Vibration Analysis of Beams

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***Abstract*—*This paper presents the numerical results of Vibration analysis of a cantilever beam with load at the tip and simply supported beam with the center load. Modal analysis of a cantilever beam and simply supported beam were carried out in ANSYS for different materials. The results were compared and it was found that for the same cross-section and for both configurations (i.e. cantilever and simply supported) structural steel gives higher natural frequencies.*

Keywords—*Vibration,Cantilever beam,Simply supported beam, FEM, Modal Analysis*

I. INTRODUCTION

Vibration problem occurs where there are rotating or moving parts inmachinery. The effects of vibration are excessive stresses, undesirable noise, looseness of parts and partial or complete failure of parts [1]. The structures designed to support heavy machines are also subjected to vibrations.There have been many cases of systems not meeting performance targets because of resonance, fatigue and excessive vibration of a component. In general, each vibrating structure has a tendency to oscillate with larger amplitude at certain frequencies. These frequencies are known as resonance frequencies or natural frequencies[2].

It is,therefore necessary to study these natural frequencies and find ways to avoid resonance. This paper deals with the modal analysis of acantilever and simply supported beam. For the same cross-sectionalarea, it is shown that how different materials give different natural frequencies and thus help us in choosing the best fit for our application as far as vibrations are concerned by finding ways to avoid natural frequencies near operating frequencies.

Modal Analysis: Modal analysis is used to determine the mode shapes and natural frequencies of a machine or a structure. It is the most basic form of dynamic analysis.The output of modal analysis can further be used to carry out a more detailed dynamic analysis like harmonic response analysis, transient analysis etc.

II. MODAL ANALYSIS

A. Modal analysis of a simplecantilever Beam

A simplecantilever beam was used for analysis. The dimensions of the beam were 550 x 50 x 5 mm. Modal analysis was carried out for four different materials of the beam i.e. Structural steel, Aluminium alloy, Copper alloy and Gray cast iron. The material properties of the materials are given in Table I.

Figure 1 shows the 3D model of the beam used. Fig. 2 shows the FE model of the beam. The boundary condition for cantilever beam is shown in Fig. 3.

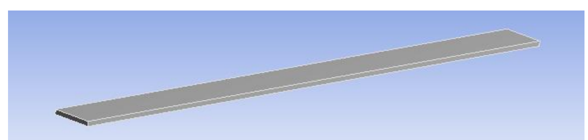


Fig. 1: 3D model of cantilever beam

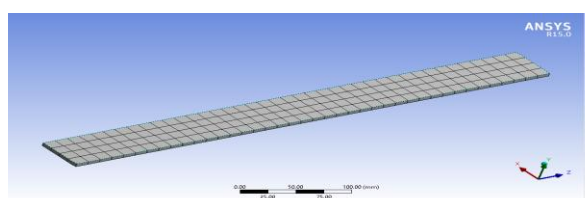


Fig. 2: FE model of the beam

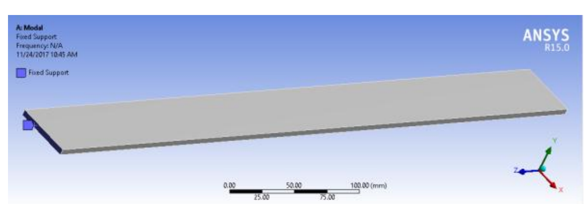
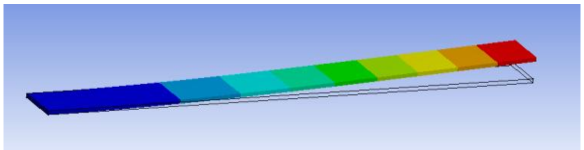
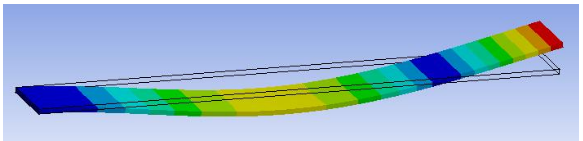


Fig. 3: Boundary condition for cantilever beam

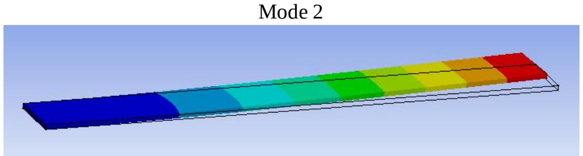
The 1st six modes of vibration were found using ANSYS, the results of the modal analysis are tabulated in Table II below.



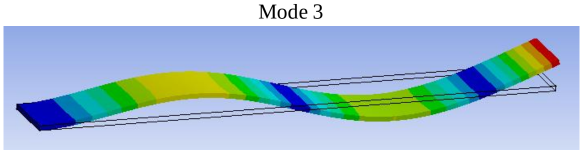
Mode 1



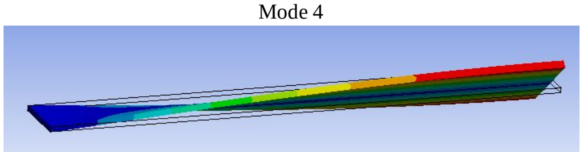
Mode 2



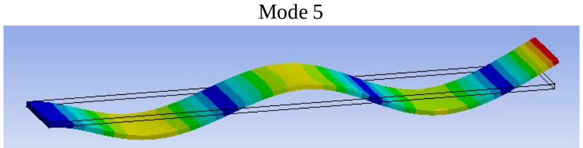
Mode 3



Mode 4



Mode 5



Mode 6

Fig. 4: Mode shapes of simplecantileverbeam

Figure 4 shows the mode shapes of simplecantilever beam from ANSYS. The wired lines in the figure show theundeformed position of the beam. Thus we can see that mode 1 is 1st bending mode, mode 2 is 2nd bending mode, mode 3 the 1st lateral bending mode, mode 4 is 3rd bending mode, mode 5 is 1st torsional mode and mode 6 is the 4th bending mode. The natural frequencies of occurrence of respective modes are given in Table II.

Modal frequencies using analytical

$$\omega_{nf} = \alpha_n \sqrt{\frac{EI}{\rho A L^4}} (1) \ldots [5], [6]$$

TABLE I: MATERIAL PROPERTIES

Materials	Young's Modulus E (GPa)	Density (kg/m^3)
Aluminium alloy	71	2770
Gray cast iron	110	7200
Structural steel	200	7850
Copper alloy	100	8300

TABLE II: NATURAL FREQUENCIES of DIFFERENT MATERIALS in (Hz)USING ANSYS

Material\ Mode No	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	13.555	13.613	9.7929	10.489
2	84.901	85.259	61.333	65.697
3	134.1	134.53	96.746	103.83
4	237.71	238.73	171.74	183.93
5	280.35	278.15	199.29	218.72
6	465.92	468.01	336.71	360.47

Where n = 1, 2, 3….for bending mode 1, mode 2 and so on.ω_{nf} = angular natural frequency of bending (rad/s), E = Young's modulus of material of the beam (Pa), I = moment of inertia of cross section (m^4), ρ = density (kg/m^3), A = area of cross section of beam, L = length of beam (m).

α_n for first five Bending modes is 1.875, 4.694, 7.855, 10.996, 14.137 respectively.

For $n > 5 \alpha_n = (2n+1)\frac{\pi}{2}$

For torsional modes, the analytical formula is given by

$$\omega_{nt} = \frac{n\pi}{2L} \sqrt{\frac{GJ_s}{J_0 \rho}} \qquad (2)$$

Where ω_{nt} = Angular natural frequency of torsion (rad/s), G = shear modulus (Pa), J_0 =Polar moment of inertia of cross section (m^4),J_s = equivalent moment of inertia of cross section due to torsion (m^4).

n = 1, 2, 3….for torsional mode 1, mode 2, mode 3 resp. and J_s for a rectangular cross section is

$$J_s = bh^3 \frac{1}{3}\left(1 - 0.63\frac{h}{b} + 0.052\left(\frac{h}{b}\right)^5\right) \qquad (3)$$

The modal frequencies using analytical are presented in Table III. We can see from Table II and Table III that the natural frequencies using modal analysis from ANSYS and using analytical formula are matching, within 3% of error thus justifying the process followed.

Comparison of Results:From Fig 5 we can observe that the natural frequency values for structural steel and aluminium alloy are almost same and on the higher side. While that of gray cast iron and copper alloy are on the lower side. The comparable frequencies of aluminium alloy and structural steel are due to their similar Young's modulus by density ratio. The higher the ratio, greater is the natural frequency.

TABLEIII:NATURAL FREQUENCIES of DIFFERENT MATERIALSin (Hz) USING ANALYTICAL

Material \ Mode No	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	13.476	13.517	9.719	10.435
2	84.458	84.713	60.914	65.402
3	134.76	135.166	97.193	104.343
4	236.508	237.222	170.578	183.146
5	274.06	271.852	194.612	213.887
6	463.47	464.87	334.273	358.9

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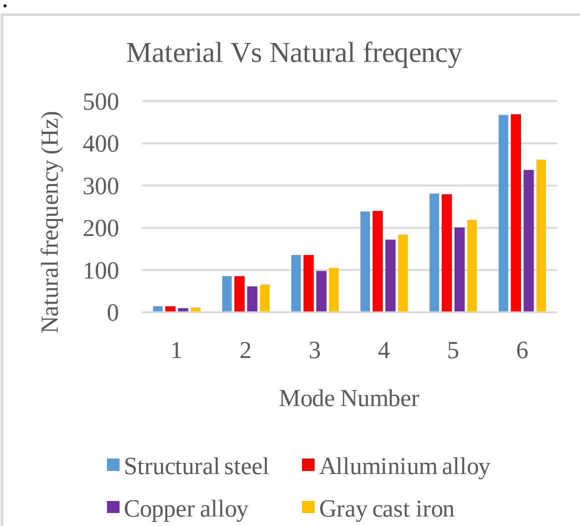


Fig. 5: Natural frequency vs mode number

B. *Modal analysis of cantilever beam with load at the tip*

Modal analysis of a cantilever beam with 0.88 kg load at the tip was carried out in ANSYS (Fig. 6). The value for weight was chosen considering a rough weight for a motor, its mounting and an eccentric weight attached to themotor shaft. This was so chosen that it coincides with a simultaneous ongoing research on Dynamic Vibration Absorber (DVA) [7].Modal analysis was carried out for the materials listed in Table I. The observed results from ANSYS are presented in Table IV.

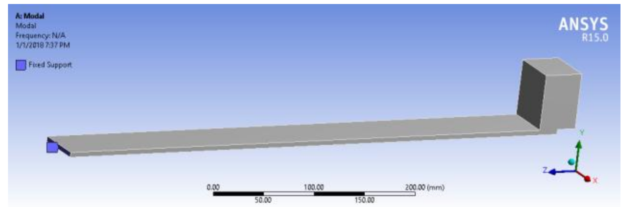


Fig. 6:Cantilever beam with load at the tip

TABLEIV:NATURAL FREQUENCIES of DIFFERENT MATERIALS in (Hz)

Material \ Mode No.	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	6.482	4.1805	4.7837	4.849
2	53.273	32.724	39.184	39.804
3	58.726	51.23	42.694	45.009
4	126.97	90.35	92.168	96.476
5	162.2	132.98	118.47	123.78
6	309.42	278.49	226.16	237.04

Comparison of Results:Natural frequency vs mode number have been compared for different materials. From Table IV and Fig. 7 we can see that there is no particular trend of increasing or decreasing order. It is observed that structural steel gives the maximum frequency.

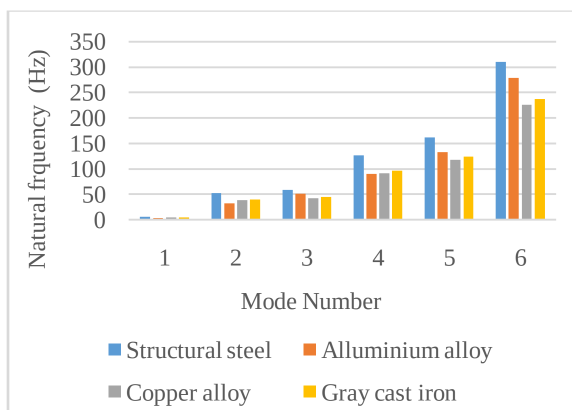


Fig. 7: Natural frequency vs mode number

C. *Modal analysis of simply supported beam*

A rectangular beam of 550x50x5 mm was used for simply supported condition analysis. Fig. 8 shows the boundary conditions that were used in

ANSYS. One end corner edge was fixed and the opposite end corner edge was given displacement constraint. The boundary conditions were so chosen to prevent rigid body motion and to get significant modes without affecting simply support condition.The modes of vibration using ANSYS are shown in Fig. 9 below.

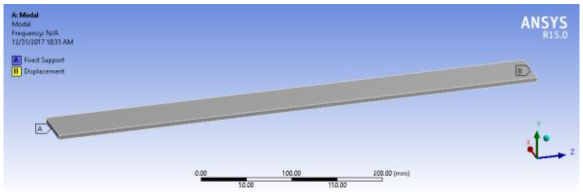
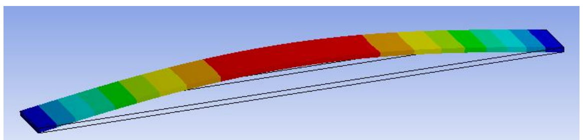
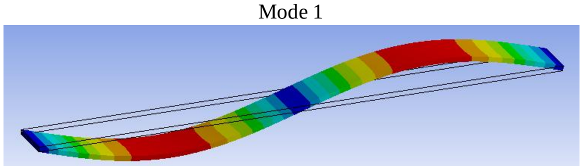


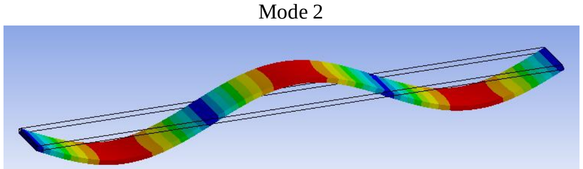
Fig. 8: Boundary condition for simply supported beam



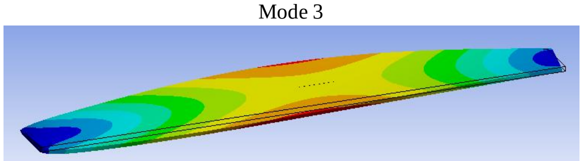
Mode 1



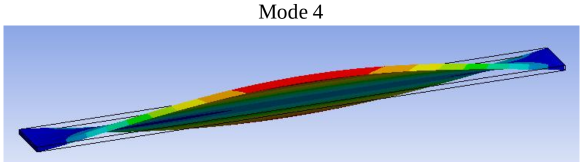
Mode 2



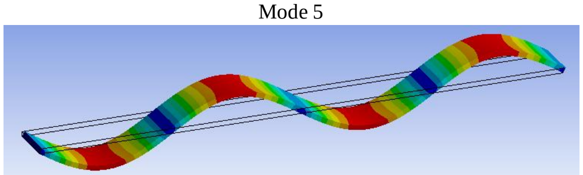
Mode 3



Mode 4



Mode 5



Mode 6

Fig. 9: Mode shapes of simply supported beam

The wired lines in the above figure showundeformed position of the beam. It can be seen that mode 1 is 1st bending mode, mode 2 is 2nd bending mode, mode 3 is 3rd bending mode, mode 4 is 1stcombined bending-torsion mode, mode 5 is 1st torsion mode and mode 6 is the 4th bending mode. The Natural Frequencies of occurrence of respective modes are tabulated in Table V.

Modal analysis was carried out for 4 different materials listed in Table I.

TABLE V:NATURAL FREQUENCIES of DIFFERENT MATERIALS in (Hz) USINGANSYS

Material \ Mode No.	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	37.833	37.95	27.289	29.296
2	151.34	151.84	109.19	117.17
3	340.48	341.7	245.75	263.57
4	497.8	497.98	357.74	386.13
5	558.32	554.95	397.89	435.11
6	604.95	607.35	436.87	468.18

Modal frequencies using analytical

$$\omega_{nf} = (n\pi)^2\sqrt{\frac{EI}{\rho AL^4}} \quad (4) \ldots [5], [6]$$

Where n = 1, 2, 3… for bending modes. For Torsional modes, the analytical formula is given by

$$\omega_{nt} = \frac{n\pi}{L}\sqrt{\frac{GJ_s}{J_0\rho}} \qquad (5)$$

Where n = 1, 2, 3….for torsional mode 1, mode 2, mode 3 resp. and J_s for a rectangular cross section is

$$J_s = bh^3\frac{1}{3}\left(1 - 0.63\frac{h}{b} + 0.052\left(\frac{h}{b}\right)^5\right) \qquad (6)$$

The modal frequencies using analytical are presented in Table VI below. The mode 4 shown in Fig. 8 which is a combination of bending and torsional mode was not found using the analytical method.

We can see from Table V and Table VI that the natural frequencies using modal analysis from ANSYS and using analytical formula are coinciding, within 3% of error thus justifying the process followed.

TABLE VI:NATURAL FREQUENCIES of DIFFERENT MATERIALS in (HZ) USING ANALYTICAL APPROACH

Material \ Mode No.	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	37.832	37.946	27.285	29.296
2	151.326	151.783	109.142	117.183
3	340.484	341.512	245.569	263.661
5	548.119	543.704	389.224	427.772
6	605.305	607.132	436.567	468.731

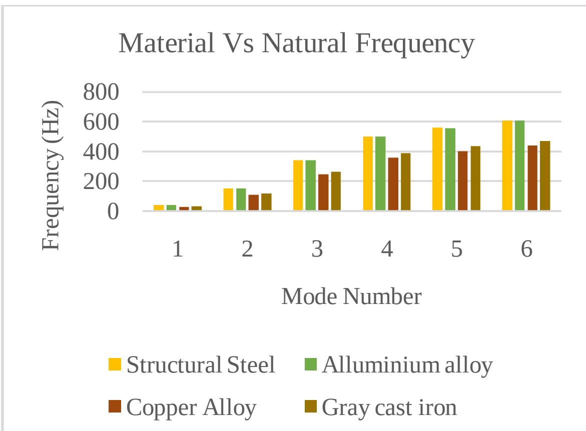


Fig. 10: Natural frequency vs mode number

Comparison of Results:Fig. 10shows that the natural frequency values for Structural steel and Aluminium alloy are almost same and on the higher side as was also observed incase of cantileverbeam. While that of gray cast iron and copper alloy are on the lower side.

D. *Modal analysis of simply supported beam with load at the center*

Modal analysis of a simply supported beam with 0.88 kg load at the center was carried out in ANSYS. Modal analysis was carried out for the materials listed in Table I. The setup for the beam is shown in Fig. 11. One edge at one corner was fixed and the other edge at the other end was given displacement constraints to prevent the rigid body modes from surfacing in the analysis without affecting the significant modes of simply support condition.

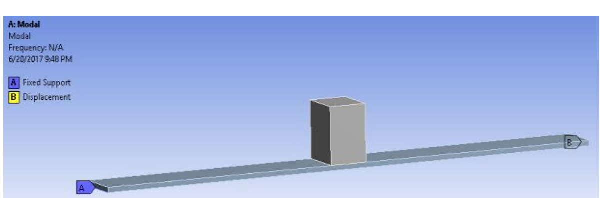


Fig. 11:Simply supported beam with center load

The first six modes of vibration for structural steel are shown below in Fig. 7.

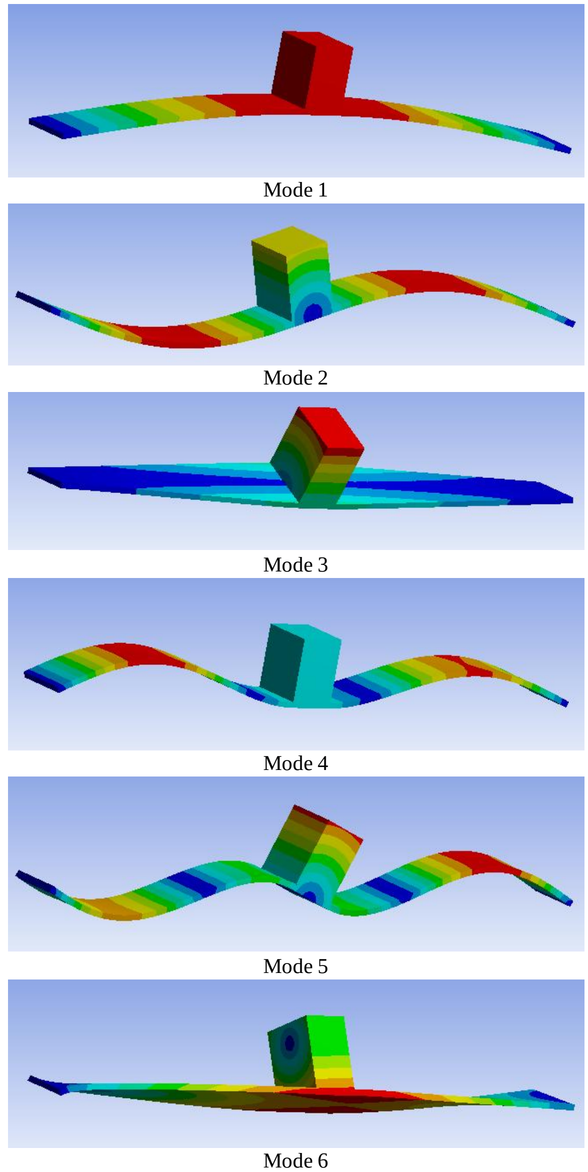


Fig. 12: Mode shapes of simply supported beam with center load

It can be seen from Fig. 7 that first mode shape is the first bending mode and occurs at a frequency of 25.576 Hz as seen from Table V. The second mode shape is the second bending mode. The third mode shape shows the first twisting mode occurring at 148.62 Hz. The fourth and fifth mode shapes correspond to third and fourth bending modes respectively while the sixth mode shape is the second twisting mode.

Table VII shows the natural frequencies against mode number for different materials.

Table VII:NATURAL FREQUENCIES of DIFFERENT MATERIALS in (Hz)

Material\ Mode No	Structural steel	Al alloy	Copper alloy	Gray cast iron
1	25.576	17.694	18.797	19.263
2	134.87	89.743	97.945	103.35
3	148.62	112.69	108.72	111.39
4	300.14	284.83	217.96	230.76
5	410.94	328.62	301.3	311.28
6	421.16	336.16	307.68	323.74

Comparison of Results:Natural frequency vs mode number have been compared for different materials. From Table VII and Fig. 13 we can see that Structural steel is found to give higher natural frequencies for a simply supported beam with load at the center. Copper alloy,on the other hand, gives lower values of natural frequencies for the same.

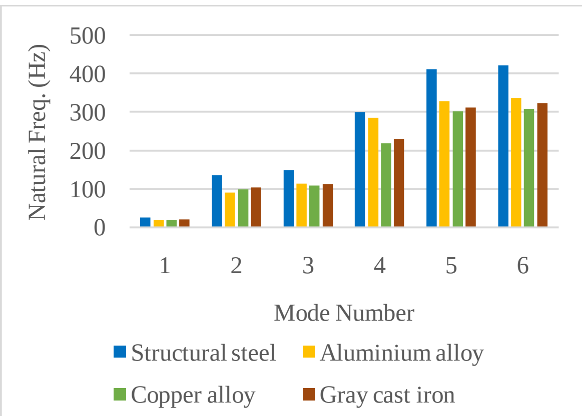


Fig. 13: Natural frequency vs mode number

III. CONCLUSION

Modal analyses of the cantilever and the simply supported beams were carried out in unloaded and with load conditions in ANSYS for 4 different materials i.e. structural steel, aluminium alloy, copper alloy and gray cast iron.

- For unloaded cantilever and simply supported condition of the rectangular beam, it was observed that structural steel and aluminium alloy consistently gave higher natural frequencies than copper alloy and gray cast iron. This is due to their similar Young's modulus to density ratio, higher this ratio greater the natural frequency.
- For cantilever beam condition with load at the tip and simply supported condition with center load structural steel gave maximum natural frequencies.
- Since the material assigned to weight was structural steel whose density is much greater than aluminium, it resulted in lower mode 2 natural frequencies of beam assigned aluminium alloy due to lower stiffness by massratio as the mass increased more compared to stiffness. (Fig. 7 & Fig. 13).

References

[1] Grover G.K., *"Mechanical Vibrations"*, Nem Chand & Bros, Eighth Edition, 2009.

[2] Rao S.S., *"Mechanical Vibrations"*, Wesley, Third Edition, 1995.

[3] ANSYS WORKBENCH, Reference guide, Chapter-1, Page-3, 2014.

[4] Ravi Prasad and D.R. Seshu, *"A study on dynamic characteristics of structural materials using modal* analysis",*Asian Journal of Civil Engineering*, Volume 9, Number 2, pp. 141-152, 2008.

[5] *Free Vibrations of a Cantilever Beam* (Continuous System) [Online]. Available:http://iitg.vlab.co.in/?sub=62&brch=175&sim=1080&cnt=1

[6] Mauro Caresta, "*Beam Vibrations"* , [Online]Available: https://www.colorado.edu/physics/phys1240/phys1240_fa15/homelabs/Beam_vibration_Mauro%20Caresta_UNSW.pdf

[7] Abdullah Ö., Mojtaba G., Akio S., Ashraf S. and Mohammed N.A.S., "*Design and Experimental Implementation of a Beam-Type Twin Dynamic Vibration Absorber for a Cantilevered Flexible Structure Carrying an Unbalanced Rotor: Numerical and Experimental Observations"*, Hindawi Publishing Corporation, *Shock and Vibration*, July 2015.