Delay-dependent passivity analysis of time-varying delayed neural networks with leakage and stochastic effects

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Abstract

This paper deals with the problem of passivity analysis for a class of stochastic neural networks (SNNs) with leakage, discrete and distributed delays. By constructing a suitable Lyapunov-Krasovskii functionals and linear matrix inequality (LMI) approach, which is combined with free-weighting matrix method and stochastic analysis technique. We obtain sufficient delay-dependent criteria for the passivity of the addressed neural networks are established in terms of LMIs, which can be verified easily by MATLAB LMI Control toolbox. In addition, few numerical examples are given to show the effectiveness and less conservatism of the approaches proposed in this paper.

Keywords: *Stochastic neural networks; Passivity; Leakage delay; Distributed delay; Linear matrix inequalities.*

1 Introduction

It is well known that passivity is a property of many physical systems which may be generally defined as energy dissipation and transformation. Therefore, it is related to the property of stability in an input-output sense, that is we say that the system is stable if bounded "input energy" supplied to the system, yields bounded "output energy". This is in contrast to Lyapunov stability, which concerns the internal stability of a system. Passivity is a widely adopted tool for analyzing the stability of dynamical systems and is used in several domains of engineering sciences, such as in the analysis of electrical circuits, mechanical systems, chemical processes, electromechanical systems, control over networks, hybrid systems and etc. In recent years, the stability of delayed stochastic neural networks has also received much attention to reducing the time delays in both theoretical and practical applications. Since time delay is frequently encountered in stochastic neural networks, and it is often a source of instability and oscillation in a system. Therefore, considerable effort has been devoted to analyzing the stability of neural networks with time delays.

In general, the stability criteria for delayed stochastic neural networks can be classified into two categories: namely, delay-dependent and delay-independent conditions. Since, it is well known that the delay-dependent (see [1, 2]) criteria gives the less conservative comparing with delay-independent (see [3, 4]) criteria, specifically when the time delay is small, because of this physical reason much attention has been paid for the delay-dependent type. The main idea of passivity is that the passive properties of a system can keep the system internally stable [24]. In [5, 6], delay-dependent passivity conditions were obtained for a class of uncertain continuous-time neural networks with discrete delay type of problems was discussed. Wu et al. [7] developed a robust dissipativity analysis of neural networks with time varying delay with the presence of randomly occurring uncertainties. Recently, a complete delay-decomposition approach was employed to study the passivity of neural networks with time-varying delays in [8]. Recently, by implementing a new improved integral inequality techniques to study the neural network problems with time delays, and proposed technique were successfully applied to a benchmark problem which was investigated by Manivannan et al. (see [9]–[14]).

Similarly, on the other hand SNNs usually have a spatial extent, there is a distribution of propagation delays over a period of time. In these circumstances the signal propagation is not instantaneous and cannot be modeled with discrete delays. Therefore, it is necessary to introduce continuously distributed delays over a certain duration of time (see [30, 15]). In addition, uncertain stochastic neural networks with discrete and distributed delays have been investigated, and significant result has been reported in [31]. For neural networks, both discrete and distributed delays, the problem of passivity analysis was addressed in [16, 17, 18]. In [32, 33, 34, 35], the authors considered stochastic perturbations on the passivity of stochastic neural networks with time delays. Recently, Song et al. [38] addressed some sufficient conditions for obtaining the passivity of uncertain neural networks with leakage delay and time varying delay. Wu et al. investigated the problem of exponential passivity of neural networks with time-varying delays in [28]. Raja et al. [29], investigated the problem of passivity analysis for a class of discrete-time stochastic BAM neural networks with time-varying delays. Recently, Li et al.[23] discussed the delay-dependent stability analysis for a class of dynamical systems with leakage delay and nonlinear perturbations.

Interestingly, speaking that in real nervous systems, the time delay in the stabilizing negative feedback terms has a tendency to destabilize a system (This kind of delays is known as leakage delays or "forgetting" delays). Hence, it is a significant importance to consider the leakage delay effects on dynamics of SNNs. Moreover, neural networks with time delay in the leakage term also have great impact on the dynamics of stochastic neural networks because time delay in the stabilizing negative feedback term has a tendency to destabilize a system (see [19, 20, 21, 22, 23]). However, most of the results are based on the assumption that the time-varying delays is differentiable, which greatly reduce the applied range of those results in practice. More recently, improved the issue and established some LMIs conditions to estimate the neuron state of mixed delayed neural networks in which the time-varying delays are non-differentiable. Unfortunately, little progress has been made towards solving the problem of analysis and synthesis for a passivity of SNNs with both discrete and distributed time-varying delay while the presence of leakage term have not been fully investigated yet, while research in this area it is clearly very important from both theoretical and practical point of view. Therefore, so far it is necessary to further investigate the problem of passivity for a class of SNNs with discrete and distributed time delays with presence of leakage delay.

Motivated by the above discussions, the main objective of this paper is to study the passivity analysis for a class of SNNs with leakage, discrete and distributed delays. We introduce a new Lyapunov-Krasovskii functional by taking the information about integral terms in leakage delay and derivative of variables into account and moreover, the leakage delay occurs in both single and double integral terms in order to derive the desirable results. All the derived conditions obtained here are expressed in terms of LMIs whose feasibility can be easily checked by using numerically efficient MATLAB LMI Control toolbox. Finally, few numerical examples are given to show the effectiveness and advantage of the present results.

Notations: The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. $\| . \|$ refers to the Euclidean vector norm. A^T represents the transpose of matrix A and the asterisk "$*$" in a matrix is used to represent the term which is induced by symmetry. I is the identity matrix with compatible dimension. $X > Y$ means that X and Y are symmetric matrices, and that $X - Y$ is positive definite. Let $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathfrak{F}_t\}_{t\geq 0}$ satisfying the usual conditions (*i.e.* it is right continuous and $\mathfrak{F}_o$ contains all $\mathcal{P}$-null sets). $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. Denote by $L^2_{\mathfrak{F}_o}([-\tau, 0], \mathbb{R}^n)$ the family of all $\mathfrak{F}_o$-measurable $C([-\tau, 0], \mathbb{R}^n)$-valued random variables $\Psi = \{\Psi(s) : s \in [-\tau, 0]\}$ such that $\sup_{s\in[-\tau,0]} \mathbb{E}\{| \Psi(s) |\} < \infty$. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2 Problem formulation and preliminaries

In this section, we consider the problem of stochastic neural networks with leakage, discrete and distributed delays can be written as:

$$\begin{aligned} dx(t) &= \Big[-Cx(t-\rho)+Af(x(t))+Bf(x(t-\tau(t)))+D\int_{t-d(t)}^{t} f(x(s))ds+u(t)\Big]dt \\ &\quad +\sigma(t,x(t),x(t-\tau(t)),x(t-d(t)))d\omega(t) \\ x(s) &= \varphi(s),\ \forall\ s\ \in[-max(\rho,\tau,d),0], \end{aligned} \tag{1}$$

where $x(t)=\big(x_1(t),x_2(t),\dots,x_n(t)\big)^T\in\mathbb{R}^n$ is the state vector of the network at time t, n corresponds to the number of neurons; C is a positive diagonal matrix; $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$ and $D=(d_{ij})_{n\times n}$ are the interconnection weight matrices; $\sigma\in\mathbb{R}^{n\times q}$ is the diffusive coefficient vector and $\omega(t)=\big(\omega_1(t),\omega_2(t),\dots,\omega_q(t)\big)^T$ is a $q-$ dimensional Brownian motion defined on a complete probability space $(\Omega,\mathfrak{F},\{\mathfrak{F}_t\}_{t\geq0},\mathcal{P})$ with a filtration $\{\mathfrak{F}_t\}_{t\geq0}$ satisfying the usual conditions (*i.e.* it is right continuous and $\mathfrak{F}_o$ contains all $\mathcal{P}$-null sets); $f(x(t))=\big(f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t))\big)^T$ denotes the neuron activation at time t; $u(t)=\big(u_1(t),u_2(t),\dots,u_n(t)\big)^T\in\mathbb{R}^n$ is a varying external input vector; $\rho>0$ is the leakage delay term, $\tau(t)>0$ and $d(t)>0$ denotes the discrete and distributed delays and is assumed to satisfying $0\leq\tau(t)\leq\tau$ and $0\leq d(t)\leq d$ respectively, where τ and d is the constant. Throughout this paper, the neuron activation functions are assumed to satisfying the following assumption:

(H1). For any $j\in\{1,2,\dots,n\}$, $f_j(0)=0$ and their exist constants F_j^- and F_j^+ such that

$$F_j^-\leq\frac{f_j(\alpha_1)-f_j(\alpha_2)}{\alpha_1-\alpha_2}\leq F_j^+,$$

for all $\alpha_1\neq\alpha_2$.

Remark 2.1 *The above assumption on the neuron activation function is more general than [5, 6, 17, 28, 34]. Since F_j^- and F_j^+ (j=1,2,...,n) may be positive, zero or negative, that is to say, the activation function under assumption* **(H1)** *may be non-monotonic, non-differentiable and unbounded. Hence, assumption* **(H1)** *in this paper is weaker than the assumption in [5, 6, 17, 28, 34]. Therefore, the passivity condition adopted in this paper is less conservative than [5, 6, 17, 28, 34].*

(H2). There exist constant matrices R_1, R_2 and R_3 of appropriate dimensions such that the following inequality

$$tr\big(\sigma^T(t,u,v,w)\sigma(t,u,v,w)\big)\leq\parallel R_1u\parallel^2+\parallel R_2v\parallel^2+\parallel R_3w\parallel^2$$

holds for all $(t,u,v,w)\in\mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$.

Definition 2.1 (Song et al. [26]). System (1) is called globally passive in the sense of expectation, if there exists a scalar $\gamma>0$ such that

$$2\mathbb{E}\Big\{\int_0^{t_p} f^T(x(s))u(s)ds\Big\}\geq-\mathbb{E}\Big\{\gamma\int_0^{t_p} u^T(s)u(s)ds\Big\},$$

for all $t_p\geq0$ and for all $x(t,0)$.
To prove our main results, it is necessary to introduce the following lemmas.

Lemma 2.1 (Samidurai et al. [30]). For any constant matrix $W\in\mathbb{R}^{m\times m}$, $W>0$, scalar $0<h(t)<h$, vector function $\omega:\big[0,h\big]\to\mathbb{R}^m$ such that the integrations concerned are well defined, then

$$-\Big(\int_0^{h(t)} w(s)ds\Big)^T W\Big(\int_0^{h(t)} w(s)ds\Big)\leq-h(t)\Big(\int_0^{h(t)} w^T(s)Ww(s)ds\Big).$$

Lemma 2.2 (Zhao et al. [25]). Let a, $b \in \mathbb{R}^n$, P be a positive definite matrix, then

$$2a^T b \leq a^T P^{-1} a + b^T P b.$$

Lemma 2.3 (Raja et al. [36]). Given constant matrices P, Q and R where $P^T = P$, $Q^T = Q$, then

$$\begin{bmatrix} P & R \\ * & -Q \end{bmatrix} < 0,$$

is equivalent to the following conditions $Q > 0$ and $P + RQ^{-1}R^T < 0$.

3 Main results

In this section, we will perform to analyze the passivity of stochastic neural networks (1). Based on Lyapunov-Krasovskii functional stability theorem and stochastic analysis approach, which shows that the system (1) is stable in the mean square if the linear matrix inequality (LMI) is feasible. For presentation convenience, in the following, we denote

$$F_1 = diag(F_1^- F_1^+, F_2^- F_2^+, \ldots, F_n^- F_n^+), \qquad F_2 = diag\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \ldots, \frac{F_n^- + F_n^+}{2}\right).$$

Theorem 3.1 *Consider the stochastic neural networks (1) satisfies the assumption* (**H1**) *and* (**H2**), *model (1) is passive in the sense of Definition 2.1, For a given scalar* τ, ρ *and* d, *if there exist symmetric positive definite matrices* $P_i (i = 1, 2, \ldots, 9)$, *positive diagonal matrices* U *and* V, *matrices* $Q_i (i = 1, 2, \ldots, 6)$ *and positive constants* $\gamma > 0$, $\lambda_i > 0$ $(i = 1, 2, 3)$ *such that the following LMIs hold:*

$$P_1 < \lambda_1 I, \tag{2}$$

$$P_3 < \lambda_2 I, \tag{3}$$

$$P_8 < \lambda_3 I, \tag{4}$$

$$\Omega = \begin{bmatrix} \Pi_{11}^1 & \Pi_{12}^1 \\ * & \Pi_{22}^1 \end{bmatrix}, \tag{5}$$

where

$$\Pi_{11}^1 = \begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & \Pi_{15} & 0 & \Pi_{17} & \Pi_{18} & \Pi_{19} & \Pi_{1_{11}} & \Pi_{1_{12}} & \Pi_{1_{13}} \\
* & \Pi_{22} & 0 & 0 & 0 & 0 & -CQ_1^T & 0 & 0 & 0 & 0 & 0 \\
* & * & \Pi_{33} & Q_4 & 0 & 0 & 0 & 0 & F_2V & 0 & 0 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Pi_{55} & Q_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{77} & Q_1A & Q_1B & 0 & Q_1D & Q_1 \\
* & * & * & * & * & * & * & \Pi_{88} & 0 & -A^TP_1C & 0 & -I \\
* & * & * & * & * & * & * & * & \Pi_{99} & -B^TP_1C & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Pi_{10_{10}} & -CP_1D & -CP_1 \\
* & * & * & * & * & * & * & * & * & * & \Pi_{11_{11}} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Pi_{12_{12}}
\end{bmatrix} < 0,$$

$$\Pi_{12}^1 = \begin{bmatrix}
Q_3 & Q_3 & Q_5 & Q_5 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & Q_4 & Q_4 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & Q_6 & Q_6 \\
* & * & * & * & * & * & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 \\
* & * & * & * & * & * & * & 0
\end{bmatrix},$$

$$\Pi_{22}^1 = diag\left(-\frac{1}{\tau}P_4, -P_3, -\frac{1}{d}P_9, -P_8, -\frac{1}{\tau}P_4, -P_3, -\frac{1}{d}P_9, -P_8\right)$$

in which

$\Pi_{11} = -P_1C - CP_1 + (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_1^TR_1 + P_2 + \rho^2P_3 + P_6 + d^2P_7 - Q_3 - Q_3^T - Q_5 - Q_5^T - F_1U + P_5,$
$\Pi_{12} = -Q_2C,\ \Pi_{13} = Q_3,\ \Pi_{15} = Q_5,\ \Pi_{17} = -Q_2,\ \Pi_{18} = P_1A + Q_2A + F_2U,\ \Pi_{19} = P_1B + Q_2B,$
$\Pi_{1_{11}} = CP_1C,\ \Pi_{1_{12}} = P_1D,\ \Pi_{1_{13}} = P_1 + Q_2,\ \Pi_{22} = -P_2,\ \Pi_{33} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_2^TR_2 - Q_4 - Q_4^T - F_1V,$
$\Pi_{44} = -P_5,\ \Pi_{55} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_3^TR_3 - Q_6 - Q_6^T,\ \Pi_{66} = -P_6,\ \Pi_{77} = dP_9 - Q_1 - Q_1^T,\ \Pi_{88} = -U,$
$\Pi_{99} = -V,\ \Pi_{10_{10}} = -P_3,\ \Pi_{11_{11}} = -P_7,\ \Pi_{12_{12}} = -\gamma I.$

Proof. Let us take our dynamical system (1) like as follows,

$$y(t) = -Cx(t-\rho) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-d(t)}^{t} f(x(s))ds + u(t) \quad (6)$$

$$\alpha(t) = \sigma(t, x(t), x(t-\tau(t)), x(t-d(t))), \quad (7)$$

then the model (1) is can be rewritten as

$$dx(t) = y(t)dt + \alpha(t)d\omega(t). \quad (8)$$

Let us consider the following Lyapunov-Krasovskii functional candidate to be

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t) + V_6(x_t) \quad (9)$$

where

$$\begin{aligned}
V_1(x_t) &= \Big(x(t) - C\int_{t-\rho}^{t} x(s)ds\Big)^T P_1\Big(x(t) - C\int_{t-\rho}^{t} x(s)ds\Big),\\
V_2(x_t) &= \int_{t-\rho}^{t} x^T(s)P_2x(s)ds + \rho\int_{-\rho}^{0}\int_{t+\xi}^{t} x^T(s)P_3x(s)dsd\xi,\\
V_3(x_t) &= \int_{-\tau}^{0}\int_{t+\xi}^{t} (tr(\alpha^T(s)P_3\alpha(s)) + y^T(s)P_4y(s))dsd\xi,\\
V_4(x_t) &= \int_{t-\tau}^{t} x^T(s)P_5x(s)ds,\\
V_5(x_t) &= \int_{t-d}^{t} x^T(s)P_6x(s)ds + d\int_{-d}^{0}\int_{t+\xi}^{t} x^T(s)P_7x(s)dsd\xi,\\
V_6(x_t) &= \int_{-d}^{0}\int_{t+\xi}^{t} (tr(\alpha^T(s)P_8\alpha(s)) + y^T(s)P_9y(s))dsd\xi.
\end{aligned}$$

By using Ito's differential rule, the mathematical expectation of the stochastic derivative of $V_1(x_t)$ along the trajectory of the system (1) can be found as follows:

$$\begin{aligned}
\mathbb{E}\Big\{dV_1(x_t)\Big\} = \mathbb{E}\Big\{2\Big(x(t) - C\int_{t-\rho}^{t} x(s)ds\Big)^T P_1\Big(&-Cx(t) + Af(x(t)) + Bf(x(t-\tau(t)))\\
&+D\int_{t-d(t)}^{t} f(x(s))ds + u(t)\Big)\\
&+tr\Big(\sigma^T\big(x(t), x(t-\tau(t)), x(t-d(t))\big)P_1\sigma\big(x(t), x(t-\tau(t)), x(t-d(t))\big)\Big)\Big]dt\Big\} \quad (10)
\end{aligned}$$

In above equation using assumption (**H2**) then can write the following inequality,

$$\begin{aligned}
&tr\Big(\sigma^T\big(x(t), x(t-\tau(t)), x(t-d(t))\big)P_1\sigma\big(x(t), x(t-\tau(t)), x(t-d(t))\big)\Big)\\
&\quad\le \lambda_1\Big[x^T(t)R_1^TR_1x(t) + x^T(t-\tau(t))R_2^TR_2x(t-\tau(t)) + x^T(t-d(t))R_3^TR_3x(t-d(t))\Big]. \quad (11)
\end{aligned}$$

Make use of (11) in (10), we get

$$\leq \Big\{x^T(t)\big(-P_1C - CP_1 + \lambda_1 R_1^T R_1\big)x(t) + 2x^T(t)P_1Af(x(t)) + 2x^T(t)P_1Bf(x(t-\tau(t)))$$
$$+2x^T(t)P_1u(t) + 2x^T(t)P_1D\int_{t-d(t)}^{t} f(x(s))ds + 2\int_{t-\rho}^{t} x^T(s)dsCP_1Cx(t)$$
$$-2\int_{t-\rho}^{t} x^T(s)dsCP_1Af(x(t)) - 2\int_{t-\rho}^{t} x^T(s)dsCP_1Bf(x(t-\tau(t)))$$
$$-2\int_{t-\rho}^{t} x^T(s)dsCP_1D\int_{t-d(t)}^{t} f(x(s))ds - 2\int_{t-\rho}^{t} x^T(s)dsCP_1u(t)$$
$$+x^T(t-\tau(t))\lambda_1 R_2^T R_2 x(t-\tau(t)) + x^T(t-d(t))\lambda_1 R_3^T R_3 x(t-d(t))\Big]dt\Big\}. \quad (12)$$

Computing the derivative of $V_2(x_t)$, $V_3(x_t)$, $V_4(x_t)$, $V_5(x_t)$ and $V_6(x_t)$, and using Lemma 2.1 and inequality (12), we get the following

$$dV_2(x_t) = \Big[x^T(t)P_2x(t) - x^T(t-\rho)P_2x(t-\rho) + \rho^2 x^T(t)P_3x(t) - \rho\int_{t-\rho}^{t} x^T(s)P_3x(s)ds\Big]dt$$

$$\leq \Big[x^T(t)\big(P_2 + \rho^2 P_3\big)x(t) - x^T(t-\rho)P_2x(t-\rho) - \Big(\int_{t-\rho}^{t} x(s)ds\Big)^T P_3\Big(\int_{t-\rho}^{t} x(s)ds\Big)\Big]dt \quad (13)$$

$$dV_3(x_t) = \Big[\tau tr\big(\alpha^T(t)P_3\alpha(t)\big) - \int_{t-\tau}^{t} tr\big(\alpha^T(s)P_3\alpha(s)\big)ds + \tau y^T(t)P_4y(t) - \int_{t-\tau}^{t} y^T(s)P_4y(s)ds\Big]dt$$

$$\leq \Big[x^T(t)\tau\lambda_2 R_1^T R_1 x(t) + x^T(t-\tau(t))\tau\lambda_2 R_2^T R_2 x(t-\tau(t)) + x^T(t-d(t))\tau\lambda_2 R_3^T R_3 x(t-d(t))$$
$$+y^T(t)\tau P_4 y(t) - \int_{t-\tau}^{t} tr\big(\alpha^T(s)P_3\alpha(s)\big)ds - \int_{t-\tau}^{t} y^T(s)P_4y(s)ds\Big]dt \quad (14)$$

$$dV_4(x_t) = \big[x^T(t)P_5x(t) - x^T(t-\tau)P_5x(t-\tau)\big]dt \quad (15)$$

$$dV_5(x_t) = \Big[x^T(t)P_6x(t) - x^T(t-d)P_6x(t-d) + d^2x^T(t)P_7x(t) - d\int_{t-d}^{t} x^T(s)P_7x(s)ds\Big]dt$$

$$\leq \Big[x^T(t)\big(P_6 + d^2P_7\big)x(t) - x^T(t-d)P_6x(t-d) - \Big(\int_{t-d(t)}^{t} x(s)ds\Big)^T P_7\Big(\int_{t-d(t)}^{t} x(s)ds\Big)\Big]dt \quad (16)$$

$$dV_6(x_t) = \Big[d\ tr\big(\alpha^T(t)P_8\alpha(t)\big) + d\ y^T(t)P_9y(t) - \int_{t-d}^{t} tr\big(\alpha^T(s)P_8\alpha(s)\big)ds - \int_{t-d}^{t} y^T(s)P_9y(s)ds\Big]dt$$

$$\leq \Big[x^T(t)d\lambda_3 R_1^T R_1 x(t) + x^T(t-\tau(t))d\lambda_3 R_2^T R_2 x(t-\tau(t)) + x^T(t-d(t))dR_3^T R_3 x(t-d(t))$$
$$+y^T(t)dP_9y(t) - \int_{t-d}^{t} tr\big(\alpha^T(s)P_8\alpha(s)\big)ds - \int_{t-d}^{t} y^T(s)P_9y(s)ds\Big]dt. \quad (17)$$

From the definition of $y(t)$, we have

$$\begin{aligned} 0 &= 2\big(y^T(t)Q_1 + x^T(t)Q_2\big)\Big[-y(t) - Cx(t-\rho) + Af(x(t)) + Bf(x(t-\tau(t))) \\ &\quad + D\int_{t-d(t)}^{t} f(x(s))ds + u(t)\Big] \end{aligned}$$

$$\begin{aligned} 0 &= y^T(t)\big(-Q_1 - Q_1^T\big)y(t) - 2y^T(t)Q_1Cx(t-\rho) + 2y^T(t)Q_1Af(x(t)) \\ &\quad + 2y^T(t)Q_1Bf(x(t-\tau(t))) + 2y^T(t)Q_1D\int_{t-d(t)}^{t} f(x(s))ds + 2y^T(t)Q_1u(t) \\ &\quad - 2x^T(t)Q_2y(t) - 2x^T(t)Q_2Cx(t-\rho) + 2x^T(t)Q_2Af(x(t)) \\ &\quad + 2x^T(t)Q_2Bf(x(t-\tau(t))) + 2x^T(t)Q_2D\int_{t-d(t)}^{t} f(x(s))ds + 2x^T(t)Q_2u(t) \end{aligned} \tag{18}$$

Integrating both sides of (8) from taking limits $t-\tau(t)$ to t and $t-d(t)$ to t following respectively, we have

$$x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} \alpha(s)d\omega(s) = 0,$$

and

$$x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} y(s)ds - \int_{t-d(t)}^{t} \alpha(s)d\omega(s) = 0,$$

By using Lemma 2.2, and noting that $0 \le \tau(t) \le \tau$ and $0 \le d(t) \le d$ respectively, we get the following

$$\begin{aligned} 0 &= -2x^T(t)Q_3\Big[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\Big] \\ 0 &\le x^T(t)\big(-Q_3 - Q_3^T + \tau Q_3P_4^{-1}Q_3^T + Q_3P_3^{-1}Q_3^T\big)x(t) + 2x^T(t)Q_3x(t-\tau(t)) \\ &\quad + \int_{t-\tau(t)}^{t} y^T(s)P_4y(s)ds + \Big(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\Big)^T P_3\Big(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\Big), \end{aligned} \tag{19}$$

and

$$\begin{aligned} 0 &= -2x^T(t)Q_5\Big[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} y(s)ds - \int_{t-d(t)}^{t} \alpha(s)d\omega(s)\Big] \\ 0 &\le x^T(t)\big(-Q_5 - Q_5^T + dQ_5P_9^{-1}Q_5^T + Q_5P_8^{-1}Q_5^T\big)x(t) + 2x^T(t)Q_5x(t-d(t)) \\ &\quad + \int_{t-d(t)}^{t} y^T(s)P_9y(s)ds + \Big(\int_{t-d(t)}^{t} \alpha(s)d\omega(s)\Big)^T P_8\Big(\int_{t-d(t)}^{t} \alpha(s)d\omega(s)\Big). \end{aligned} \tag{20}$$

Similarly, integrating both sides of (8) taking limits from $t-\tau$ to $t-\tau(t)$ and $t-d$ to $t-d(t)$ respectively, we get

$$\begin{aligned} 0 &= -2x^T(t-\tau(t))Q_4\Big[x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} y(s)ds - \int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\Big] \\ 0 &\le x^T(t-\tau(t))\big(-Q_4 - Q_4^T + \tau Q_4P_4^{-1}Q_4^T + Q_4P_3^{-1}Q_4^T\big)x(t-\tau(t)) + 2x^T(t-\tau(t))Q_4x(t-\tau) \\ &\quad + \int_{t-\tau}^{t-\tau(t)} y^T(s)P_4y(s)ds + \Big(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\Big)^T P_3\Big(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\Big), \end{aligned} \tag{21}$$

and

$$
\begin{aligned}
0 &= -2x^T(t-d(t))Q_6\Big[-x(t-d(t))+x(t-d)+\int_{t-d}^{t-d(t)} y(s)ds-\int_{t-d}^{t-d(t)} \alpha(s)d\omega(s)\Big] \\
0 &\leq x^T(t-d(t))\big(-Q_6-Q_6^T+dQ_6P_9^{-1}Q_6^T+Q_6P_8^{-1}Q_6^T\big)x(t-d(t))+2x^T(t-d(t))Q_6x(t-d) \\
&\quad +\int_{t-d}^{t-d(t)} y^T(s)P_9y(s)ds+\Big(\int_{t-d}^{t-d(t)} \alpha(s)d\omega(s)\Big)^T P_8\Big(\int_{t-d}^{t-d(t)} \alpha(s)d\omega(s)\Big). \qquad (22)
\end{aligned}
$$

From the proof of [37], we can get that

$$\mathbb{E}\Big\{\Big(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\Big)^T P_3\Big(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\Big)\Big\}=\mathbb{E}\Big\{\int_{t-\tau(t)}^{t} tr\big(\alpha^T(s)P_3\alpha(s)\big)ds\Big\}, \qquad (23)$$

$$\mathbb{E}\Big\{\Big(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\Big)^T P_3\Big(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\Big)\Big\}=\mathbb{E}\Big\{\int_{t-\tau}^{t-\tau(t)} tr\big(\alpha^T(s)P_3\alpha(s)\big)ds\Big\}, \qquad (24)$$

$$\mathbb{E}\Big\{\Big(\int_{t-d(t)}^{t} \alpha(s)d\omega(s)\Big)^T P_8\Big(\int_{t-d(t)}^{t} \alpha(s)d\omega(s)\Big)\Big\}=\mathbb{E}\Big\{\int_{t-d(t)}^{t} tr\big(\alpha^T(s)P_8\alpha(s)\big)ds\Big\}, \qquad (25)$$

$$\mathbb{E}\Big\{\Big(\int_{t-d}^{t-d(t)} \alpha(s)d\omega(s)\Big)^T P_8\Big(\int_{t-d}^{t-d(t)} \alpha(s)d\omega(s)\Big)\Big\}=\mathbb{E}\Big\{\int_{t-d}^{t-d(t)} tr\big(\alpha^T(s)P_8\alpha(s)\big)ds\Big\}. \qquad (26)$$

For three positive diagonal matrices U and V, we can get from assumption (**H1**), we have

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_1U & -F_2U \\ -F_2U & U \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \leq 0, \qquad (27)$$

and

$$\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} F_1V & -F_2V \\ -F_2V & V \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \leq 0. \qquad (28)$$

Combining from (10)–(28), we get

$$\mathbb{E}\Big\{dV(t)-2f^T(x(t))u(t)dt-\gamma u^T(t)u(t)dt\Big\} \leq \mathbb{E}\Big\{\zeta^T(t)\ \Gamma\ \zeta(t)dt\Big\}, \qquad (29)$$

where

$$
\begin{aligned}
\zeta(t) = \Big\{& x^T(t)\ x^T(t-\rho)\ x^T(t-\tau(t))\ x^T(t-\tau)\ x^T(t-d(t))\ x^T(t-d)\ y^T(t)\ f^T(x(t)) \\
& f^T(t-\tau(t))\ \int_{t-\rho}^{t} x^T(s)ds\ \int_{t-d(t)}^{t} f^T(x(s))ds\ u^T(t)\Big\},
\end{aligned}
$$

and

$$\Gamma=\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 & \Gamma_{15} & 0 & \Gamma_{17} & \Gamma_{18} & \Gamma_{19} & \Gamma_{1_{11}} & \Gamma_{1_{12}} & \Gamma_{1_{13}} \\
* & \Gamma_{22} & 0 & 0 & 0 & 0 & -CQ_1^T & 0 & 0 & 0 & 0 & 0 \\
* & * & \Gamma_{33} & Q_4 & 0 & 0 & 0 & 0 & F_2V & 0 & 0 & 0 \\
* & * & * & \Gamma_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Gamma_{55} & Q_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Gamma_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Gamma_{77} & Q_1A & Q_1B & 0 & Q_1D & Q_1 \\
* & * & * & * & * & * & * & \Gamma_{88} & 0 & -A^TP_1C & 0 & -I \\
* & * & * & * & * & * & * & * & \Gamma_{99} & -B^TP_1C & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Gamma_{10_{10}} & -CP_1D & -CP_1 \\
* & * & * & * & * & * & * & * & * & * & \Gamma_{11_{11}} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Gamma_{12_{12}}
\end{bmatrix} < 0$$

$$\begin{aligned}
&\Gamma_{11} = -P_1C - CP_1 + (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_1^TR_1 + P_2 + \rho^2P_3 + P_6 + d^2P_7 - Q_3 - Q_3^T + \\
&\quad \tau Q_3P_4^{-1}Q_3^T + Q_3P_3^{-1}Q_3^T - Q_5 - Q_5^T + dQ_5P_9^{-1}Q_5^T + Q_5P_8^{-1}Q_5 - F_1U + P_5, \\
&\Gamma_{12} = -Q_2C,\ \Gamma_{13} = Q_3,\ \Gamma_{15} = Q_5,\ \Gamma_{17} = -Q_2,\ \Gamma_{18} = P_1A + Q_2A + F_2U, \\
&\Gamma_{19} = P_1B + Q_2B,\ \Gamma_{1_{11}} = CP_1C,\ \Gamma_{1_{12}} = P_1D,\ \Gamma_{1_{13}} = P_1 + Q_2,\ \Gamma_{22} = -P_2, \\
&\Gamma_{33} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_2^TR_2 - Q_4 - Q_4^T + \tau Q_4P_4^{-1}Q_4^T + Q_4P_3^{-1}Q_4^T - F_1V,\ \Gamma_{44} = -P_5, \\
&\Gamma_{55} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_3^TR_3 - Q_6 - Q_6^T + dQ_6P_9^{-1}Q_6^T + Q_6P_8^{-1}Q_6^T,\ \Gamma_{66} = -P_6, \\
&\Gamma_{77} = dP_9 - Q_1 - Q_1^T,\ \Gamma_{88} = -U,\ \Gamma_{99} = -V,\ \Gamma_{10_{10}} = -P_3,\ \Gamma_{11_{11}} = -P_7,\ \Gamma_{12_{12}} = -\gamma I.
\end{aligned}$$

Therefore, it is easy to verify the equivalence of $\Gamma < 0$ and $\Omega < 0$ by using Lemma 2.3. Thus, one can derive from (8) and (29) we get

$$\frac{\mathbb{E}\big\{dV(t)\big\}}{dt} - \mathbb{E}\big\{2f^T(x(t))u(t) + \gamma u^T(t)u(t)\big\} \leq 0. \tag{30}$$

It follows from (30) the definition $V(t, x(t))$, we can conclude that

$$2\mathbb{E}\bigg\{\int_0^{t_p} f^T(x(s))u(s)ds\bigg\} \geq -\mathbb{E}\bigg\{\gamma\int_0^{t_p} u^T(s)u(s)ds\bigg\}. \tag{31}$$

From Definition 2.1, we know that the stochastic neural networks (1) is globally passive in the sense of Definition 2.1. This completes the proof of Theorem 3.1. □

It is worth pointing out that, following the similar fashion of the proof Theorem 3.1. It is not difficult to prove the following Corollaries can be obtained. Therefore the proof is omitted for lack of space.

If there is no stochastic effect means, then the neural networks (1) becomes as follows:

$$dx(t) \;=\; \big[-Cx(t-\rho) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-d(t)}^{t} f(x(s))ds + u(t)\big]dt. \tag{32}$$

Corollary 3.1 Consider the neural networks (32) which satisfies the assumption (**H1**), model (32) is passive in the sense of Definition 2.1, for a given scalars τ, η and d, if there exist symmetric positive definite matrices $P_i(i = 1, 2, \ldots, 9)$, positive diagonal matrices U and V, matrices $Q_i(i = 1, 2, \ldots, 6)$ and a positive constant $\gamma > 0$, such that the following LMI holds:

$$\Pi = \begin{bmatrix} \Pi_{11}^1 & \Pi_{12}^1 \\ * & \Pi_{22}^1 \end{bmatrix} < 0, \tag{33}$$

where

$$\Pi_{11}^1 = \begin{bmatrix}
\Phi_{11} & \Pi_{12} & \Pi_{13} & 0 & \Pi_{15} & 0 & \Pi_{17} & \Pi_{18} & \Pi_{19} & \Pi_{1_{11}} & \Pi_{1_{12}} & \Pi_{1_{13}} \\
* & \Pi_{22} & 0 & 0 & 0 & 0 & -CQ_1^T & 0 & 0 & 0 & 0 & 0 \\
* & * & \Phi_{33} & Q_4 & 0 & 0 & 0 & 0 & F_2 & 0 & 0 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Phi_{55} & Q_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{77} & Q_1A & Q_1B & 0 & Q_1D & Q_1 \\
* & * & * & * & * & * & * & \Pi_{88} & 0 & -A^TP_1C & 0 & -I \\
* & * & * & * & * & * & * & * & \Pi_{99} & -B^TP_1C & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Pi_{10_{10}} & -CP_1D & -CP_1 \\
* & * & * & * & * & * & * & * & * & * & \Pi_{11_{11}} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Pi_{12_{12}}
\end{bmatrix}$$

in which

$$\Phi_{11} = -P_1C - CP_1 + P_2 + \rho^2 P_3 + P_6 + d^2 P_7 - Q_3 - Q_3^T - Q_5 - Q_5^T - F_1U + P_5,$$
$$\Phi_{33} = -Q_4 - Q_4^T - F_1V,\ \Phi_{55} = -Q_6 - Q_6^T,$$

and $\Pi_{12}, \Pi_{13}, \Pi_{15}, \Pi_{17}, \Pi_{18}, \Pi_{19}, \Pi_{1_{11}}, \Pi_{1_{12}}, \Pi_{1_{13}}, \Pi_{22}, \Pi_{44}, \Pi_{66}, \Pi_{77}, \Pi_{88}, \Pi_{99}, \Pi_{10_{10}}, \Pi_{11_{11}}, \Pi_{12_{12}}, \Pi_{12}^1, \Pi_{22}^1$ are defined in Theorem 3.1.□.

Incase there is no effect of leakage delay means, then the stochastic neural networks (1) becomes as follows:

$$dx(t) = \Big[- Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-d(t)}^{t} f(x(s))ds + u(t)\Big]dt + \sigma\big(t, x(t), x(t-\tau(t)), x(t-d(t))\big)d\omega(t). \tag{34}$$

Corollary 3.2 Consider the stochastic neural networks (34) satisfies the assumption (**H1**) and (**H2**), model (34) is passive in the sense of Definition 2.1, For a given scalars τ and d, if there exist symmetric positive definite matrices $P_1, P_3, P_4, P_5, P_6, P_7, P_8$ and P_9, positive diagonal matrices U and V, matrices $Q_i (i = 1, 2, \dots, 6)$ and positive constants $\gamma > 0$, $\lambda_i > 0$ $(i = 1, 2, 3)$ such that the following LMIs hold:

$$P_1 < \lambda_1 I, \tag{35}$$
$$P_3 < \lambda_2 I, \tag{36}$$
$$P_8 < \lambda_3 I, \tag{37}$$

$$\Theta = \begin{bmatrix} \Theta_{11}^1 & \Pi_{12}^1 \\ * & \Pi_{22}^1 \end{bmatrix}, \tag{38}$$

where

$$\Theta_{11}^1 = \begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & \Theta_{14} & 0 & \Theta_{16} & \Theta_{17} & \Theta_{18} & \Theta_{1_{10}} & \Theta_{1_{11}} \\ * & \Theta_{22} & Q_4 & 0 & 0 & 0 & 0 & F_2V & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & Q_6 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & Q_1A & Q_1B & Q_1D & Q_1 \\ * & * & * & * & * & * & \Theta_{77} & 0 & 0 & -I \\ * & * & * & * & * & * & * & \Theta_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Theta_{9_9} & 0 \\ * & * & * & * & * & * & * & * & * & \Theta_{10_{10}} \end{bmatrix} < 0,$$

in which

$$\Theta_{11} = -P_1C - CP_1 + (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_1^T R_1 + P_6 + d^2P_7 - Q_3 - Q_3^T - Q_5 - Q_5^T - F_1U + P_5,$$
$$\Theta_{12} = Q_3,\ \Theta_{14} = Q_5,\ \Theta_{16} = -Q_2,\ \Theta_{17} = P_1A + Q_2A + F_2U,\ \Theta_{18} = P_1B + Q_2B,\ \Theta_{1_{10}} = P_1D,$$
$$\Theta_{1_{11}} = P_1 + Q_2,\ \Theta_{22} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_2^T R_2 - Q_4 - Q_4^T - F_1V,\ \Theta_{33} = -P_5,$$
$$\Theta_{44} = (\lambda_1 + \tau\lambda_2 + d\lambda_3)R_3^T R_3 - Q_6 - Q_6^T,\ \Theta_{55} = -P_6,\ \Theta_{66} = dP_9 - Q_1 - Q_1^T,$$
$$\Pi_{77} = -U,\ \Theta_{88} = -V,\ \Theta_{9_9} = -P_7,\ \Theta_{10_{10}} = -\gamma I,$$

and Π_{12}^1, Π_{22}^1 are defined in Theorem 3.1.□.

Remark 3.1 *It should be noted that, if we let there is no distributed delay, leakage delay and stochastic effect means, then the dynamical system (1) is reduced to in [5, 6]. Hence, the following Corollary 3.3 gives the new passivity criteria LMI technique, to reduce the conservatism when compared to those results in [5, 6]. Table 1 provides, the merits and improvements of our method.*

$$dx(t) = \big[- Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + u(t)\big]dt. \tag{39}$$

Corollary 3.3 Consider the neural networks (39) satisfying the assumption (**H1**), model (39) is passive in the sense of Definition 2.1, For a given scalar τ, if there exist symmetric positive definite matrices P_1, P_3, P_4 and P_5, positive diagonal matrices U and V, matrices $Q_i(i = 1, 2, \dots, 4)$ and positive constant $\gamma > 0$, such that the following LMI holds: where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & Q_3 & Q_3 & 0 & 0 \\ * & \Xi_{22} & Q_4 & 0 & 0 & F_2V & 0 & 0 & 0 & Q_4 & Q_4 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & Q_1A & Q_1B & Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{10_{10}} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Xi_{11_{11}} \end{bmatrix} < 0, \tag{40}$$

in which

$$\Xi_{11} = -P_1C - CP_1 - Q_3 - Q_3^T - F_1U + P_5,\ \Xi_{12} = Q_3,\ \Xi_{14} = -Q_2,\ \Xi_{15} = P_1A + Q_2A + F_2U,$$
$$\Xi_{16} = P_1B + Q_2B,\ \Xi_{17} = P_1 + Q_2,\ \Xi_{22} = -Q_4 - Q_4^T - F_1V,\ \Xi_{33} = -P_5,\ \Xi_{44} = -Q_1 - Q_1^T,$$
$$\Pi_{55} = -U,\ \Xi_{66} = -V,\ \Xi_{77} = -\gamma I,\ \Xi_{88} = -\frac{1}{\tau}P_4,\ \Xi_{99} = -P_3,\ \Xi_{10_{10}} = -\frac{1}{\tau}P_4,\ \Xi_{11_{11}} = -P_3.\Box.$$

Incase there is no distributed delay and stochastic effect means, then the stochastic neural networks (1) becomes as follows:

$$dx(t) \ = \ \big[- Cx(t-\rho) + Af(x(t)) + Bf(x(t-\tau(t))) + u(t) \big] dt. \tag{41}$$

Corollary 3.4 Consider the neural networks (41) which satisfies the assumption (**H1**), model (41) is passive in the sense of Definition 2.1, for a given scalars τ and η, if there exist symmetric positive definite matrices $P_i(i = 1, 2, \dots, 5)$, positive diagonal matrices U and V, matrices $Q_i(i = 1, 2, \dots, 4)$ and a positive constant $\gamma > 0$, such that the following LMI holds:

$$\Psi_{11}^1 = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & Q_3 & Q_3 & 0 & 0 \\ * & \Psi_{22} & 0 & 0 & -CQ_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & Q_4 & 0 & 0 & F_2V & 0 & 0 & 0 & 0 & Q_4 & Q_4 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & Q_1A & Q_1B & 0 & Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & -A^TP_1C & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & -B^TP_1C & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & -CP_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{10_{10}} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{11_{11}} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Psi_{12_{12}} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \Psi_{13_{13}} \end{bmatrix} \tag{42}$$

in which

$$\Psi_{11} = -P_1C - CP_1 + P_2 + \rho^2P_3 - Q_3 - Q_3^T - F_1U + P_5,\ \Psi_{12} = -Q_2C,\ \Psi_{13} = Q_3,\ \Psi_{15} = -Q_2,$$
$$\Psi_{16} = P_1A + Q_2A + F_2U,\ \Psi_{17} = P_1B + Q_2B,\ \Psi_{18} = CP_1C,\ \Psi_{19} = P_1 + Q_2,\ \Psi_{22} = -P_2,$$
$$\Psi_{33} = -Q_4 - Q_4^T - F_1V,\ \Psi_{44} = -P_5,\ \Psi_{55} = -Q_1 - Q_1^T,\ \Psi_{66} = -U,\ \Pi_{77} = -V,\ \Psi_{88} = -P_3,$$
$$\Psi_{99} = -\gamma I,\ \Psi_{10_{10}} = -\frac{1}{\tau}P_4,\ \Psi_{11_{11}} = -P_3,\ \Psi_{12_{12}} = -\frac{1}{\tau}P_4,\ \Psi_{13_{13}} = -P_3.\Box.$$

4 Numerical examples

In this section, we provide following numerical examples to demonstrate the effectiveness of our stability results.

Example 4.1 Consider a stochastic neural networks (1) with the following parameters:

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.4 & -0.2 \\ 0.3 & 0.2 \end{bmatrix}, \quad R1 = \begin{bmatrix} 0 & 0.01 \\ 0.01 & -0.01 \end{bmatrix}$$

$$R2 = \begin{bmatrix} -0.01 & 0.01 \\ 0 & 0.01 \end{bmatrix}, \quad R3 = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad F1 = 0.$$

The activation functions are assumed to be

$$f_i(x_i) = 0.5\big(|x_i + 1| - |x_i - 1|\big), \ i = 1, 2.$$

and which satisfies the assumption (**H1**) with $F_j^- = 0$, $F_j^+ = 1$, $j = 1, 2.$ when $\tau = 0.6$, $d = 0.3$ and $\rho = 0.2$, using MATLAB LMI control Toolbox and by solving the LMIs in Theorem 3.1 in our paper, we find that the stochastic neural networks (1) is globally passive in the mean square, and we obtain the following feasible solutions:

$$P1 = \begin{bmatrix} 7.6396 & 1.4157 \\ 1.4157 & 3.2644 \end{bmatrix}, \quad P2 = \begin{bmatrix} 0.6660 & 0.0329 \\ 0.0329 & 0.4889 \end{bmatrix}, \quad P3 = \begin{bmatrix} 20.0652 & 1.8128 \\ 1.8128 & 7.4207 \end{bmatrix},$$

$$P4 = \begin{bmatrix} 62.7423 & 11.7620 \\ 11.7620 & 19.2943 \end{bmatrix}, \quad P5 = \begin{bmatrix} 2.9133 & -0.0301 \\ -0.0301 & 1.1854 \end{bmatrix}, \quad P6 = \begin{bmatrix} 1.1761 & 0.4933 \\ 0.4933 & 0.6194 \end{bmatrix},$$

$$P7 = \begin{bmatrix} 14.9387 & -3.7063 \\ -3.7063 & 2.2667 \end{bmatrix}, \quad P8 = \begin{bmatrix} 6.9909 & 0.7905 \\ 0.7905 & 6.1680 \end{bmatrix}, \quad P9 = \begin{bmatrix} 0.5844 & -0.0142 \\ -0.0142 & 0.5434 \end{bmatrix},$$

$$Q1 = \begin{bmatrix} 0.1803 & -0.0414 \\ -0.0414 & 0.1537 \end{bmatrix}, \quad Q2 = \begin{bmatrix} 0.0545 & 0.0067 \\ 0.0067 & 0.1646 \end{bmatrix}, \quad Q3 = \begin{bmatrix} 0.3195 & -0.2613 \\ -0.2613 & 0.4392 \end{bmatrix},$$

$$Q4 = \begin{bmatrix} 2.4757 & 0.0172 \\ 0.0172 & 0.9970 \end{bmatrix}, \quad Q5 = \begin{bmatrix} 0.1314 & -0.1908 \\ -0.1908 & 0.2911 \end{bmatrix}, \quad Q6 = \begin{bmatrix} 0.6356 & 0.1860 \\ 0.1860 & 0.4103 \end{bmatrix},$$

$$U = \begin{bmatrix} 9.8074 & 0 \\ 0 & 6.7995 \end{bmatrix}, \quad V = \begin{bmatrix} 5.4499 & 0 \\ 0 & 0.3040 \end{bmatrix}, \quad W = \begin{bmatrix} 0.0136 & 0 \\ 0 & 0.0296 \end{bmatrix},$$

$\lambda_1 = 8.0915, \ \lambda_2 = 20.3758, \ \lambda_3 = 7.5881, \ \gamma = 279.2441.$

Example 4.2 Consider a neural network (32) with the following parameters:

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.4 & -0.2 \\ 0.3 & 0.2 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad F1 = 0.$$

Using Corollary 3.1, it can be easily verify that the maximum allowable upper bounds $\tau = 2.0$, $\rho = 0.4$ and $d = 0.6$. By using the Matlab LMI Control Toolbox, we obtain the following feasible solutions:

$$P1 = \begin{bmatrix} 26.0739 & -6.9992 \\ -6.9992 & 10.2794 \end{bmatrix}, \quad P2 = \begin{bmatrix} 0.7916 & -0.8067 \\ -0.8067 & 0.9408 \end{bmatrix}, \quad P3 = \begin{bmatrix} 76.3084 & -22.5596 \\ -22.5596 & 23.7668 \end{bmatrix},$$

$$P4 = \begin{bmatrix} 495.3907 & -8.5974 \\ -8.5974 & 103.8188 \end{bmatrix}, \quad P5 = \begin{bmatrix} 9.3796 & -2.5765 \\ -2.5765 & 2.9426 \end{bmatrix}, \quad P6 = \begin{bmatrix} 0.4551 & -0.4701 \\ -0.4701 & 0.5794 \end{bmatrix},$$

$$P7 = \begin{bmatrix} 22.1105 & -12.4555 \\ -12.4555 & 9.6308 \end{bmatrix}, \quad P8 = \begin{bmatrix} 50.1195 & -0.3370 \\ -0.3370 & 50.3594 \end{bmatrix}, \quad P9 = \begin{bmatrix} 0.3044 & -0.3556 \\ -0.3556 & 0.4700 \end{bmatrix},$$

$$Q1 = \begin{bmatrix} 0.1904 & -0.2169 \\ -0.2169 & 0.2784 \end{bmatrix}, \quad Q2 = \begin{bmatrix} 0.1207 & -0.1453 \\ -0.1453 & 0.1979 \end{bmatrix}, \quad Q3 = \begin{bmatrix} 1.1682 & -0.7936 \\ -0.7936 & 0.7741 \end{bmatrix},$$

$$Q4 = \begin{bmatrix} 8.0318 & -2.1760 \\ -2.1760 & 2.5184 \end{bmatrix}, \quad Q5 = \begin{bmatrix} 0.1945 & -0.2202 \\ -0.2202 & 0.2700 \end{bmatrix}, \quad Q6 = \begin{bmatrix} 0.2669 & -0.2946 \\ -0.2946 & 0.3739 \end{bmatrix},$$

$$U = \begin{bmatrix} 17.8834 & 0 \\ 0 & 8.4885 \end{bmatrix}, \quad V = \begin{bmatrix} 11.7050 & 0 \\ 0 & 2.6811 \end{bmatrix}, \quad W = \begin{bmatrix} 0.0408 & 0 \\ 0 & 0.0545 \end{bmatrix},$$

$\gamma = 759.5810.$

Table 1: The maximum allowable delay τ of Example 4.4

Methods	τ
[5] Theorem 1	0.4683
[6] Corollary 1	1.3027
In this paper Corollary 3.3	3.0125

Example 4.3 Consider a stochastic neural network (34) with the following parameters:

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.4 & -0.2 \\ 0.3 & 0.2 \end{bmatrix}, \quad R1 = \begin{bmatrix} 0 & 0.01 \\ 0.01 & -0.01 \end{bmatrix}$$
$$R2 = \begin{bmatrix} -0.01 & 0.01 \\ 0 & 0.01 \end{bmatrix}, \quad R3 = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad F1 = 0.$$

Using Corollary 3.2, it can be easily verify that the maximum allowable upper bounds $\tau = 0.8$ and $d = 0.6$. By using the Matlab LMI Control Toolbox, we obtain the following feasible solutions:

$$P1 = \begin{bmatrix} 6.1052 & 0.4388 \\ 0.4388 & 1.9381 \end{bmatrix}, \quad P3 = \begin{bmatrix} 6.4955 & 0.0565 \\ 0.0565 & 6.2335 \end{bmatrix}, \quad P4 = \begin{bmatrix} 217.3796 & 33.9971 \\ 33.9971 & 58.8342 \end{bmatrix},$$
$$P5 = \begin{bmatrix} 3.0030 & -0.4035 \\ -0.4035 & 1.2833 \end{bmatrix}, \quad P6 = \begin{bmatrix} 0.7352 & 0.2864 \\ 0.2864 & 0.3512 \end{bmatrix}, \quad P7 = \begin{bmatrix} 7.6039 & -2.8076 \\ -2.8076 & 1.7267 \end{bmatrix},$$
$$P8 = \begin{bmatrix} 2.8645 & 0.2504 \\ 0.2504 & 2.5842 \end{bmatrix}, \quad P9 = \begin{bmatrix} 0.9083 & -0.2384 \\ -0.2384 & 0.9267 \end{bmatrix}, Q1 = \begin{bmatrix} 0.3880 & -0.1767 \\ -0.1767 & 0.4027 \end{bmatrix},$$
$$Q2 = \begin{bmatrix} 0.2180 & 0.0557 \\ 0.0557 & 0.1996 \end{bmatrix}, \quad Q3 = \begin{bmatrix} 0.1455 & -0.2819 \\ -0.2819 & 0.5548 \end{bmatrix}, \quad Q4 = \begin{bmatrix} 2.0251 & -0.2195 \\ 0.2195 & 1.0401 \end{bmatrix},$$
$$Q5 = \begin{bmatrix} 0.0927 & -0.1264 \\ -0.1264 & 0.1754 \end{bmatrix}, \quad Q6 = \begin{bmatrix} 0.3855 & 0.0977 \\ 0.0977 & 0.2322 \end{bmatrix}, \quad U = \begin{bmatrix} 8.8242 & 0 \\ 0 & 5.1605 \end{bmatrix},$$
$$V = \begin{bmatrix} 4.2645 & 0 \\ 0 & 0.6083 \end{bmatrix}, \quad W = \begin{bmatrix} 0.0033 & 0 \\ 0 & 0.0060 \end{bmatrix},$$
$$\lambda_1 = 6.1586, \ \lambda_2 = 6.5171, \ \lambda_3 = 3.0243, \ \gamma = 708.3960.$$

Example 4.4 Consider a neural network (39) with the following parameters:

$$C = \begin{bmatrix} 2.2 & 0 \\ 0 & 1.8 \end{bmatrix}, \quad A = \begin{bmatrix} 1.2 & 1 \\ -0.2 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 & 0.4 \\ -0.2 & 0.1 \end{bmatrix},$$

The activation functions are mentioned in Example 4.1 which satisfies the assumption (**H1**) with $F_j^- = 0$, $F_j^+ = 1$, $j = 1, 2$. Using Corollary 3.3, we get the maximum allowable upper bound τ is shown in Table 1 with the those results obtained in [5, 6]. Hence the system (39) is globally passive in the sense of Definition 2.1 and also, it is clear that this method gives less conservative than those results in [5, 6] based on the upper bound techniques. Also we obtain the following feasible solutions:

$$P1 = \begin{bmatrix} 22.5481 & 0.2351 \\ 0.2351 & 39.5518 \end{bmatrix}, \quad P3 = \begin{bmatrix} 73.5673 & -0.0176 \\ -0.0176 & 72.6795 \end{bmatrix}, \quad P4 = \begin{bmatrix} 141.1621 & -0.0375 \\ -0.0375 & 139.2510 \end{bmatrix},$$
$$P5 = \begin{bmatrix} 41.2902 & 0.4701 \\ 0.4701 & 84.0040 \end{bmatrix}, \quad Q1 = \begin{bmatrix} 14.1621 & -0.0375 \\ -0.0375 & 139.2510 \end{bmatrix}, \quad Q2 = \begin{bmatrix} 8.3195 & 0.2504 \\ 0.2504 & 22.8553 \end{bmatrix},$$
$$Q3 = \begin{bmatrix} 14.9579 & 0.0379 \\ 0.0379 & 15.9362 \end{bmatrix}, \quad Q4 = \begin{bmatrix} 18.5912 & 0.0323 \\ 0.0323 & 20.0852 \end{bmatrix}, \quad U = \begin{bmatrix} 98.7417 & 0 \\ 0 & 91.2982 \end{bmatrix},$$

$$V = \begin{bmatrix} 82.3262 & 0 \\ 0 & 69.1792 \end{bmatrix}, \quad \gamma = 107.1970.$$

Example 4.5 Consider a neural network (41) with the following parameters:

$$C = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad A = \begin{bmatrix} 1.3 & -1.7 \\ -1.6 & 1.2 \end{bmatrix}, \quad B = \begin{bmatrix} 1.4 & 1.9 \\ 0.6 & -1.2 \end{bmatrix}.$$

Using Corollary 3.4, it can be verified that the maximum allowable upper bounds $\rho = 1.1$ and $\tau = 2.15$ By using the Matlab LMI Control Toolbox, we obtain the following feasible solutions:

$$P1 = \begin{bmatrix} 48.0516 & 15.9019 \\ 15.9019 & 69.5546 \end{bmatrix}, \quad P2 = \begin{bmatrix} 0.4533 & 0.8886 \\ 0.8886 & 5.7652 \end{bmatrix}, \quad P3 = \begin{bmatrix} 35.1573 & 10.5252 \\ 10.5252 & 37.7845 \end{bmatrix},$$

$$P4 = \begin{bmatrix} 317.4531 & 12.2965 \\ 12.2965 & 305.5506 \end{bmatrix}, \quad P5 = \begin{bmatrix} 1.5512 & 1.4716 \\ 1.4716 & 9.9385 \end{bmatrix}, \ Q1 = \begin{bmatrix} 0.6099 & 1.1581 \\ 1.1581 & 6.8119 \end{bmatrix},$$

$$Q2 = \begin{bmatrix} 0.1689 & 0.2993 \\ 0.2993 & 1.9245 \end{bmatrix}, \quad Q3 = \begin{bmatrix} 1.3707 & 1.0103 \\ 1.0103 & 6.5258 \end{bmatrix}, \quad Q4 = \begin{bmatrix} 1.4603 & 1.2162 \\ 1.2162 & 8.0489 \end{bmatrix},$$

$$U = \begin{bmatrix} 807.0247 & 0 \\ 0 & 781.1181 \end{bmatrix}, \quad V = \begin{bmatrix} 369.4984 & 0 \\ 0 & 824.5048 \end{bmatrix}, \quad \gamma = 478.9829.$$

5 Conclusion

In this paper, the passivity for stochastic neural networks with both discrete and distributed time-varying delays has been investigated without assuming the differentiability of the time-varying delays. By utilizing a combination of Lyapunov functionals, Ito's differential rule, free-weighting matrix method, inequality technique and stochastic analysis approach, several delay-dependent criteria for checking the passivity of addressing neural networks have been established under the weaker assumptions of neuron activation functions and it is expressed in terms of LMIs, which can be easy to check numerically using the effective LMI toolbox in MATLAB. A numerical example has been given to demonstrate the effectiveness and less conservatism which gives merits of the proposed criteria.

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