Survey of Orthogonal Frequency Division Multiplexing

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Abstract

In the development of wireless communication technology Orthogonal frequency division multiplexing (OFDM) played significant roll. In this paper author is trying to incorporate the complete journey of development of OFDM technique since its inception. This complete development probably helps researchers to understand the growth of wireless Communication. Probably inclusion of limitations will also help to understand the simultaneous development of other technologies parallel to OFDM.

Key Words: *OFDM, ISI, ICI, PAPR*

1. Introduction

For a long period of time OFDM was the most popular method for broadband wired and wireless communication. The OFDM development started in late 1950's with the introduction of frequency division multiplexing. OFDM was the popular choice of point to point communication. It faces challenges in implementation of multi user communication network. Orthogonal frequency division multiple access (OFDMA) network was developed for the purpose of implementation of multi user network. In 1966 the structure and concept of OFDM was developed by using orthogonal overlapping multi-tone signals. The target area of development in multi user communication was perfect carrier synchronization between users to avoid overlapping. It created complexity at the receiver side in terms of order of magnitude of signals.

In 1971 the idea of implementation of generation and reception of OFDM signals introduced by using Discrete Fourier Transform (DFT).It was eliminated the requirement of banks of analog subcarrier oscillators. In further growing steps OFDM implementation was done with the help of FFT. Limited performance of OFDM observed over a set of noncontiguous frequency band known as carrier aggregation. During the journey of development of OFDM signal few important parameters have been analyzed and the performance optimization of these parameters developed the better performing OFDM system.

Organization of this paper is according to the development of optimization of few important parameters like Inter Symbol Interference (ISI), Inter Carrier Interference (ICI) and Peak Average Power Ratio (PAPR) that will reflect performance OFDM.

2. Inter Symbol Interference (ISI)

Interference of symbols whenever continuous symbols transmitted through transmitter is called Inter symbol Interference (ISI). To understand the ISI let us consider dispersive channel model[1]:

$$r(t) = u(t) * h_c(t) + n(t) \qquad (1)$$

Where $u(t)$ is the transmitted signal, $h_c(t)$ is the impulse response of the channel and $n(t)$ is Additive white Gaussian noise (AWGN) with power spectral density of $N_0 \,/\, 2$. The transmission of sequence of symbols with the basic waveform $u(t)$; the nth symbol b_n sent as $b_n u(t - nT)$, where T is the symbol interval. Based on the dispersive channel model.

$$r(t) = \sum b_n v(t - nT) + n(t) \qquad (2)$$

Where $v(t) = u(t) * h_c(t)$ is the received symbol. Demodulation of symbol is done by matched filter at the receiving end. Therefore for the sequence of symbols output of matched filter at $t = mT$ is

$$z_m = \sum b_n * \tilde{v}(mT - nT) + n_m \qquad (3)$$

$$= b_m \|v\|^2 + \sum_{n \neq m} b_n * \tilde{v}(mT - nT) + n_m \qquad (4)$$

Where n_m is zero mean Gaussian random variable with variance $N_0\, \|v\|^2/2$. The first term of above equation shows desired signal contribution due to symbol b_m and the second term shows contribution from the other symbols in the same time duration.

This unwanted contribution from the other symbols are called inter symbol Interference. OFDM signals easily overcome the issue of ISI by using guard bands that avoid overlapping of symbols.

3. Inter Carrier Interference (ICI)

OFDM signals are complex baseband signals[2], Let

$$S(t) = S_I(t) + S_Q(t) \tag{5}$$

Where $S_I(t)$ is the inphase component of the baseband signal and $S_Q(t)$ is the quadrature component of baseband signal. Therefore,

$$S(t) = \sum_{k=0}^{N-1}(R\{A_k\}cos2\pi f_k t - I\{A_k\}sin2\pi f_k t) + j\sum_{k=0}^{N-1}(I\{A_k\}cos2\pi f_k t + RAksin2\pi fkt \tag{6}$$

where, $R\{A_k\}$ and $I\{A_k\}$ represents real and imaginary parts of the symbolA_k. Before the transmission of baseband signal it is modulated by carrier signal and pass-band selected for the transmission will be

$$S_p(t) = \sum_{k=0}^{N-1}(R\{A_k\}cos2\pi(f_c + f_k)t - I\{A_k\}sin2\pi(f_c + f_k)t) \tag{7}$$

OFDM system is sensitive to phase offset and frequency offset caused by Doppler frequency shift and multipath fading. Consider OFDM signal

$$S(t) = \sum_{k=0}^{N-1} A_k e^{i2\pi f_k t} \qquad 0 \le t \le T_s \tag{8}$$

where, $f_k = f_0 + K\Delta f$. Multiplicative time varying distortion $\gamma(t)$ caused by either frequency offset or Doppler spread. Then the received signal is multiplication of time varying distortion and transmitted OFDM signal. Demodulation of the signal is DFT operation but due to large number of samples integral function is considered here

$$R_m = \frac{1}{T_s}\int_0^{T_s} r(t)e^{-j2\pi f_m t}dt \tag{9}$$

$$R_m = \frac{1}{T_s}\int_0^{T_s} \gamma(t)\sum_{k=0}^{N-1} A_k e^{i2\pi f_k t}\, e^{-j2\pi f_m t}dt \tag{10}$$

$$R_m \sum_{k=0}^{N-1}\left\{\frac{1}{T_s}\int_0^{T_s}\gamma(t)e^{-j2\pi(f_m-f_k)t}dt\right\}A_k \tag{11}$$

$$R_m = a_0 A_m + \sum_{k\neq m} a_{m-k}A_k \quad (m-k=l) \tag{12}$$

where, $a_l \triangleq \frac{1}{T_s}\int_0^{T_s}\gamma(t)e^{-j2\pi l\Delta f t}dt$ and a_0 is a complex number, whose amplitude and phase represent the attenuation and phase shift of the desired signal. If $\gamma(t)$ is changes with respect to the time then a_l for all $l \neq 0$ are complex gain of the ICI. Since $\gamma(t)$ is a stochastic process and a_l is a random variable with zero mean and variance$\sigma_l{}^2 = E\{|a_l|^2\}$. Total power of ICI is

$$P_{ICI} \triangleq E|\textstyle\sum_{l\neq 0} a_l A_{m-l}|^2 \tag{13}$$

There are four different approaches are presented to mitigate the effect of ICI, which includes CFO estimation and removal [3-5], frequency domain equalization [6], time domain windowing [7] and ICI cancellation. CFO estimation scheme uses training sequences such as pilot symbols [8-10]; performed in two steps coarse and fine frequency estimation. The frequency domain equalization scheme requires highly complex matrix inversion whereas time domain windowing uses pulse shaping scheme for reducing the effect of ICI. In a similar manner prototype filter design using high side-lobe falloff rate (SLFOR) combinational window function with a bisection type optimization algorithm[11] and pseudo-transmultiplexer using Blackman window family[12] also gives optimized performance of ICI.

ICI cancellation scheme have two broad areas which includes ICI self-cancellation[13-14], ICI conjugate cancellation [15]. In ICI self-cancellation data symbols are repeated on multiple adjacent subcarriers using polynomial coding, but degradation of CIR has been found for higher values of CFO. To improve the performance of CIR new ICI self-cancellation scheme is used that repeats the data symbols on two symmetrically located subcarrier using polynomial coding. It can be further improved by windowing technique at the transmitter and receiver. In ICI conjugate cancellation scheme time domain OFDM signal and its conjugate are transmitted over two different paths and combined at the receiver end to improve the Bit Error Rate (BER) performance of the system.

ICI cancellation for OFDM with delay diversity[16] was proposed. In this method modified delay diversity called time domain self-Interference cancellation (TDSIC) was proposed. It was a new diversity collection scheme at the receiver end, which can be used to improve the system performance by suppressing ICI in time variant channel. In this particular method OFDM symbols repeated by predefined delay (D). In the above paper OFDM with Delay diversity is compared with the $1/2$ rate polynomial cancellation coding (PCC-OFDM); fD_{max} beyond 0.25 TDSIC systems has given better performance than $1/2$PCC-OFDM. It can also be seen that better ICI performance gain can be achieved with loss of frequency efficiency by 50%. TDSIC system provides better frequency efficiency than PCC-OFDM for same number of sub-carriers.

Conventional ICI self-cancellation scheme modulates a single data with group of subcarriers. A kernel based ICI self-cancellation scheme using constrained subcarrier combiners (CSC) is proposed[17]. In this scheme kernel-based precoder is used to map the data symbols and on the receiver side Carrier-frequency-Offset (CFO) estimation develop a set of CSC to eliminate intra group interference. Interference is classify into two parts: i.e. Inter-group interference and Intra-group interference. Suppression of inter-group interference is achieved by smooth response, while the suppression of intra-group interference multiple-signal-classification (MUSIC) algorithm is used to estimate the CFO and based on this estimation CSC is developed.

It is found that for the throughput of 2.4 bits/subcarrier the performance of this method is 60% better than conventional method. It is useful to achieve high spectral efficiency in an OFDM system by mapping $L_P \leq L-1$ data symbols to a group of L consecutive subcarriers.

On the basis of similar methodology, a Novel Nonlinear Constellation Precoding for OFDM systems with Subcarrier grouping [18] is proposed. In this scheme full set of subcarriers are splitted into smaller groups coded by maximum distance separable (MDS) encoder, A maximum-likelihood (ML) decoder is used to decode the subcarriers.

A novel NCP technique based on MDS code provides the flexibility of supporting any number of diversity channels and desired diversity order. Diversity order is basically minimum hamming distance between two vectors of constellation points. In OFDM system, three diversity channel joint modulation techniques (3-DCJMT-OFDM) considered for novel NCP. The analysis of BER performance of novel NCP based 3-DCJMT-OFDM design with and without ICI for $S_3 - Type1, S_3 - Type2, S_3 - Type3$ [19].

Another latest technique of ICI reduction in OFDM system using IMBH (Improved Modified Bartlett-Hanning) pulse shape [20]. Here IMBH pulse shapes give an improvement of 2 to 3 dB in ICI with different β(window shape parameter), and α(roll − off factor), These parameter decide the shape of window and its performance in an OFDM respectively. To evaluate performance of the OFDM system normalized frequency offset (ε) and exponential power of function (n) are also considered along with β and α. The performance of OFDM system with 64 sub-carrierare better than BTRC (better than raised cosine) at $0 < \varepsilon < 0.175$, for average ICI power IMBH pulse shape at $n = 1$ and $\beta = 1.04$.

4. Peak Average Power Ratio (PAPR)

A multicarrier signal is the sum of many independent signals modulated onto the sub-channel of equal bandwidth [21]. The collection of all data symbol is $A_k, k = 0, 1, 2, \ldots, N-1$, as a vector $A = [A_0, A_1, A_2, \ldots, A_{N-1}]$that will be termed a data block. The multicarrier signal consisting of N subcarrier is given by,

$$a(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} A_n e^{j2\pi m \Delta f t}, 0 \leq t \geq NT \qquad (14)$$

where $j = \sqrt{-1}$, Δf is the subcarrier spacing and NT is useful data block period. In OFDM the subcarriers are orthogonal, i.e. $\Delta f = 1/NT$. The PAPR of the transmit signal is defined as

$$PAPR = \frac{\max_{0 \leq t < NT} |a(t)|^2}{1/_{NT} \int_0^{NT} |a(t)|^2 \, dt} \qquad (15)$$

For the further analysis of PAPR; approximation applied in which NL equidistant samples of $a(t)$ will be considered where L is an integer (L ≥ 1). These time domain signal samples are represented as a vector $A = [A_0, A_1, A_2, \ldots A_{N-1}]^T$ and can be calculated as

$$a_k = a(k\ T/L) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{\frac{j2\pi kn\ \Delta f T}{L}} \qquad (16)$$

where, k = 0, 1,…NL-1. The variable a_k can be interpreted as the inverse discrete fourier transform (IDFT) of data block A with (L-1)N zero padding. Precise calculation of PAPR of continuous-time signal by the use of Nyquist rate sampling cannot possible. The PAPR computed from the L-times oversampled time domain signal samples is given by

$$PAPR = \frac{\max_{0 \leq t \geq NL-1} |a_k|^2}{E[|a_k|^2]} \qquad (17)$$

where E[.] denotes expectation of average value. The mathematical definition of ICI and PAPR is discussed in section II and III. However, in practice some optimization tool is required to get optimum solution. The next section is discussed the conventional optimization techniques.

The simplest technique of PAPR reduction is amplitude clipping in which peak amplitude limited by clipping it with some predefined value [22-23]. But distortion caused by amplitude clipping can be considered as another source of noise. To avoid this noise clipping and filtering method is used; but it is again responsible for the regrowth of amplitude. In coding method codes are transmitted to minimize the effect of PAPR [24-25]. However, this method suffers from the search of the best code and to store large look-up tables for encoding and decoding. Tone reservation (TR) and Tone injection (TI) [26] are based on adding a data block in time domain to the actual multi-carrier signal to reduce its peak. These time domain signals can be easily computed in transmitting end and easily removed at the receiving end.

In Selected Mapping (SLM), many independent frequency domain OFDM signals are created after multiplication of information data block with a phase sequence set, in which lowest PAPR is selected for transmission [27-28]. In Partial Transmits Sequence (PTS), Information data block is partitioned into different sub-blocks and these blocks multiplied with phase rotation factor then combined to minimize the PAPR [29-30]. Comparison chart of these methods are given in Table 1, on the basis of Power increment in transmit signals, and increase in BER at the receiver and loss in data rate.

Table 1 Comparison chart between different methods of PAPR reduction

Sr. No.	Method	Distortion in signal	Power increment in signal transmission	Loss in data rate
1.	Clipping and filtering	Yes	No	No
2.	Coding	No	No	Yes
3.	Tone Reservation (TR)	No	Yes	Yes
4	Tone Injection (TI)	No	Yes	No
5.	Selected Mapping (SLM)	No	No	Yes
6.	Partial transmit signal (PTS)	No	No	Yes

Combination of SLM and PTS schemes with minimum computational complexity using hybrid techniques [31] was proposed. Basically hybrid techniques are available for Conventional Hybrid schemes(CH), Additional Hybrid schemes(AH), Switching Hybrid schemes(SH) and modified Hybrid schemes(MH). Here Complementary cumulative distribution function (CCDF), which denotes the probability that the PAPR of an OFDM symbol exceeds the given threshold PAPR0. In the given hybrid schemes original OFDM symbol is multiplied with the phase sequences and then each of new OFDM symbols is partitioned into pairwise disjoint sub-blocks. The MH scheme is having better PAPR reduction capability than all other scheme. It is important to mention here that SLM and PTS are the promising schemes of PAPR reduction. In these schemes information about the phase factor is required at the receiver known as side information (SI), The SI has to be recovered from the received OFDM signal for data recovery.

Now there is another technique that is precoding based PAPR reduction, which is a linear technique to implement without the need of any side information (SI). Walsh-Hadamard transform (WHT)[32], Discrete-Hartley transform (DHT)[33], and Zadoff-chu matrix transform (ZCMT) [34] are the some other new techniques for reduction of PAPR.A new precoding technique has been proposed based on vandermonde-like matrix (VLM) and SLM [35]. In this technique VLM precoding reduces the auto-correlation of input sequences while SLM utilizes the sensitivity of the signal towards to the phase shift. This technique can achieve 0.2dB PAPR gain over DCT-SLM, 1.3dB gain over WHT-SLM and 2.3dB gain over conventional SLM at CCDF = 10-3.

5. Conclusion

The complete study discussed above is about the development of OFDM. Still the work on the behavior of OFDM is going on to develop a better wireless communication system. Mean while work on Filter Bank Multicarrier (FBMC) is also going on as an alternative of OFDM technology. It will support the emerging areas of multiuser and Massive MIMO applications which is thrust area of wireless communication technologies now days. Probably this study helps the research to understand the development of OFDM communication system.

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