Convectional stability of thermohaline fluid in solar pond under vertical magnetic field

Anoop Kumar[#1], Surjeet Singh[*2], V. Kanwar[#3]

[#]*M.E. (Mechanical) UIET Panjab University Chandigarh, India*
[#]*Assistant Proff. UIET Panjab University Chandigarh, India*
[#]*Assistant Proff. UIET Panjab University Chandigarh, India*

ABSTRACT -
In the present paper, we study working of solar pond and discussed the principle of exchange of stability with reference to boundary conditions of a solar pond with lower boundary rigid and upper boundary dynamically free and there is continuous vertical upward magnetic field knows as generalized magnetohydrodynamicBérnard convection problem in the case when Lewis number ($\tau = \frac{\eta_1}{\kappa}$) not equal to zero, means there are both thermal as well as solute (salt) diffusion.

We derived new modified Rayleigh number denoted by $R = \left[R_1(1-\alpha_2 T_0) - \frac{R_2}{\tau} + T_0 \hat{\alpha}_2 R_1 R_3\right]$ and extended modified Rayleigh number $R' = R_1(1-\alpha_2 T_0 + T_0 \hat{\alpha}_2 R_3) - \frac{R_2}{\tau} - Q\Pi^2]$, we further study the effect of coupled magnetic field, temperature and concentration gradient on convectional stability of thermohaline fluid and found that role of magnetic field and concentration gradient is to stabilize and temperature gradient causes both stabilization and destabilization depending upon ratio of coefficient of specific temperature and concentration is greater or less than R_3.

***Index Terms*—**Solar *pond, Magnetic field, Rayleigh number, Lewis number, Bérnard convection.*

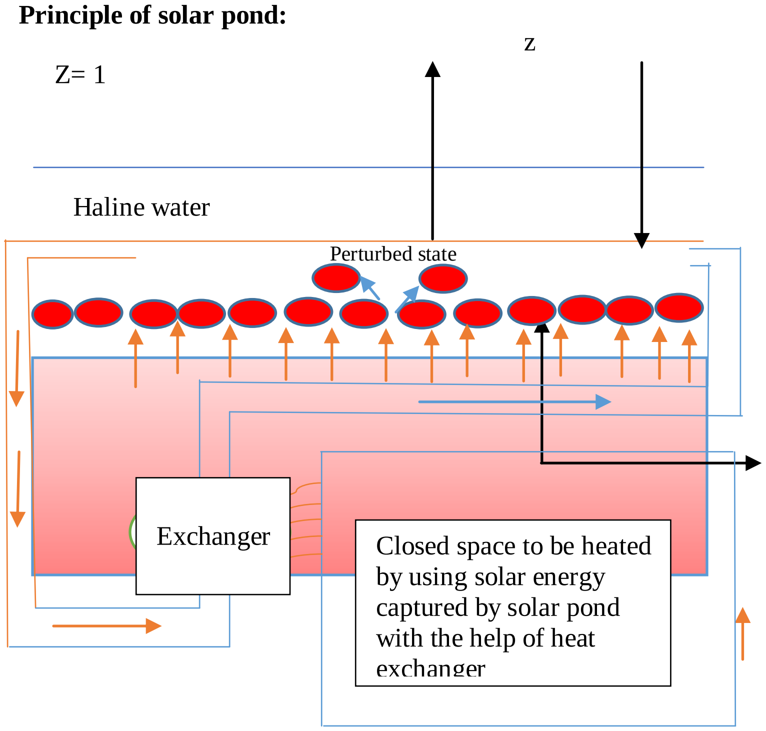
I. INTRODUCTION:

In today's scenario renewable energy is becoming an important source of energy because of its zero environment effect and ease of availability at every part of earth, In India our most of electricity is generated through thermal power plant in which primary source of heat is fossil fuel, At the same time burning of fossil fuels releases carbon dioxide and other toxic gases results in greenhouse effect as a result increase in average temperature of earth and damage to the environment is long lasting. With only finite source of fossil fuel and exponential increase in the demand of power due to increase in the human population, power generation from renewable energy promises a sustainable future for mankind.

India's most of energy requirement is fulfil through coal and hydroelectricity as a result we are facing a challenge of global warming so now it is very important to reduce the consumption of non-renewable resource and move towards renewable energy resources. Renewable energy sources have the ability to replace fossil fuels and fulfil our energy requirement, lot of research are ongoing on renewable energy sources one of them is sun light called solar energy

Solar Energy: Solar energy is the energy received from sun in the form of solar radiation. Sun can be assumed as infinite source of solar energy (life 4.6 billion years) as compare to fossil fuels and it is available at all parts of earth, the countries which are in tropical zone have special advantage of solar energy, tropical region capture around 70-80% of solar radiation incoming from sun and India being one of tropical country have clear future vision if we shift renewable energy. For trapping of solar energy now a days solar ponds are becoming popular because of their simple structure and have ability to store solar radiation energy for long time at low input cost.

Principle of solar pond:



Solar pond physical system and geometry

Solar pond consists of mainly three layers they are -

(a) Upper convecting layer: It is a layer of fresh water and having temperature near to ambient temperature. It is lowest density layer comparing to other two layer.
(b) Non convecting layer: In this layer density as well as temperature of fluid increases downward. There exist temperature and salinity gradient.
(c) Bottom convecting zone: It is the lowermost part of pond having maximum salinity, below this level black surface painting is done to absorb maximumamount of radiation, temperature in this region reaches up to 90 degree centigrade.

In solar pond heavy saline water is present at the bottom of the pond and its density is highest out of three layers, sun light is trapped by lower layer of fluid but it will not rise upward due to its high density compared to middle fluid layer, middle fluid layer act as a non convecting zone which avoids intermixing of upper and lower layer and maintain high temperature below it. When heating of lower layer continues its temperature increase up to 90 degree centigrade, at this temperature chance of intermixing increases to avoid this energy of hot fluid present at the bottom is removed through conduction by supplying of cold water or other fluid through it.
In solar pond double diffusion of salt and heat takes place heat diffuses from bottom to top side and salt also diffuses from high density to low density either from bottom to top, diffusion of salt generates instability in the system and affects the performance of solar pond so in present paper we try to study exchange of stability of thermohaline fluid in presence of magnetic field also known as principal of exchange of stability of magneto hydrodynamics thermohaline fluid.

[2] Literature review:

Convection is the concerted collective movements of groups or aggregate of molecules in the fluid. Bérnard [1] studied the onset of thermal instability and the role of viscosity in thermal convection. Rayleigh–Bérnard [2] convection is a type of natural convection, occurring in a plane horizontal layer of fluid heated from below, in which the fluid develops a regular pattern of convection cells known as Bernard cells. In 1916, Lord Rayleigh [3] conducted a theoretical study and developed a mathematical theory for the convection phenomenon and found that Instability is found to be due to the buoyancy effect arising when heating is from below and give a dimensionless number Rayleigh number(Ra). Rayleigh number is a dimensionless number associated with buoyancy-driven flow, also known as free convection or natural convection. When the Rayleigh number is below a critical value for that fluid, heat transfer is primarily in the form of conduction, when it exceeds the critical value, heat transfer is primarily in the form of convection. Two fundamental configurations have been studied in the context of thermohaline instability problem, the first one by stern [4] wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and second one by Veronis [5], wherein the temperature gradient is stabilizing and concentration gradient is destabilizing. The main results derived by Veronis and Stern for their respective configurations are that both allow the occurrence of a stationary convection or an oscillatory convection of growing amplitude, provided the destabilizing temperature gradient or concentration gradient is sufficiently. Veronis focuses his work on free boundaries, whereas Stern's work uses the "principle of exchange of instabilities. Following from Stern's and Veronis' studies on oscillatory motion, Gupta *et al.* [6] went on further to prescribe upper limits for these oscillatory motions in both these configurations. Furthermore, they showed sufficient conditions for stability using the thermal Rayleigh number, R_t and the salinity Rayleigh number (R_s). Chandrasekhar's [7] interest in the effect of magnetic field on the dynamic stability of a convective system led to investigations of the effect on the Rayleigh-Taylor [8] instability and the Kelvin Helmholtz [9] instability. In the end he organized and compiled his results in a monumental tome entitled Hydrodynamic and Hydro magnetic Stability(Chandrasekhar 1961).Nielsen and Rabal [10] and Rabal [11] have studied the time as a key factor for convective stability in the solar pond. They applied Weinberger's stability criteria to determine the growth and shrinkage of convective layers. Gupta and Rana [11] derived that principle of exchange of stabilities is not valid for generalized magneto hydrodynamic Bernard convection for the case where mass diffusivity is neglected either ($\tau = 0$).
In the present paper, we discussed the case of Gupta and Rana with reference of solar pond when mass diffusivity is taken i.e. ($\tau \neq 0$) in the presence of vertical magnetic field.

[3]Mathematical formulation and analysis:

Modified thermohaline instability problem of Veronis type in the presence of a uniform magnetic field acting parallel to the gravity in non-dimension form are given by -

$$(D^2 - a^2)(D^2 - a^2 - \frac{p}{\sigma})\omega$$

$$= R_1 a^2 \theta - \frac{R_2}{R_3} a^2 \phi - QD(D^2$$

$$- a^2)h_z \quad (1)$$

$$[(D^2 - a^2) - p(1 - \alpha T_0)]\theta - (T_0\ \hat{\alpha}_2 p\phi)$$

$$= (1 - \alpha_2 T_0)\omega - (T_0 \hat{\alpha}_2 R_3 \omega) \quad (2)$$

$$[\tau(D^2 - a^2) - p]\phi = -R_3\omega \quad (3)$$

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)h_z = -D\omega \quad (4)$$

Possible Boundary conditions for different boundaries are given as:

$$\omega = 0 = D^2\omega = D\theta = D\phi = h_z \quad at \ z = -1 \ and \ z = +1$$

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$$\omega = 0 = D^2\omega = D\theta = D\phi \ \ and \ Dh_z = \pm az \quad at \ z = -1 \ and \ z = +1$$

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With $\omega = 0 = \theta = \phi$ on both the boundaries, $D^2\omega = 0$ is true everywhere on the boundary which is tangent stress free, $D\omega = 0$ on a rigid boundary, $h_z = 0$ holds true if the regions outside the fluid is perfectly conducting, $Dh_z = \pm az \ at \ z = -1 \ and \ z = +1$ only when regions outside the fluid are insulating.

Meaning of the symbol in the given boundary conditions are as follows - z is the vertical coordinate where $z = -1 \ and \ z = +1$ represent the two boundaries, $D = (d/dz)$ is differential in the z direction, $\omega \rightarrow$ vertical velocity, $a^2 \rightarrow$ square of horizontal wave number, $h_z \rightarrow$ vertical magnetic field, $\theta \rightarrow$ temperature, $\phi \rightarrow$ concentration, $\sigma = \nu/\kappa \rightarrow$ thermal Prandtl number, $\sigma_1 = \nu/\eta$ is the magnetic Prandtl number $Q = (\mu^2 H^2 \sigma d^2)/(\rho\nu) \rightarrow$ Chandrasekhar number, $R_1 =\rightarrow$ Rayleigh number, $R_2 \rightarrow$ concentration Rayleigh number, $R_3 \rightarrow$ ratio of concentration gradient to temperature gradient, $\tau = \eta_1/\kappa \rightarrow$ Lewis number which is equal to the ratio of mass diffusivity to heat diffusivity, $\alpha_2 \& \hat{\alpha}_2$ are respectively the coefficient of specific variation due to temperature and concentration variations.

$p = p_r + ip_i$ is the complex growth number where p_r and p_i are real constants and a given normal mode is stable, neutral or unstable if p_r is negative, zero or positive respectively. If $p_r = 0$ for all $a^2 > 0$, then the principle of exchange of stability is valid otherwise, we will have overstability at least when instability sets in as certain modes. We assume solar pond length in x and y direction large enough such that all parameter does not change with time in these directions we only study in z direction with definite of solar pond in this direction is d and also assume that density gradient of solar pond is constant at steady state in z direction.

Theorem 1: If $(p, \omega, \theta, h_z, \phi)$ is a solution of the equation (1) to (4) together with either of boundary condition and $\tau \neq 0$ then the principal of exchange of stability is valid if

$$R_1 > \frac{Q\pi^2 + \frac{R_2}{\tau}}{1 + T_0(\hat{\alpha}_2 R_3 - \alpha_2)}$$

Proof: For $\tau \neq 0$, let if possible principal of exchange of stability is valid either $p = 0$ then equation (1) to (4) reduce to

$$(D^2 - a^2)\omega = R_1 a^2\theta - \frac{R_2}{R_3}a^2\phi - QD(D^2 - a^2)h_z \qquad (5)$$

$$(D^2 - a^2)\theta = -(1 - \alpha_2 T_0)\omega - T_0\hat{\alpha}_2 R_3\omega \qquad (6)$$

$$(D^2 - a^2)\phi = -\frac{R_3}{\tau}\omega \qquad (7)$$

$$(D^2 - a^2)h_z = -D\omega \qquad (8)$$

Using boundary condition and transformation $\psi = R_1\theta - (R_2/R_3)\phi$ then equation (5) to (8) reduce to –

$$(D^2 - a^2)\omega = a^2\psi + QD^2\omega \qquad (9)$$

(Putting equation (8) into equation (5))

$$(D^2 - a^2)\psi = -\left[R_1(1 - \alpha_2 T_0) - \frac{R_2}{\tau} + T_0\hat{\alpha}_2 R_1 R_3\right]\omega$$

$$(D^2 - a^2)\psi = -R\omega \qquad (10)$$

Where

$R = \left[R_1(1 - \alpha_2 T_0) - \frac{R_2}{\tau} + T_0\hat{\alpha}_2 R_1 R_3\right]$ is a modified thermohaline Rayleigh number

Now multiplying equation (9) by $\omega^\star$ and integrating the equation so obtained over the vertical range of z by parts finite number of times and using the boundary conditions we get -

$$\int_{-1}^{1} \omega^\star (D^2 - a^2)^2 \omega dz = a^2 \int_{-1}^{1} \omega^\star \psi dz + Q\int_{-1}^{1} \omega^\star D^2 \omega dz \qquad (11)$$

$$\int_{-1}^{1} [|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2]dz = a^2 \int_{-1}^{1} \omega^\star \psi dz - Q\int_{-1}^{1} |D\omega|^2 dz \quad (12)$$

From equation (10), we can write

$$\int_{-1}^{1} \psi (D^2 - a^2)\psi^\star dz = -R\int_{-1}^{1} \psi\omega^\star dz$$

$$\frac{1}{R}\int_{-1}^{1}[|D\psi|^2 + a^2|\psi|^2]dz = \int_{-1}^{1}\omega^{\star}\psi dz \quad (13)$$

Where using (13) in (12), we get

$$\int_{-1}^{1}[|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2]dz = \frac{a^2}{R}\int_{-1}^{1}[|D\psi|^2 + a^2\psi^2]dz - Q\int_{-1}^{1}|D\omega|^2 dz$$

Or

$$\int_{-1}^{1}[|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2]dz - \frac{a^2}{R}\int_{-1}^{1}[|D\psi|^2 + a^2\psi^2]dz - Q\int_{-1}^{1}|D\omega|^2 dz = 0 \qquad (14)$$

Again from (10) we have

$$\int_{-1}^{1}(D^2 - a^2)\psi(D^2 - a^2)\psi^{\star}dz = \mathrm{R}^2\int_{-1}^{1}|\omega|^2 dz$$

$$\int_{-1}^{1}[|D^2\psi|^2 + 2a^2|D\psi|^2 + a^4|\psi|^2 dz] = \mathrm{R}^2\int_{-1}^{1}|\omega|^2 dz$$

$$\int_{-1}^{1}[|D\psi| + a^2|\psi|^2]dz \leq \frac{R^2}{a^2}\int_{-1}^{1}|\omega|^2 dz$$

$$\int_{-1}^{1}[|D\psi| + a^2|\psi|^2]dz \leq \frac{R^2}{a^2\Pi}\int_{-1}^{1}|D\omega|^2 dz \qquad (15)$$

Where (14) is obtained by using Rayleigh Ritz inequality now using

Now using (15) in (14), we get

$$\int_{-1}^{1}[|D^2\omega|^2 + 2a^2|D\omega|^2 + a^4|\omega|^2]dz + [Q - \frac{R}{\Pi^2}]\int_{-1}^{1}|D\omega|^2 dz \leq 0$$

It is clear that equation is not possible until

$$\left[Q - \frac{R}{\Pi^2}\right] < 0$$

$$Q\Pi^2 < R$$

Or

$$Q\Pi^2 < \left[R_1(1-\alpha_2 T_0) - \frac{R_2}{\tau} + T_0\hat{\alpha}_2 R_1 R_3\right]$$

$$Q\Pi^2 < R_1(1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3) - \frac{R_2}{\tau}$$

Which further implies that

$$R_1(1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3) - \frac{R_2}{\tau} - Q\Pi^2 > 0 \quad or \quad R' > 0 \qquad (16)$$

$$R' = R_1(1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3) - \frac{R_2}{\tau} - Q\Pi^2 \qquad (17)$$

Where R' is our new extended modified Rayleigh number.

Using equation (16), we can say that the principle of exchange of stability is valid only if

$$R' > 0 \quad or \quad R_1 > \frac{Q\Pi^2 + \frac{R_2}{\tau}}{1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3} \qquad (18)$$

And therefore, we can conclude that principal of exchange of stability is not valid if

$$R_1 \leq \frac{Q\Pi^2 + \frac{R_2}{\tau}}{1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3}$$

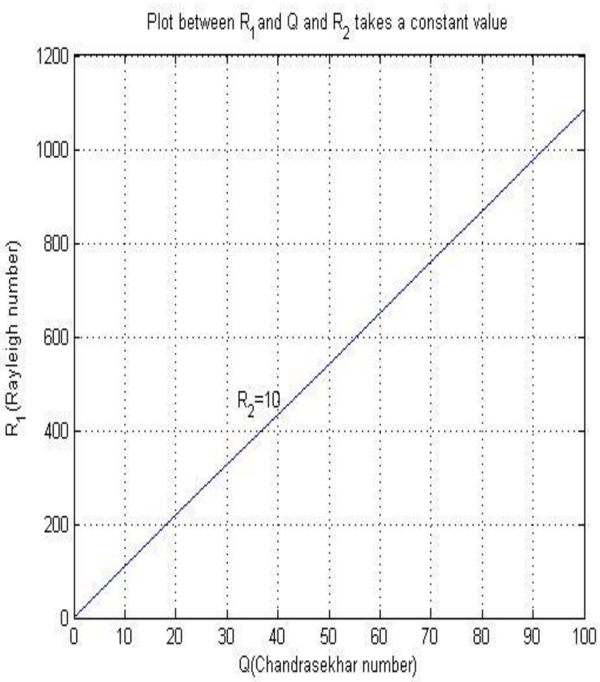
So the critical condition for the exchange of stability

$$R_1 = \frac{Q\Pi^2 + \frac{R_2}{\tau}}{1-\alpha_2 T_0 + T_0\hat{\alpha}_2 R_3} \qquad (19)$$

Now we try to find out thermal, magnetic and concentration effect on stability of thermohaline fluid in the solar pond with the help of equation (19).

From equation (19), we derive the following results with graphical representation –

1)

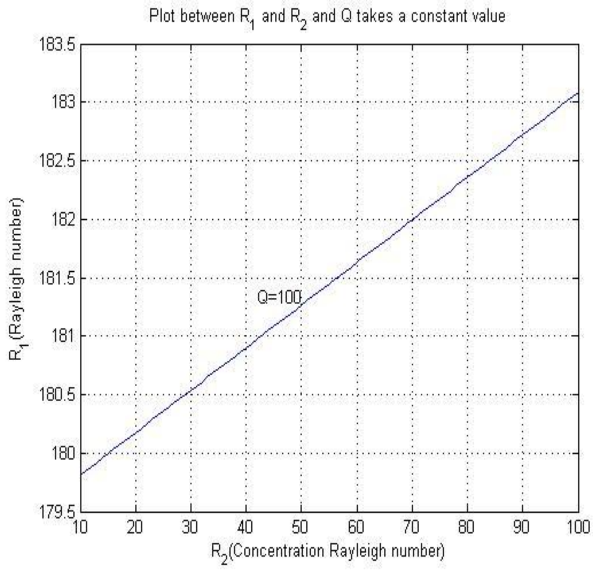


Parameter	Range of parameter	Value of parameter taken
$\boldsymbol{\alpha_2}$	$(\mathbf{10^{-3}, 10^{-2}})$	**0.01**
$\boldsymbol{\hat{\alpha}_2}$	$(\mathbf{10^{-4}, 10^{-3}})$	**0.001**
$\boldsymbol{\tau}$	$(\mathbf{10^{0}, 10^{1}})$	**10**
$\boldsymbol{R_3}$	$(\mathbf{10^{-1}, 10^{0}})$	**1**
$\boldsymbol{T_0}$	$(\mathbf{10^{0}, 10^{1}})$	**80**
$\boldsymbol{Q}$	$(\mathbf{10^{0}, 10^{2}})$	$(\mathbf{10^{0}, 10^{2}})$
$\boldsymbol{R_2}$	**10, 20, 40**	**10**

Fig. 1 The graph has been drawn for thermal Rayleigh number versus Q (Chandrasekhar number) by giving the constant values to different parameters

As Q increases and other parameters are kept constant, R_1 increases, so we can conclude that the role of magnetic field is to stabilize the fluid layer.

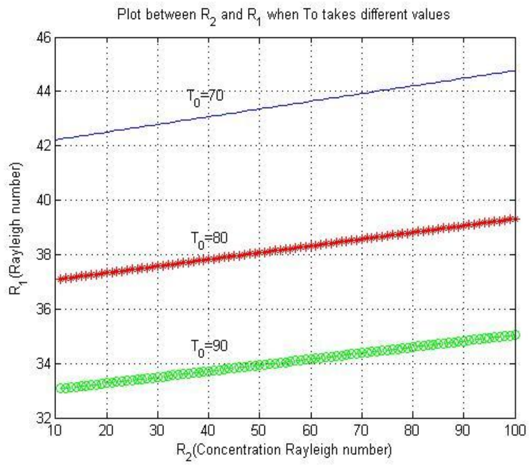
2)



Parameter	Range of parameter	Value of parameter taken
$\boldsymbol{\alpha_2}$	$(\mathbf{10^{-3}, 10^{-2}})$	**0.001**
$\boldsymbol{\hat{\alpha}_2}$	$(\mathbf{10^{-2}, 10^{-1}})$	**0.1**
$\boldsymbol{\tau}$	$(\mathbf{10^{0}, 10^{1}})$	**5**
$\boldsymbol{R_3}$	$(\mathbf{10^{0}, 10^{1}})$	**1**
$\boldsymbol{T_0}$	$(\mathbf{10^{1}, 10^{2}})$	**80**
$\boldsymbol{Q}$	**10, 100, 150**	**10**
$\boldsymbol{R_2}$	$(\mathbf{10, 10^{2}})$	$(\mathbf{10, 10^{2}})$

Fig. 2 The graph has been drawn for thermal Rayleigh number versus concentration Rayleigh number R_2 by giving the constant values to different parameters
As R_2 increases and other parameter are given fixed values, the value of R_1 increases. It means that in Veronis type instability problem, the role of concentration gradient is to stabilize the flow.

3)



Parameter	Range of parameter	Value of parameter taken
$\boldsymbol{\alpha_2}$	$(\mathbf{10^{-3}, 10^{-2}})$	**0.001**
$\boldsymbol{\hat{\alpha}_2}$	$(\mathbf{10^{-2}, 10^{-1}})$	**0.1**
$\boldsymbol{\tau}$	$(\mathbf{10^{0}, 10^{1}})$	**1**
$\boldsymbol{R_3}$	$(\mathbf{10^{0}, 10^{1}})$	**5**
$\boldsymbol{T_0}$	**70, 80, 100**	**70, 80, 90**
$\boldsymbol{Q}$	$(\mathbf{10^{2}, 10^{3}})$	**150**
$\boldsymbol{R_2}$	$(\mathbf{10^{1}, 10^{2}})$	$(\mathbf{10^{1}, 10^{2}})$

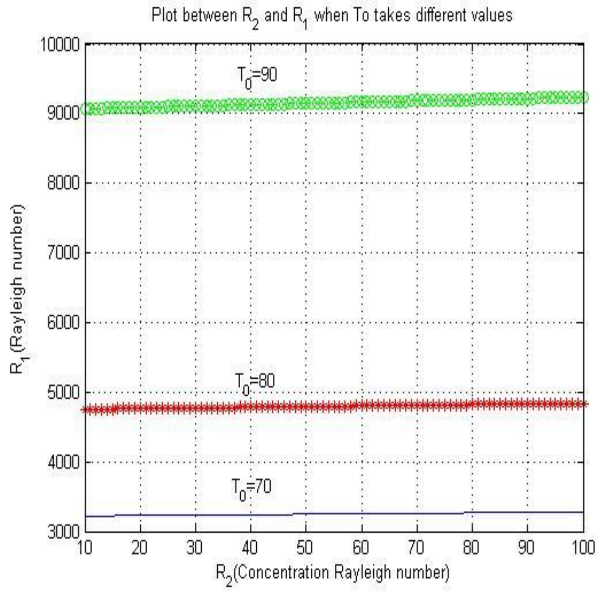
Fig.3 The graph has been drawn for thermal Rayleigh number R_1 versus concentration Rayleigh number R_2 by giving constant values to different parameters

For given $Q > 0, R_2 > 0, \tau > 0$ as the temperature T_0 increases, R_1 decreases provided

$$R_3 > \frac{\alpha_2}{\hat{\alpha}_2}$$

Which indicate that hotter the liquid layer, the instability is preponed.

4)



Parameter	Range of parameter	Value of parameter taken
α_2	$(10^{-3}, 10^{-2})$	0.01
$\hat{\alpha}_2$	$(10^{-4}, 10^{-3})$	0.001
τ	$(10^{0}, 10^{1})$	5
R_3	$(10^{-1}, 10^{0})$	1
T_0	50, 80, 100	70, 80, 90
Q	$(10^{2}, 10^{3})$	100
R_2	$(10^{1}, 10^{2})$	$(10^{1}, 10^{2})$

Fig. 4 The graph has been drawn for thermal Rayleigh number R_1 versus concentration Rayleigh number R_2 by giving constant values to different parameters

For given $Q > 0, R_2 > 0, \tau > 0$ as the temperature T_0 increases, R_1 increases provided $R_3 < \frac{\alpha_2}{\hat{\alpha}_2}$, Which indicate that hotter the liquid layer, the instability is postponed.

[4]Conclusion:

(1) We derive the condition of the validity of principle of exchange of stability for solar pond and found that the result holds if extended modified Rayleigh number

$$R' > 0 \quad or \quad R_1 > \frac{Q\Pi^2 + \frac{R_2}{\tau}}{1 - \alpha_2 T_0 + T_0 \hat{\alpha}_2 R_3}$$

(2) In a Veronis type instability problem, the role of concentration gradient is to stabilize the flow by postponing the instability and here also in solar pond concentration gradient is stabilizing the system by generating heavierhaline density layer downward in the pond.

(3) The role of magnetic field is to make the system stable by delaying instability in the in the solar pond.

(4) The role of temperature is to destabilize the fluid layer provided$R_3 > \alpha_2/\hat{\alpha}_2$.

(5) The temperature is making the fluid flow stable when $R_3 < \alpha_2/\hat{\alpha}_2$.

ACKNOWLEDGEMENT:

I am grateful to Associate Professor VinayKanwar and Assistant Professor Surjeet Singh, Department of Mechanical Engineering, Panjab University Chandigarh.

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