# Solving Transitive Fuzzy Travelling Salesman Problem using Yager's Ranking Function

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**Abstract:**

*Fuzzy numbers can be used to solve many real life problems like Travelling Salesman problems, Assignment Problems, Allocation problems etc. In this paper a new method is proposed for solving travelling salesman problems using transitive fuzzy numbers. The transitive trapezoidal fuzzy numbers is used to solve a general travelling salesman problem with an optimal solution. The efficiency of this method is proved by solving a numerical problem.*

**General Terms:**

*Fuzzy numbers, transitive trapezoidal fuzzy numbers, α-level cut, Yager's ranking function, Hungarian method.*

## 1. Introduction [6]:

The Traveling Salesman Problem (TSP) was first proposed by Irish mathematician W.R Hamilton in the 19th century. A large number of techniques were developed to solve the problem. The objective or goal of the problem is to invent the shortest route of the salesman starting from a given city, visiting all other cities only once and finally return to the same city where he started.

Traveling salesman problems are classical and widely studied in Combinatorial Optimization. It is an important problem in Artificial Intelligence and Operations Research domain and has been studied intensively, in both Operations Research and Computer Science since 1950's, as a result of which a large number of techniques were developed to solve this problem.

The conventional linear programming deals with crisp parameters. However, information available in real life system is of vague, imprecise and uncertain nature. The impreciseness and uncertainty aspects are handled using fuzzy sets to obtain optimal solutions. Multi-objective linear programming effectively deals with flexible aspiration levels or goals. Fuzzy linear programming enhances the effectiveness of solutions with acceptable solutions through fuzzy constraints. If the cost or time or distance is not crisp values, then it becomes a fuzzy TSP.

In this paper, transitive trapezoidal fuzzy numbers are used for solving TSP with the help of Yager's ranking function. Here an example of TSP with transitive trapezoidal Fuzzy parameters is also discussed.

In recent years, Fuzzy TSP has got great attention and the problems in Fuzzy TSP have been approached using several techniques.

## 2. Preliminaries:

A fuzzy number A is a mapping, $\mu_A(x) : R \rightarrow [0,1]$ with the following properties:

1. μA is an upper semi-continuous function on R.
2. μA (x) = 0 outside of some interval $[a_1, b_2] \epsilon R$
3. There are real numbers $a_2, b_1$, such that $a_1 \leq a_2 \leq b_1 \leq b_2$ and
   i. μA (x) is a monotonic increasing function on $[a_1, a_2]$
   ii. μA (x) is a monotonic decreasing function on $[b_1, b_2]$
   iii. μA (x) = 1 in $[a_2, b_1]$.

### 2.1 Membership function, $\mu_A(x)$ for trapezoidal fuzzy number:

Let A = (a, b, c, d) be a fuzzy set of the real line R then the membership function $\mu_A(x) : R \rightarrow [0,1]$ is defined as

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & : a \leq x \leq b \\ 1 & : b \leq x \leq c \\ \frac{d-x}{d-c} & : c \leq x \leq d \\ 0 & : oherwise \end{cases}$$

### 2.2 Transitive Trapezoidal Fuzzy Number [6]:

If $a_1 \approx a_4$, then the trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$ is called transitive trapezoidal fuzzy number. It is denoted by $A = (a_1, a_4)$.

### 2.3 α- cut :

The α- cut of a fuzzy number A (x) is defined as,

$$A(\alpha) = \{x / \mu(x) \geq \alpha \; ; \alpha \in [0,1]\}$$

### 2.4 Ranking of Trapezoidal Fuzzy Number using Yager's index:

For every fuzzy number $A = (a_1, a_2, a_3, a_4) \epsilon F(R)$, the ranking function $\rho(A): F(R) \rightarrow R$ is defined as

$$\rho(A) = \int_0^1 0.5\,(a_\alpha^l,\; a_\alpha^m, a_\alpha^u)\, d\alpha$$

where $(a_\alpha^l, a_\alpha^m, a_\alpha^u) = a_\alpha^l + a_\alpha^m + a_\alpha^u$ is the α-cut of A and

$a_\alpha^l = (a_2 - a_1)\alpha + a_1$
$a_\alpha^m = (a_3 - a_2)\alpha + a_2$
$a_\alpha^u = -(a_4 - a_3)\alpha + a_4$

### 3. Linear Programming Formulation of Fuzzy Travelling Salesman Problems [12][19] :

Suppose a person has to visit n cities. He starts from a particular city, visits each city once and then returns to the starting point. The fuzzy travelling cost from $i^{th}$ city to $j^{th}$ city is given by $C_{ij}$. The fuzzy travelling salesman problem in the following matrix may be formulated as

**Matrix of fuzzy Travelling Costs between cities**

| City → ↓ | 1 | 2 | ..... | j | ....... | n |
|---|---|---|---|---|---|---|
| 1 | ∞ | $C_{12}$ | ...... | $C_{1j}$ | ....... | $C_{1n}$ |
| 2 | $C_{21}$ | ∞ | ...... | $C_{2j}$ | ....... | $C_{2n}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| j | $C_{j1}$ | $C_{j2}$ | | ∞ | | $C_{jn}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| n | $C_{n1}$ | $C_{n2}$ | ....... | $C_{nj}$ | ...... | ∞ |

Where $C_{iJ} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$

The linear programming problem of the above fuzzy matrix can be formulated as,

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$

subject to,

$\sum_{i=1}^{n} x_{ij} = 1$, i=1,2….n, i ≠ j j=1,2,……n
$\sum_{j=1}^{n} x_{Ij} = 1$, j=1,2….n , i ≠ j i=1,2,……n
$x_{ij} = 0$ or 1

### 4. Numerical Example:

Consider the following fuzzy matrix of travelling costs

| City → ↓ | A | B | C | D |
|---|---|---|---|---|
| **A** | ∞ | **(9 , 10 , 1 ,9)** | **(6 ,8 ,3 ,6)** | **(8 ,9 ,1 ,8)** |
| **B** | **(9, 10 ,2 ,9)** | ∞ | **(10, 11 ,3 ,10)** | **(4 , 5 , 1 , 4)** |
| **C** | **(7 , 8 ,1 , 7)** | **(10 ,11 ,3 ,10)** | ∞ | **( 7 ,8 , 2 ,7)** |
| **D** | **(9 ,10 ,3 ,9)** | **(9 ,11 ,3 , 9)** | **( 6 ,8 , 1 , 6)** | ∞ |

It's solution for an optimal route (minimum cost of travel) using the above method is as follows,

We have from Yager's ranking function, the rank of A, $\rho(A) = \int_0^1 0.5\,(a_\alpha^l, a_\alpha^m, a_\alpha^u)\,d\alpha$

Therefore for $A_{12} = (9, 10, 1, 9)$

$a_\alpha^l = (10 - 9)\alpha + 9 = \alpha + 9$
$a_\alpha^m = (1 - 10)\alpha + 10 = -\alpha + 10$
$a_\alpha^u = -(9 - 1)\alpha + 9 = -\alpha + 9$

Hence, $\rho(A_{12}) = \int_0^1 0.5\,(\alpha+9, -\alpha+10, -\alpha+9)\,d\alpha$
$= \int_0^1 0.5\,(-4\alpha+14)\,d\alpha = 12$

Similarly,

$\rho(A_{13}) = 8.5$; $\rho(A_{14}) = 10$;
$\rho(A_{21}) = 11.5$; $\rho(A_{23}) = 12$; $\rho(A_{24}) = 5$
$\rho(A_{31}) = 8$; $\rho(A_{32}) = 12$; $\rho(A_{34}) = 8.5$;
$\rho(A_{41}) = 11$ ; $\rho(A_{42}) = 11.5$; $\rho(A_{43}) = 7.5$;

Ranked matrix is,

| City → ↓ | A | B | C | D |
|---|---|---|---|---|
| **A** | ∞ | **12** | **8.** 5 | **10** |
| **B** | **11.** 5 | ∞ | **12** | 5 |
| **C** | **8** | **12** | ∞ | **8.** 5 |
| D | **11** | **11.** 5 | **7.** 5 | ∞ |

Step -1 Row Reduction

| City → ↓ | A | B | C | D |
|---|---|---|---|---|
| **A** | ∞ | **3.** 5 | **0** | **1.** 5 |
| **B** | **6.** 5 | ∞ | **7** | 0 |
| **C** | **0** | 4 | ∞ | **0.** 5 |
| **D** | **3.** 5 | 4 | **0** | ∞ |

Step -2 Column Reduction

| City → ↓ | A | B | C | D |
|---|---|---|---|---|
| **A** | ∞ | (0) | **0** | **1.** 5 |
| **B** | **6.** 5 | ∞ | **7** | (0) |
| **C** | (0) | 0. 5 | ∞ | **0.** 5 |
| D | **3.** 5 | 0. 5 | (0) | ∞ |

Therefore the optimum solution is,

A →B→D →C→A = 32. 5

## 4. Conclusion:

Numerical example shows that, this method can be used to get an optimal solution for the transitive fuzzy travelling salesman problem. This method is highly effective and is very easy to understand due to its natural similarity to classical method of solving TSP. The example shown above guarantees the correctness and effectiveness of the working procedure of this method. The result obtained match with the existing techniques that satisfy the regular TSP conditions.

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