Special Fuzzy Boolean Ring

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Abstract— *The set of all mappings from a finite set X into a closed interval $[0,1]$ is the set of fuzzy sets denoted by F. This set F is closed under the binary operation absolute difference, Δ of fuzzy set F satisfies the axioms, closure, commutativity, identity and inverse law under the binary operation Δ. The associative law is not satisfied by F. In this article, we wish to introduce the subset B of F with binary operation absolute difference Δ and fuzzy intersection $\cap$, as a special fuzzy Boolean ring briefly denoted by SFBR.*

Keywords— *Special fuzzy Boolean ring (SFBR), absolute difference, subs SFBR, Isomorphic SFBR, Divisor of empty fuzzy set.*

I. Introduction

The set F of fuzzy sets with binary operations fuzzy union, fuzzy intersection and unary operation complementation is not a Boolean algebra. F does not satisfy the complement laws. i.e if A is a fuzzy set of F and its complement is A', then $A \cup A' \neq U$ (Universal set) and $A \cap A' \neq \phi$ (empty set),
where:

$$U = \{(x, U(x) = 1) \text{ for all } x \in F\} \text{ and}$$

$$\phi = \{(x, \phi(x) = 0) \text{ for all } x \in F\}$$

In this regard, in the article [4], we take a subset of F, which holds the complement laws and forms a fuzzy Boolean algebra under some limitations of identity. The complement operation has been redefined. The fuzzy Boolean algebra formed by the subset of F was defined as special fuzzy Boolean algebra.

The operation of absolute difference for integers has been applied by Talukdar D to introduce notions of many algebraic structures like Smarandache Groupoid, Smarandache Group, and Smarandache ring etc. in his articles [5,10].

Our aim in this article is to form special fuzzy Boolean ring simply SFBR. If B is a subset of F and $(B, \cup, \cap, ', \phi, U)$ is a special fuzzy Boolean algebra, then $(B, \Delta, \cap)$ is a special fuzzy Boolean ring (SFBR). Properties of SFBR are discussed here.

II. PRILIMINARIES

We recall the following ideas before introducing the special fuzzy Boolean ring with the help of absolute difference operation of fuzzy sets.

A. Absolute Difference

The absolute difference of two fuzzy sets a and b of F denoted by Δ and defined as:

$a \Delta b = \{(x_i, |a(x_i) - b(x_i)|)\}$, where $a = \{(x_i, a(x_i))\}$ and $b = \{(x_i, b(x_i))\}$, for $i = 0,1,2,3,\ldots n-1$.

Consider $X = \{x_1, x_2, x_3, x_4\}$ and the mappings from X into $[0,1]$ are fuzzy sets. Any two fuzzy sets A and B are given below:

$A = \{(x_1, .1), (x_2, .3), (x_3, .1), (x_4, 1)\}$, and

$B = \{(x_1, 0), (x_2, .2), (x_3, .3), (x_4, .6)\}$.

Then the absolute difference of the fuzzy sets A and B is denoted by $A \Delta B$, where,

$$A\Delta B(x_1) = |.2 - 0| = .2$$

$$A\Delta B(x_2) = |.3 - .2| = .1$$

$$A\Delta B(x_3) = |.1 - .3| = .2$$

$$A\Delta B(x_4) = |1 - .6| = .4$$

Hence, $A\Delta B(x) = \{(x_1, .1), (x_2, .1), (x_3, .2), (x_4, .4)\}$

B. Special fuzzy Boolean algebra

Let, $X = \{x_0, x_1, \ldots x_{n-1}\}$ be a finite set and

$$M = \left\{0, \frac{1}{p-1}, \frac{2}{p-1}, \frac{3}{p-1}, \ldots, \frac{p-1}{p-1} = 1\right\}$$

$= \{0, h, 2h, 3h, \ldots, (p-1)h = 1\}$ be an ordered subset of the closed interval $[0,1]$, where p is any positive integer greater than 1.

Then the family of fuzzy subsets obtain from the mappings from X into M is unable to form boolean algebra since the complement law don't hold.

But if a subset $M_k = \{0, kh\}$, $k = 0,1,2,3,\ldots(p-1)$ of M is considered, then the set B of fuzzy sets, that is the mappings from X to M_k forms a Boolean algebra, which is defined as special fuzzy Boolean algebra. For $k = 1,2,3,\ldots, p-1$, we get special fuzzy

Boolean algebras $B_1, B_2, B_3, \ldots B_{p-1}$ which are isomorphic to each other [1].

III.SPECIAL FUZZY BOOLEAN RING

The set B_1 of all mappings from X to $M_1 = \{0, h\}$ is a ring. B_1 satisfies all the postulates of an abelian group under the binary operation Δ, absolute difference of fuzzy sets. Again B_1 is a semi group under binary operation $\cap$, fuzzy intersection. Again the operation $\cap$, fuzzy intersection is distributive over the binary operation Δ, absolute difference. Also, it is a Boolean ring, because the elements of B_1 are idempotent with respect to $\cap$. We defined this as **special fuzzy Boolean ring (SFBR)**. Let a, b, c be any three fuzzy sets of B_1, then we get:

$$a \cap (b \Delta c) = (a \cap b) \Delta (a \cap c)$$

$$(b \Delta c) \cap a = (b \cap a) \Delta (c \cap a)$$

B_1 is called a special fuzzy Boolean ring, i.e, SFBR. The zero element of B_1, is $\{(x_1,0),(x_2,0),(x_3,0),\ldots(x_n,0)\}$ and the identity element of B_1, is $\{(x_1,h),(x_2,h),(x_3,h),\ldots(x_n,h)\}$. For $k = 1,2,3,\ldots, p-1$, we get SFBRs $B_1, B_2, B_3, \ldots, B_{p-1}$.

Proposition: In a special fuzzy Boolean ring (SFBR), B,

i) $a \Delta a = \phi$ for all $a \in B$, and

ii) $a \Delta b = \phi \Rightarrow a = b$ for all $a, b \in B$

Proof:

i) We know that $a \in B \Rightarrow a \Delta a \in B$

Now, $(a \Delta a) \cap (a \Delta a) = (a \Delta a)$

$$\Rightarrow (a \Delta a) \cap a \Delta (a \Delta a) \cap a = (a \Delta a)$$

$$\Rightarrow (a \cap a) \Delta (a \cap a) \Delta (a \cap a) \Delta (a \cap a) = (a \Delta a)$$

$$\Rightarrow (a \Delta a) \Delta (a \Delta a) = (a \Delta a) + \phi$$

$$\Rightarrow a \Delta a = \phi$$

ii) Here $a \Delta b = \phi$

$$\Rightarrow a \Delta b = a \Delta a$$

$$\Rightarrow a \Delta (a \Delta b) = a \Delta (a \Delta a)$$

$$\Rightarrow (a \Delta a) \Delta b = (a \Delta a) \Delta a$$

$$\Rightarrow \phi \Delta b = \phi \Delta a$$

$$\Rightarrow b = a$$

Proposition: The special fuzzy Boolean ring is commutative.

Proof: Let a and b be any two elements of a special fuzzy Boolean ring B, then $a \Delta b \in B$.

Now, $(a \Delta b) \cap (a \Delta b) = (a \Delta b)$

$$\Rightarrow a \cap (a \Delta b) \Delta b \cap (a \Delta b) = (a \Delta b)$$

$$\Rightarrow (a \cap a) \Delta (a \cap b) \Delta (b \cap a) \Delta (b \cap b) = (a \Delta b)$$

$$\Rightarrow a \Delta (a \cap b) \Delta (b \cap a) \Delta b = (a \Delta b)$$

$$\Rightarrow (a \cap b) \Delta (b \cap a) = \phi$$

$$\Rightarrow a \cap b = b \cap a$$

Hence a special fuzzy Boolean ring is commutative.

Definition of Isomorphism of SFBRs:

A mapping f of a $SFBR$ B to a SFBR B' is called an isomorphism if for all $a, b \in B$,

i) $f(a \Delta b) = f(a) \Delta f(b)$

ii) $f(a \cap b) = f(a) \cap f(b)$

iii) f is one-one and onto.

Proposition: The SFBRs $B_1, B_2, B_3, \ldots, B_{p-1}$ given above are isomorphic to each other.

Proposition: The relation isomorphism of SFBRs of the same finite set forms equivalence relation.

Sub-SFBR: A non empty sub set H of a SFBR B is called a sub-SFBR of B if:

i) H contains zero and identity element of B

ii) For any $a, b \in H$, $a \Delta b \in H$ and $a \cap b \in H$.

iii) H itself is a SFBR under binary operations absolute difference Δ and fuzzy intersection $\cap$.

Proposition: The intersection of two sub-SFBRs of a SFBR B is a sub SFBR of B.

Divisors of empty fuzzy set

If a and b be any two non empty fuzzy sets of $SFBR$ B such that $a \cap b = 0$ (empty fuzzy set) then a and b are called divisors of empty fuzzy set.

The fuzzy sets a and its complement a' of an SFBR are the ϕ (Zero) divisors of SFBR., i.e, $a \cap a' = \phi$.

Proposition: A fuzzy set of a SFBR is the divisor of itself.

Proposition: Let $(B, \Delta, \cap)$ be a SFBR and (B', Δ) is a special fuzzy Boolean group, then the mapping $B \times B' \to B'$ satisfies the following conditions.

For $a, b \in B$ and $\alpha, \beta \in B'$,

i) $a \cap (\alpha \Delta \beta) = (a \cap \alpha) \Delta (a \cap \beta)$

ii) $(a \Delta b) \cap \alpha = (a \cap \alpha) \Delta (b \cap \alpha)$

iii) $a \cap (b \cap \alpha) = (a \cap b) \cap \alpha$

iv) $I \cap \alpha = \alpha$, where I is the identity element of B.

IV. CONCLUSIONS

This article has introduced a kind of family of fuzzy subsets which forms Boolean ring. In this article, we have studied different characteristics of the special fuzzy Boolean ring. These concepts can lead to a new direction for further study and there is a lot of potential growth in this direction.

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