Probabilistic Analysis of a Standby System with Provision of Concomitant Working

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Abstract -- *In the present study, the model is analyzed probabilistically which comprises of one main unit and one standby unit. Initially, there is one main unit and one cold standby unit. The facility of concomitant working is provided in it, where both units may do concomitant working in order to fulfill the demand. When both main and standby unit stops functioning, the system goes to failed state. For repair there is only single repairman facility. The graphical interpretation is done for the model. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique.*

Keywords: *Standby systems; semi Markov process; Regenerative point technique.*

I. INTRODUCTION

Reliability of a component can be defined as its ability to perform some task under given circumstances. The standby systems play a vital role in the field of reliability engineering. Reliability models have been extensively used by various researchers under different situations [1-7] . Most of the studies deal with the systems having standby units, so that the system may function efficiently with the operation of standby unit under the situation of failure in the main unit. The literature still lacks behind considering the situation where both main and standby unit may become operative in order to fulfill the increased demand. Our motive is to study this situation and thus filling the gap. The concept of concomitant working of both the units is discussed in the present paper.

The system consists of one main unit and one standby unit. in the beginning and only the main unit is in operative state. If the demand increases, both the units (main unit as well as standby unit) may become operative in order to share the increased load. There is only one repairman available to do the job. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique. The graphical interpretation has also been done depicting the reliability and profit of the model.

II. NOTATIONS

λ	Constant failure rate of main unit (Unit 1)
λ_1	Constant failure rate of cold standby units (Unit 2)
α	Constant rate of Unit 2 to become operative from standby state
α_1	Constant rate of Unit 2 to become standby from operative state
$g(t)/G(t)$	pdf/ cdf of repair time of the main unit at failed state (Unit 1)
$g_1(t)/G_1(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 2)
a	probability that after the repair of a unit, workload is only for one unit
b	probability that after the repair of a unit, workload is for both units (main and standby unit)
O_I/ O_{II}	Unit 1/2 is in operative state
CS_{II}	Unit 2 is in cold standby state
F_{rI}/F_{rII}	Unit 1/2 is under repair respectively
F_{wrI}/F_{wrII}	Unit 1/2 is waiting for repair respectively
F_{RI}/F_{RIII}	Unit 1/2 is under repair respectively from the previous state, i.e.,repair is continuing from previous state

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0,1,2 and 3 are regenerative points and thus these are regenerative states. The states 4 and 5 are failed states.

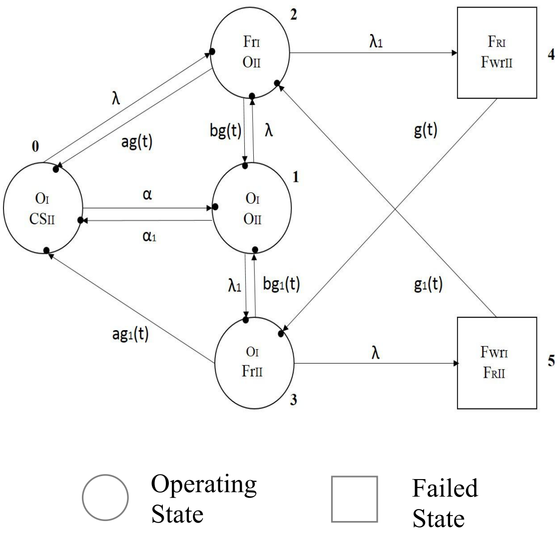

Fig. 1

The non-zero elements p_{ij} , are obtained as under :

$$p_{01} = \frac{\alpha}{\alpha+\lambda} \qquad p_{02} = \frac{\lambda}{\alpha+\lambda}$$

$$p_{10} = \frac{\alpha_1}{\lambda+\lambda_1+\alpha_1} \qquad p_{12} = \frac{\lambda}{\lambda+\lambda_1+\alpha_1}$$

$$p_{13} = \frac{\lambda_1}{\lambda+\lambda_1+\alpha_1} \qquad p_{20} = ag^*(\lambda_1)$$

$$p_{21} = bg^*(\lambda_1) \qquad p_{24} = \frac{\lambda_1[1-g^*(\lambda_1)]}{\lambda_1} = p_{23}^{(4)}$$

$$p_{30} = ag_1^*(\lambda) \qquad p_{31} = bg_1^*(\lambda)$$

$$p_{35} = 1-g_1^*(\lambda) = p_{32}^{(5)} \qquad p_{43} = g^*(0)$$

$$p_{52} = g_1^*(0)$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} = 1 \qquad p_{10} + p_{12} + p_{13} = 1$$

$$p_{20} + p_{21} + p_{24} = 1 \qquad p_{20} + p_{21} + p_{23}^{(4)} = 1$$

$$p_{30} + p_{31} + p_{35} = 1 \qquad p_{30} + p_{31} + p_{32}^{(5)} = 1$$

$$p_{43} = 1 \qquad p_{52} = 1$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as -

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*'}(0), Thus -$$

$$m_{01} + m_{02} = \mu_0 \qquad m_{10} + m_{12} + m_{13} = \mu_1$$

$$m_{20} + m_{21} + m_{24} = \mu_2 \qquad m_{20} + m_{21} + m_{23}^{(4)} = k$$

$$m_{30} + m_{31} + m_{35} = \mu_3 \qquad m_{30} + m_{31} + m_{32}^{(5)} = k_1$$

where,

$$k = \int_0^\infty \overline{G}(t)dt \qquad k_1 = \int_0^\infty \overline{G}_1(t)dt$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$$\mu_0 = \frac{1}{\lambda+\alpha} \qquad \mu_1 = \frac{1}{\lambda+\lambda_1+\alpha_1}$$

$$\mu_2 = \frac{1-g^*(\lambda_1)}{\lambda_1} \qquad \mu_3 = \frac{1-g_1^*(\lambda)}{\lambda}$$

$$\mu_4 = -g^*(0) \qquad \mu_5 = -g_1^*(0)$$

IV. MEAN TIME TO SYSTEM FAILURE

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \frac{N}{D}$$

$$N = \mu_0[1-p_{12}p_{21}-p_{13}p_{31}]+\mu_1[p_{01}+p_{02}p_{21}]$$
$$+\mu_2[p_{02}+p_{01}p_{12}-p_{02}p_{13}p_{31}]+\mu_3[p_{01}p_{13}+p_{02}p_{13}p_{21}]$$
$$D = 1-p_{01}p_{10}-p_{02}p_{20}-p_{12}p_{21}-p_{13}p_{31}-p_{02}p_{10}p_{21}$$
$$-p_{01}p_{12}p_{20}-p_{01}p_{13}p_{30}-p_{02}p_{13}p_{21}p_{30}+p_{02}p_{13}p_{31}p_{20}$$

V. EXPECTED UP-TIME OF THE SYSTEM

The steady state availability of the system is given by

$$A_0 = \frac{N_1}{D_1}$$

$$
\begin{aligned}
N_1 = \mu_0[1 - p_{13}p_{31} - p_{21}(p_{12} + p_{13}p_{32}^{(5)}) - p_{23}^{(4)}(p_{32}^{(5)} + p_{31}p_{12})] \\
+\mu_1[p_{01}(1 - p_{23}^{(4)}p_{32}^{(5)}) + p_{02}(p_{21} + p_{31}p_{23}^{(4)})] \\
+\mu_2[p_{01}(p_{12} + p_{13}p_{32}^{(5)}) + p_{02}(1 - p_{13}p_{31})] \\
+\mu_3[p_{01}(p_{13} + p_{12}p_{23}^{(4)}) + p_{02}(p_{13}p_{21} + p_{23}^{(4)})]
\end{aligned}
$$

$$
\begin{aligned}
D_1 = \mu_0[p_{10}(1 - p_{23}^{(4)}p_{32}^{(5)}) + p_{12}(p_{20} + p_{30}p_{23}^{(4)}) + p_{13}(p_{30} + p_{20}p_{32}^{(5)})] \\
+\mu_1[p_{01}(1 - p_{23}^{(4)}p_{32}^{(5)}) + p_{02}(p_{21} + p_{31}p_{23}^{(4)})] \\
+k[1 - p_{13}p_{31} - p_{01}(p_{10} + p_{13}p_{30})] \\
+k_1[p_{01}(p_{13} + p_{12}p_{23}^{(4)}) + p_{02}(p_{23}^{(4)} + p_{13}p_{21})]
\end{aligned}
$$

VI. BUSY PERIOD OF A REPAIRMAN

The steady state busy period of the system is given by :

$$B_R = \frac{N_2}{D_1}$$

$$
\begin{aligned}
N_2 = W_2[p_{01}(p_{12} + p_{13}p_{32}^{(5)}) + p_{02}(1 - p_{13}p_{31})] \\
+W_3[p_{01}(p_{13} + p_{12}p_{23}^{(4)}) + p_{02}(p_{13}p_{21} + p_{23}^{(4)})]
\end{aligned}
$$

Where D_1 is already specified.

VII. EXPECTED NO. OF VISITS OF REPAIRMAN

The steady state expected no. of visits of the repairman is given by :

$$V_R = \frac{N_3}{D_1}$$

$$
\begin{aligned}
N_3 = [1 - p_{01}p_{10}][1 - p_{23}^{(4)}p_{32}^{(5)}] \\
+p_{02}[-p_{13}p_{31}(1 - p_{23}^{(4)}) + p_{21}p_{13}(1 - p_{32}^{(5)})]
\end{aligned}
$$

Where D_1 is already specified.

VIII. PROFIT ANALYSIS

The expected profit incurred of the system is -

$$P = C_0A_0 - C_1B_R - C_2V_R$$

C_0 = Revenue per unit up time of the system
C_1 = Cost per unit up time for which the repairman is busy in repair
C_2 = Cost per visit of the repairman

IX. GRAPHICAL INTERPRETATION

For graphical analysis following particular cases are considered :

$$g(t) = \beta e^{-\beta t} \qquad g_1(t) = \beta_1 e^{-\beta_1 t}$$

As a particular case, when all the distributions are considered as exponential and by taking numerical values to the considered rates and costs, the graphical study has been made for the MTSF and the profit with respect to failure rate of main unit (λ), revenue per unit uptime of the system (C_0) for different values of rate of failure rate of main unit (λ).

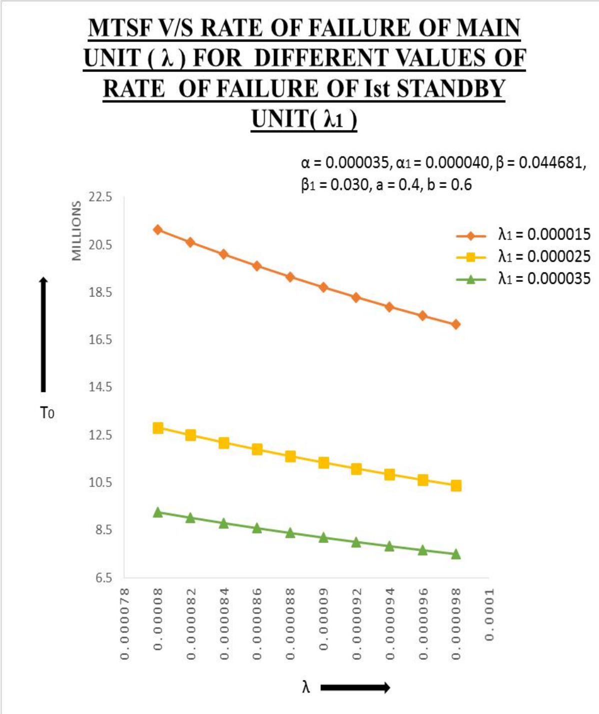

Fig. 2

Fig. 2 shows the behaviour of MTSF w.r.t. failure rate of main unit (λ) for different values of rate of failure of 1st standby unit (λ_1). It is clear from the graph that MTSF decreases with the increase in the values of the failure rate of main unit (λ). Also, the MTSF decreases as failure rate of 1st standby unit (λ_1) increases.

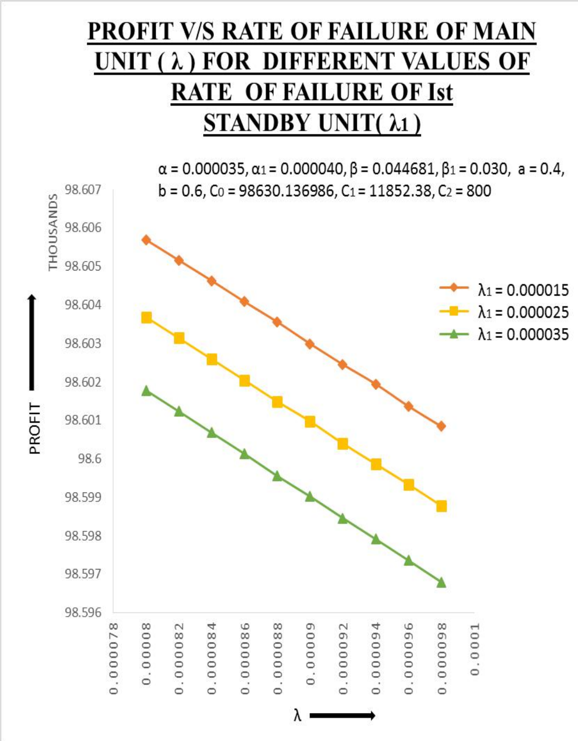

Fig. 3

The above Fig. 3 depicts the behaviour of profit w.r.t. to failure rate of main unit (λ) for different values of failure rate of 1st standby unit (λ_1). As the values of failure rate of main unit (λ) increases, the profit decreases. Also, the profit decreases as failure rate of 1st standby unit (λ_1) increases.

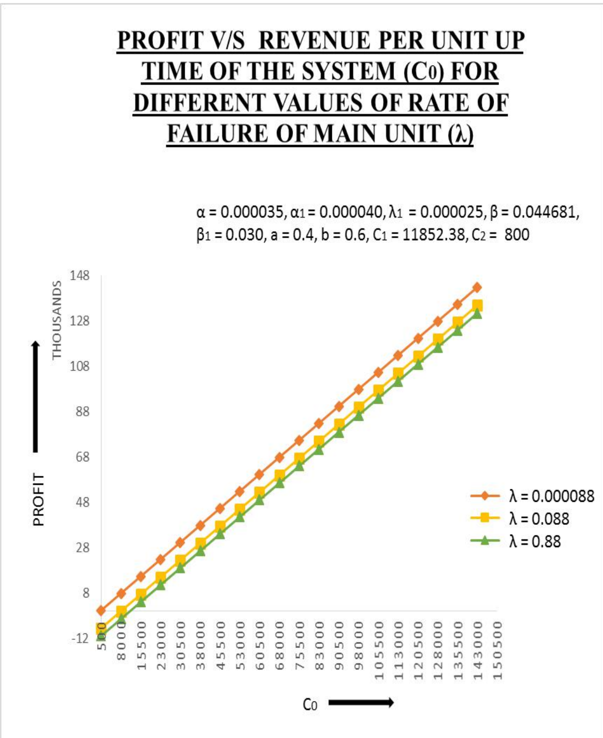

Fig. 4

Fig. 4 interprets the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of main unit (λ). It can be concluded that the profit increases with increase in the values of C_0. Following conclusions can be drawn from the graph:

1. For λ = 0.000088, profit is positive according as C_0 i.e. revenue per unit uptime of the system increases.
2. For λ = 0.088, profit is > or = or < according as C_0 > or = or < 7889.10 , i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 8156.8 to get positive profit.
3. For λ = 0.88, profit is > or = or < according as C_0 > or = or < 11314, i.e. , i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 11365 to get positive profit.

X. CONCLUSIONS AND SUGGESTIONS

The above graphical conclusions have been drawn on the basis of a particular case and the data collected. However, our model can be useful to anyone having similar system and by putting values of parameters of his/her system in the general expressions of our model and can draw the conclusions in the similar fashion.

Further, we are concentrating on the development of some more realistic models (e.g., hot standby system) related to the given system. Our aim will be to increase the uptime of the system and to reduce the cost involved in system for increasing the profit for the system.

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