaviour of the systems. The model predicted the temperature agreement with the physical realities and needed a low computational time. From the comparative study, we concluded that amongst our studied stoves, the best heating device is that with an inverted and truncated cone (configuration 2) and is best for cooking since it reduces fuel consumption and therefore has a positive impact on deforestation.

Keywords — Traditional oven, local stove, heat

transfer, thermal system, Modelling, Simulation.

I. INTRODUCTION

Nowadays, plagues such as the exponential increase in population, deforestation and the use of fuels as combustible materials cause serious environmental and economic damages. So, reducing world fuel consumption of thermal systems such as refrigerators, heating systems or ovens is imperative to face the challenges in the fields of energy consumption and sustainable development [1,2].

According to [3,4], many works have been carried out in the domain of some thermal systems modelling. An electrical description of thermal systems was done by März et al. [5] after Robertson

et al. [6] who described analogous elements of thermal and electrical systems for Transient Heat-Flow Analysis. A technique was developed, that helps to subdivide the thermal system into a number of finite subvolumes called nodes and thermalelectric system analogy [7]. Farid Chejne et al. [8] developed a mathematical model that includes the three different heat transfer mechanisms between different surfaces of the solar stove and its environment. In 2014, Ramirez et al. [1] developed a new method to build grey-box thermal models based on electrical equivalent circuits which gives information about temperature evolution, and presented a new heat and mass transfer model for an electric oven and the load placed inside [9]. Despite the fact that these researchers worked on thermal modelling of heating systems, the traditional oven still has limited literature and thus is an active field of research.

The study undertaken in this work concerns the mathematical modelling and numerical simulation on three different configurations of traditional stoves commonly used in rural areas of developing countries by means of the thermal network approach followed by a comparative study. To do it, differential systems of equations governing the heat transfer in the oven configurations considered were set up, and used for evaluating the systems node temperatures and their efficiencies. After a literature review in section 2, the ovens are described and modelled in section 3 while section 4 is consecrated to the simulation results and discussion.

II. LITERATURE REVIEW

Since the beginning of human history, open fires and primitive stoves have been used for cooking [10]. Across the whole world, more than 2.6 billion people cook on inefficient stoves that burn solid fuels such as wood and charcoal. Without any policies addressing this challenge, this number will increase to more than 2.7 billion by 2030 due to a growing population mainly in developing countries [11,12]. In developing countries, cooking stoves account for more than half of the total residential energy use and in many poor countries more than 80% of the household energy consumption is for the purpose of cooking [12,13]. Although the threestone fire is very common it is often modified in many ways.

There is no universally accepted definition of "cookstoves" linked to performance or technical standards. Access to such improved cooking stoves is even more limited in less developed countries and Sub-Saharan Africa, where only 6% of people who use traditional biomass and coal for cooking have access to improved stoves [14, 15]. A report of the World Bank in 2011 [16] has reviewed the cookstove intervention sector with a new look according to the recent scientific evidences on the environmental and health-related impacts. The authors proposed a classification of stoves without indicating measureable benchmarks for each category. Open fires, cookstoves constructed by either artisans or household members that are not energy efficient and have poor combustion features are referred by the term *traditional stove* whereas improved cookstove is used in the historical sense for cookstoves installed in "legacy" programs, usually with a firebox and chimney, but without standards and with poor quality control [17]. The more recent manufactured cookstoves that are based on higher levels of technical research are called advanced biomass cookstoves; they are generally more expensive and are based on higher, but as yet not well defined, standards including safety, efficiency, emissions and durability [17]. Among others, they might include wood, charcoal, pellet and gasified cookstoves. Finally, the effective improved cookstove, cheaper but close in performance to advanced biomass cookstoves, is assembled on site by qualified installers adhering to standards [17]. Sanchez in 2010 proposed that an "improved energy source" for cooking is one which requires less than four person hours per week per household to collect fuel and meets the recommendations of the World Health Organization (WHO) for air quality (maximum concentration of CO of 30 mg/m³ for a 24 hours period and less than 10 mg/m³ for a period of 8 hours of exposure) [18]. Compared to the traditional three-stone fire, a good improved household stove if it is properly used, can save up to 60% of fuel and is designed to minimize the generation of products of incomplete combustion, many of which have a high global-warming potential [17].

The first attempts to improve traditional solid biomass stoves were made in India in the 1950's. These stoves were designed with a chimney to remove smoke from the kitchens. In the 1970's the oil crisis brought energy issues back to the top of the agenda and improved cooking stove programmes were considered as a solution to the fuel wood crisis, the deforestation and desertification around the world [12,19]; and during this period the research was focused on the technical aspects like thermodynamic and heat transfer. All over the world and particularly in Africa, Asia and Latin America, various international donors improved and promoted cooking devices [19] even if their effects have often been short-lived basically as a result of neglecting the requirements of users. But since then a shift of the core target from environmental protection to human welfare improvement has taken place [12]. The needs of the users gained more attention in the course of this shift and afterwards, many different models of improved solid biomass stoves with hundreds of variations exist [10]. They have come in various sizes and styles, having been adapted to a very large number of cultures and food preparation methods; and as society has progressed more sophisticated stove models have been developed [10].

The most basic type of cooking with biomass is the so called "three-stone fire", which is made by arranging three stones, in such a way that it is possible to place a pot for cooking above it. Although this type of biomass cooking is the most inefficient and bears serious risks to human health and the environment, it has been around for thousands of years and is still the most prevalent way of cooking in the world [14, 20]. Traditional fuels such as biomass are quite difficult to burn completely in a simple household sized stove and their use together with inefficient cooking stoves impacts negatively on the health of household members, especially women and children, when burned indoors without either a proper stove to help control the generation of smoke or a chimney to vent the smoke outside [11, 21]. Household air pollution accounts for about 4 million deaths a year, where 3.5 is due to direct exposure and 0.5 million deaths from outdoor air pollution attributed to the impact of household emissions on ambient air quality [11.12.22] and causes indoor concentrations of important pollutants, such as carbon monoxide, benzene and formaldehyde. Such exposures are linked to acute respiratory infections, chronic obstructive lung diseases, low birth weights, lung cancer and eye problems [23]. Therefore, accelerated technological effort is required to improve coalstove and coal's environmental performance [24].

From an environmental point of view, Cooking with wood and wood charcoal, done by 90% Tanzanians [25] and about 70% of Africa's population [26], presents two major challenges that require action such as deforestation and indoor air pollution from cooking smoke [27]. Annual consumption of charcoal in Tanzania, nearly all for cooking, averages a drastically unsustainable 2 million metric tons, consuming the equivalent of 327,190 ha of forest/woodland per year [27, 28]. Improved stoves are estimated to save up to 33% of fuel when compared to traditional stoves and have a significant impact on deforestation, savings in greenhouse gas emissions and aid biodiversity [29].

III.METHODOLOGY

A. Description of the ovens

We considered three ovens made of the same type of material (Aluminum) and operating with firewood. In each configuration, the pot has a cylindrical shape, the same dimensions and contains air. The main difference amongst these ovens is the radius of the lower surface, and hence the volume of the heating device; with its lowest value in configuration 2 (see Fig.1.b) and its highest value in configuration 3 (see Fig.1.c). The previous surface is the support of the heating source. The Figure 1 and Table 1 show respectively the oven schemes followed by an illustration of the position of each node where the heat balance is made, and the corresponding dimensions [4,8].

B. Theoretical modelling

The novel idea presented in this article is to build a mathematical model of some traditional ovens by the help of the analogy existing between thermal systems and electrical circuits.

The following assumptions are made to simplify the modelling:

- The heat produced by the heating source (fire) is used to first increase the temperature of the heating element itself and therefore those of the components of the system.
- We considered each part of the ovens to be modeled as nodal point (table 2) and isothermal [3].
- The ambient temperature and the energy produced by the heating source are not time-dependent.
- The heat produced by the heating source acts as input [1].

The thermal network model for the systems that compose the ovens can been seen in Figure 2 where, the different nodes are interconnected to one another by thermal resistances; each resistance corresponding here to a particular type of heat transfer (radiation, convection, or conduction) between the different nodes as in the work of Farid Cheine et al. [8]. For example, node 1 after receiving the input energy, exchanges by conduction with node 8, by convection with node 3 and by radiation with both node 2 and 4; while node 2 exchanges by convection by both node 3 and 9, and by radiation with node 1, 4 and 9. Following this route, we easily built our thermal model which facilitates the mathematical modelling of the systems.

According to the thermal network model previously built and to the shape factor calculated as in the references [30-33], the mathematical modelling of the energy balance equations at different nodes and for each configuration are:

Energy balance at node 1 (combustible):

$$\frac{dT_{1}}{dt} = \left(\frac{1}{m_{c}cp_{c}}\right) \begin{bmatrix} A_{1}F_{12}\sigma(T_{2}^{4}-T_{1}^{4}) + h_{1}A_{1}(T_{3}-T_{1}) + \\ + A_{1}F_{14}\sigma(T_{4}^{4}-T_{1}^{4}) + \\ + U_{c,1-8}A_{1}(T_{8}-T_{1}) + Q_{fuel} \end{bmatrix}$$
(1)

Energy balance at node 2 (wall):

$$\frac{dT_{2}}{dt} = \left(\frac{1}{m_{w1}cp_{w1}}\right) \begin{bmatrix} A_{2}F_{21}\sigma(T_{1}^{4}-T_{2}^{4}) + h_{1}A_{2}(T_{3}-T_{2}) + \\ + A_{2}F_{24}\sigma(T_{4}^{4}-T_{2}^{4}) + h_{3}A_{3}(T_{9}-T_{2}) \\ + A_{3}F\sigma(T_{9}^{4}-T_{2}^{4}) \end{bmatrix} (2)$$

Energy balance at node 3 (air1):

$$\frac{d T_{3}}{d t} = \left(\frac{1}{m_{Air1} c p_{Air1}}\right) \begin{bmatrix} h_{1}A_{1}(T_{1} - T_{3}) + \\ + h_{1}A_{2}(T_{2} - T_{3}) + \\ + h_{1}A_{4}(T_{4} - T_{3}) \end{bmatrix} (3)$$

Energy balance at node 4 (the base of the pot):

$$\frac{dT_{4}}{dt} = \left(\frac{1}{m_{b,pot}}cp_{b,pot}\right) \begin{bmatrix} A_{4}F_{41}\sigma(T_{1}^{4}-T_{4}^{4})+h_{1}A_{4}(T_{3}-T_{4})+\\ +A_{4}F_{42}\sigma(T_{2}^{4}-T_{4}^{4})+A_{5}F_{45}\sigma(T_{5}^{4}-T_{4}^{4})+\\ h_{2}A_{5}(T_{6}-T_{4})+A_{5}F_{47}\sigma(T_{7}^{4}-T_{4}^{4}) \end{bmatrix} (4)$$

Energy balance at node 5 (the side of the pot):

$$\frac{dT_{5}}{dt} = \left(\frac{1}{m_{s,pot}cp_{s,pot}}\right) \left| \begin{array}{c} F_{54}A_{6}\sigma\left(T_{4}^{4}-T_{5}^{4}\right) + h_{2}A_{6}\left(T_{6}-T_{5}\right) + \\ + F_{57}A_{6}\sigma\left(T_{7}^{4}-T_{5}^{4}\right) + h_{3}A_{7}\left(T_{9}-T_{5}\right) + \\ + A_{7}F\sigma\left(T_{9}^{4}-T_{5}^{4}\right) \end{array} \right| (5)$$

Energy balance at node 6 (inside the pot):

$$\frac{d T_{6}}{d t} = \left(\frac{1}{m_{Air2} c p_{Air2}}\right) \begin{bmatrix} h_{2} A_{5} (T_{4} - T_{6}) + \\ + h_{2} A_{6} (T_{5} - T_{6}) + \\ + h_{2} A_{8} (T_{7} - T_{6}) \end{bmatrix} (6)$$

Energy balance at node 7 (cover):

$$\frac{dT_{7}}{dt} = \left(\frac{1}{m_{cover}cp_{cover}}\right) \begin{bmatrix} h_{2}A_{8}\left(T_{6}-T_{7}\right) + A_{8}F_{75}\sigma\left(T_{5}^{4}-T_{7}^{4}\right) + \\ + A_{8}F_{74}\sigma\left(T_{4}^{4}-T_{7}^{4}\right) + h_{3}A_{9}\left(T_{9}-T_{7}\right) + \\ + A_{9}F\sigma\left(T_{9}^{4}-T_{7}^{4}\right) \end{bmatrix} (7)$$



Fig. 1 Oven scheme: (a) Configuration 1, (b) Configuration 2, (c) Configuration 3.

| | 1 uote 11 D untensions of the ovensi | | | | |
|----------------|--------------------------------------|-----------------|-----------------|-----------------|--|
| | Description | Configuration 1 | Configuration 2 | Configuration 3 | |
| Heating device | Height | 0.500 m | 0.500 m | 0.500 m | |
| | Upper inner radius | 0.120 m | 0.120 m | 0.120 m | |
| | Lower inner radius | 0.120 m | 0.090 m | 0.150 m | |
| | Wall thickness | 0.001 m | 0.001 m | 0.001 m | |
| | Base thickness | 0.001 m | 0.001 m | 0.001 m | |
| pot | Height | 0.250 m | 0.250 m | 0.250 m | |
| | Inner radius | 0.149 m | 0.149 m | 0.149 m | |
| | Outer radius | 0.150 m | 0.150 m | 0.150 m | |
| | Cover radius | 0.150 m | 0.150 m | 0.150 m | |
| | Thickness | 0.002 m | 0.002 m | 0.002 m | |

Table II: Description of nodes

| Node | Description | Temperature | Remark |
|------|--------------------------------|----------------|---|
| 1 | Heating source (fired-wood) | T1 | Thermal energy input to the node Q _{fuel} |
| 2 | Oven wall (Aluminum) | T ₂ | |
| 3 | Oven's inner air | T ₃ | |
| 4 | Pot's base wall | T ₄ | |
| 5 | Pot's vertical wall | T ₅ | |
| 6 | Pot's inner air | T ₆ | |
| 7 | Pot's cover | T ₇ | |
| 8 | Oven base wall | T ₈ | |
| 9 | Ambient | T ₉ | |



Fig. 2 Thermal network model for each of the above oven configurations.

Where the shape factor is given by the following relation

$$F_{ijeq} \approx \frac{1}{\frac{1}{\varepsilon_i} - 1 + \frac{1}{F_{ij}} + \frac{S_i}{S_j} \left(\frac{1}{\varepsilon_j} - 1\right)}$$
(9)

and U_{c,1-8} =
$$\frac{K_{base}}{e_{basse}}$$
 (10)

C. Energetic efficiency

Here, the instantaneous thermal efficiency of the cooker is defined as a ratio of the actual useful energy collected (Q_u) by both the pot and its inner air (nodes 4-7) to the thermal energy input of the heating source (Q_{fuel}), and is calculated as follows [34,35]:

$$\eta_{en} = \frac{Q_u}{Q_{fuel}}$$
(11)

where the instantaneous usable energy collected by both the pot and its inner air given by (equation 12)

$$Q_{u} = \frac{\left[m_{b,pot} C p_{b,pot} (T_{4} - T_{0}) + m_{s,pot} C p_{s,pot} (T_{5} - T_{0}) + \right]}{t}$$
(12)
and
$$Q_{fuel} = \eta \dot{m} PC$$
(13)

The differential system of equations (equations 1-8) obtained for any of our oven configurations, was numerically solved in Matlab software by the Runge-Kutta algorithm. Table 1 and 3 give the values of the parameters used during the simulation.

| Parameter | Value | reference |
|-----------------------|--|-----------|
| h1 | $40 (Wm^{-2} K^{-1})$ | [36] |
| h ₂ | $40 (Wm^{-2} K^{-1}).$ | [36] |
| h ₃ | 5.7+3.8V (Wm ⁻² K ⁻¹) | [35] |
| v | 0.1 m/s | [36] |
| ε | 0.8 | [35] |
| K _{alu} | 200 (W/m K) | [37] |
| ŋ | 0.75 | [38] |
| ṁ | 2.0 ^{-10⁻⁴} (Kg/s) | [39] |
| PC | 13300000 (J/Kg) | [38] |
| T ₀ | 305 (K) | [39] |
| T9 | 305 (K) | [39] |
| ρ | 740 (Kg/m ³) | [40] |
| $\mathbf{\rho}_{alu}$ | 2700 (Kg/m ³) | [37] |
| ρ_{air} | 0.9 (Kg/m ³) | [37] |
| Cp _{alu} | 896 (J/kg K) | [35] |
| Cp _{air} | 1004 (J/kg K) | [37] |
| Cpc | 1112 (J/kg K) | [37] |
| σ | $5.6 \cdot 10^{-8} (W/m^2 K^4)$ | [35] |

Table III: Simulation parameters.

IV. RESULTS AND DISCUSSION

Figures 3, 4 and 5 show respectively the plots of the temperature at each of the configurations 1, 2 and 3 of the considered ovens. In the figures 3b, 4b and 5b, we can clearly observe the thermal behaviour of the pot via the nodes 4, 5, 7 and of that of its inner air (node 6) for our different oven configurations respectively. Figures 6 and 7 display a comparative study of the thermal behaviour made at nodes 1-8 for each of the three configurations. The instantaneous comparison of the energetic efficiencies' profiles for different the 3 configurations of oven is displayed in figures 8. Finally the figure 9 shows the instantaneous energetic efficiencies' profiles for different values of the lower inner radius of the heating device.



Fig. 3 Temperature profiles at (a) all the nodes, (b) the pot nodes, for the configuration 1.



Fig. 4 Temperature profiles at (a) all the nodes, (b) the pot nodes, for the configuration 2.



Fig. 5 Temperature profiles at (a) all the nodes, (b) the pot nodes, for the configuration 3.



Fig. 6 Comparison of the temperature profiles of the (a) heating source, (b) oven's base, (c) oven wall and (d) oven's inner air, for the 3 different configurations of oven.



Fig. 7 Comparison of the (a) pot base, (b) pot side, (c) pot's inner air, and (d) pot cover temperature profiles for the 3 different configurations of oven



Fig. 8 Comparison of the instantaneous energetic efficiency profiles for the 3 different configurations of oven



Fig. 9 Instantaneous energetic efficiency profiles for different values of the lower inner radius of the heating device ($r_b=0.005m$, $r_b=0.010m$, $r_b=0.020m$, $r_b=0.120m$, $r_b=0.150m$)

According to the numerical results, from figures 3-5, we see that the highest temperature is T_1 followed by T₈ respectively for the heating source and its support surface; this is due to the energy supply at node 1 and obviously to the conductive heat transfer occurring between both node 1 and 8. It is also evident from those figures to observe that the lowest temperatures are that of the cover (see T_7), the pot inner air temperature (T_6) and the pot wall temperature (T_5) respectively. This is mainly due to the heat loses either to the ambient (by both convection and radiation) or to increase the other nodes' temperatures by the heating device during the process. The base of the pot (node 4 with the temperature T_4) is hotter than the other pot nodes because it exchanges directly with the heating source. But, the previous node remains in this situation less hot than the heating device's wall (node 2 with the temperature T_2) and its inner air (node 2 with the temperature T_2). This testifies that these results presented describe almost the behaviour of such a physical system and allow us to trust our numerical study together with the approach undertaken in this work.

The increase of the temperature with time displayed by our results is in agreement with the realities of physics as expected. According to the curves displayed in the figures 6 and 7, for each node at a particular moment, the highest temperature evolution is the one of the configuration 2 of the ovens. This is mainly due to the value of the shape factor which is higher in the said oven configuration. Observing the figure 8, we confirm that the configuration 2 of oven is the most efficient among the 3 configurations; since its instantaneous energetic efficiency profile is greater than the others' and can reach the maximum value of 10.34 % rather than 10.03 % and 9.41 % for the configuration 1 and 3 respectively; in comparison with the 13 % for a three-stone-oven tested in Uganda [41] and which generally range between 9-15 % [14]. The efficiencies of our ovens are relatively low in this situation (between 9.41 to 10.34%) due to the fact that the pots are empty (they contain only air instead of food or water). So, cooking the same dishes in the same conditions in these ovens will lead to the result that the food of the configuration 2 will cook faster than those of the configuration 1 and 3 respectively. Well observing figure 9, we realize that the efficiencies are maximum for the value $r_b=0.09m$ of the lower inner radius of the heating device, and decrease with the increase of r_b (when $r_b > 0.09m$) or with its decease (if $0 < r_b < 0.09m$).

V. CONCLUSIONS

The mathematical modelling of the heating process in a traditional firewood oven has been developed and presented by building the differential system of equations governing the heat transfer in the oven configurations considered, and using it for evaluating the systems nodes temperatures and their efficiencies. The model based on the thermal network method and the simulation, help to estimate the temperature at the considered nodes. Our results obtained are entirely satisfactory. Making a comparative study amongst the three oven configurations; it was found that the model provides a right prediction of the thermal behaviour and that the configuration 2 is the most efficient. Since the study conducted in the same conditions shows that amongst the three configurations, the thermal performances (instantaneous energetic efficiencies) obtained with the configuration 2 is always greater than the two others. Raison for which, the presented mathematical modelling is reliable for designing such rural ovens.

NOMENCLATURE

 A_1 Area of the inner base surface of the heating device (m²).

 A_2 Area of the inner wall surface of the heating device (m²).

 A_3 Area of the outer wall surface of the heating device (m²).

- A_4 Area of the outer base of the pot (m²).
- A_5 Area of the inner base of the pot (m²).

 A_6 Area of the inner wall surface of the pot (m²).

 A_7 Area of the outer wall surface of the pot (m²).

 A_8 Area of the inner cover surface of the pot (m²).

 A_9 Area of the outer cover surface of the pot (m²).

 A_{10} Area of the outer base surface of the heating device (m²).

 Cp_c Heat capacity of the combustible (J kg⁻¹ K⁻¹).

 Cp_w Heat capacity of the wall of the heating device $(J kg^{-1} K^{-1})$.

 Cp_{air1} Heat capacity of the inner heating device's air $(J kg^{-1} K^{-1})$.

 Cp_{air2} Heat capacity of the inner pot's air (J kg⁻¹ K⁻¹). Cp_{b,pot}Heat capacity of the base of the pot (J kg⁻¹ K⁻¹). $Cp_{s,pot}$ Heat capacity of the side of the pot (J kg⁻¹ K⁻¹). Cp_{cover} Heat capacity of the pot's cover (J kg⁻¹ K⁻¹).

 Cp_{base} Heat capacity of the base of the heating device (J kg⁻¹ K⁻¹).

 E_{u} Usable exergy collected by both the pot and its inner air (J)

 e_{basse} Thickness of the oven base (m)

E_{fuel} Exergy input of the heating source (J)

F_{ii} Shape factor between surface i and surface j.

 h_1 Heat transfer coefficient of the inner heating device's air (Wm⁻² K⁻¹).

 h_2 Heat transfer coefficient of the inner pot air (Wm⁻² K⁻¹).

 h_3 Heat transfer coefficient of the ambient air $(Wm^{\text{-2}}\,K^{\text{-1}}).$

 K_{alu} Thermal conductivity of the aluminium (Wm⁻² K⁻¹).

 K_{basse} Thermal conductivity of the oven base (Wm⁻² K⁻¹).

m_c Masse of the combustible (kg).

m_w Masse of the wall of the heating device (kg).

m_{air1} Masse of the inner heating device's air (kg).

m_{air2} Masse of the inner pot's air (kg).

 $m_{b.pot}$ Masse of the base of the pot (kg).

 $m_{s,pot}$ Masse of the side of the pot(kg).

m_{cover} Masse of the pot's cover (kg).

 m_{base} $\,$ Masse of the base of the heating device (kg).

m Combustion rate of the wood (kg/s).

PC Thermal power (Jkg⁻¹).

Q_{fuel} Energy supplied by the heating source (J).

r_b Lower inner radius of the heating device (m).

T_i j node's temperature (K).

 T_0 Initial temperature of the system (K).

t Time interval (s).

 $U_{c,ij}$ Conduction heat transfer coefficient between i node and j node (WK⁻¹).

V Wind speed in ms^{-1} .

ε Emissivity of the aluminum.

η Combustion efficiency of wood.

 η_{en} Instantaneous energetic efficiency of the cooker.

 σ Stefan-Boltzmann constant (Wm⁻² K⁻⁴).

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