# Software for Stresses and Mohr's Circle Computation, Transformation and Sensitivity Analysis

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ABSTRACT — Software has been developed for stress and Mohr's circle computation, transformation and sensitivity analysis. It is developed to help students visualize the state of stress at a point and to understand the nature and effects of stress transformation which is one of the leading importance phenomena in the field of engineering. The software shows the effect of stresses on materials inclined at different angles and subjected to axial compressive loading. The software is developed in java programming language, because it is simple, objectoriented and user friendly. The developed software calculates normal stress, tangential stress, resultant stress as well as obliquity of resultant stress with changing orientation while Mohr circle is used to support the graphics. It also shows the behaviour of stresses on materials as the angle of inclination varies.

**Keywords** — Stress, Software, Stress transformation, Mohr's Circle, Principal stress, Tangential stress, Resultant Stress, Obliquity of resultant Stress.

## I. INTRODUCTION

Since antiquated times, people have been deliberately mindful of stress inside materials. Until the 17th century, the comprehension of stress was to a great extent natural and exact; but then it brought about some shockingly complex innovation, similar to the composite bow and glass blowing (Wai-Fah and Baladi, 1985). More than a few centuries, modelers and manufacturers, specifically, figured out how to assemble precisely formed wood shafts and stone pieces to withstand, transmit, and disperse stress in the best way, with astute gadgets, for example, the capitals, curves, vaults, trusses and the flying braces of Gothic houses of prayer (Dieter, 1989). Russell et al., (1992) defined stress as a physical quantity that expresses the internal forces that neighboring particles of a continuous material exert on each other. On the other hand, Brady and Brown (1993) stated that stress inside a body may arise by various mechanisms, such as reaction to external forces applied to the bulk material (like gravity) or to its surface (like contact forces, external pressure, or friction). Similarly, stress might exist in the absence of external forces; such built-in stress is important, for example, in prestressed

concrete and tempered glass (Davis and Selvadurai 1996).

The computation and transformation of normal stress, tangential stress, resultant stress, maximum shear stress and obliquity of the resultant stress among coordinate systems is important in static and structural analysis for engineering students. Late in the last century, Mohr introduced a graphical construction to assist in the process. At Mohr's time, the technology for graphical construction was drafting and any technology for computation was quite tedious compared with modern tools. His graphical approach to constructing his circle could be used to find approximate values for transformed stresses and thereby save what, at the time, would have been great computational effort. Indeed, as recently as 1972, the construction of Mohr's circle was couched in drafting terminology (Timoshenko, 1972).

The computation and transformation of normal stress, tangential stress, resultant stress, maximum shear stress, obliquity of the resultant stress and Mohr's circle have been a burden to students. They found it difficult in obtaining accurate results. Also they spend a lot of time in calculating for different values of stresses. Mohr's circle is among the most difficult topics for students to comprehend, based on survey of their examination results. This was in spite of having spent extra time in receiving lectures many of them still find it difficult in solving problems on stresses and Mohr circle. Students also find it difficult to know the relationship between stress and angle of inclination of a body using a straight line or curve line graph. In support of the above, Stephen (1996) noted that Mohr's circle is among the most difficult topics for students to comprehend, based on survey of their examination results. In his study, students in the engineering department's strength of materials and mechanics of materials courses were surveyed in at the end of both the first and second semester. This was in spite of having spent extra time in receiving lectures many of them still find it difficult in solving problems on stresses and Mohr circle. Similarly, James (2012) has noted that it is not clear at first learning why one would want what is fundamentally the same information in a different coordinate system. The above informed the development of the software.

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### **Basic Theoretical Considerations**

Stress in a Tensile Member

Consider Figure 1. When a bar of uniform sectional area A is subjected to an axial load P, then the stress acting on a cross-section given by LL normal to the

P

axis is  ${}^{A}$  . Considering another section given by the plane MM inclined at  $\theta$  to LL, the area cut by the

plane is  $cos\theta$  Let the normal stress across MM be  $\sigma_n$ .

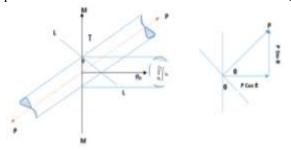


Figure 1: Stress in a Tensile Member

Resolving perpendicular to MM

$$\sigma_{n} \cdot \frac{A}{\cos \theta} = P \cos \theta$$

$$\sigma_{n} = \frac{P}{A} \cos^{2} \theta$$
(1)

Further there will be a shearing stress of T acting parallel to MM and resolving in this direction

$$\tau \cdot \frac{A}{\cos \theta} = P \sin \theta$$
$$\tau = \frac{P}{A} \sin \theta \cos \theta$$

This means that when a rod is subjected to pure tension, both tensile and shearing stresses are produced. In the material under compression, the corresponding stresses would be compressive and shearing.

Since, 
$$\tau = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{A}\frac{\sin 2\theta}{2}$$
(2a)

The greatest value of shearing stress is when  $\sin 2\theta = 1$  or  $\theta = 45^{\circ}$ 

Therefore, Maximum shearing stress produced

$$\tau_{\text{max}} = \frac{P}{2A}$$
(2b)

Stresses Due to Pure Shearing

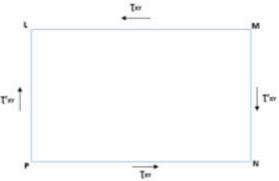


Figure 2: Stresses due to Pure Sharing

Figure 2 shows a rectangular block LMNP of unit depth perpendicular to the paper. Let shearing stresses  $T_{xy}$  act along the faces ML and PN. For equilibrium there may be neither a resultant force nor a resultant couple. Since  $T_{xy}$  ML =  $T_{xy}$  PN there is no resultant horizontal force, but the couple due to these is  $T_{xy}$  X ML X MN. If shearing stresses  $T_{xy}$  are introduced on the surfaces PL and MN to balance the outstanding couple for equilibrium, then

$$\begin{array}{ll}
T'_{xy} MN ML = T_{xy} ML MN. & (3) \\
Or, & T_{xy} = T'_{xy}
\end{array}$$

Hence, for equilibrium, complementary shearing stresses  $T_{xy}$ must be introduced.

Refer to Fig. 3. If an arbitrary plane AA cuts the block at an angle  $\theta$  to LM, the stresses acting across the plane can be determined by resolution. Let the direct and shearing stresses across AA be $B_n$ and T respectively.

Resolving perpendicular to AA:  $\sigma_n \ x \ LN = \prod_{xy} LM \ Sin \ \theta \ + \prod_{xy} MN \cos \theta \ .$   $\sigma_n = \prod_{xy} \frac{LM}{LN} \sin \theta \ + \prod_{xy} \frac{MN}{LN} \cos \theta \ .$   $= \prod_{xy} \cos \theta \ \sin \theta \ + \prod_{xy} \sin \theta \cos \theta \ .$   $= 2 \prod_{xy} \cos \theta \sin \theta \ .$   $= 2 \prod_{xy} \sin 2\theta \ .$  (4)

The maximum value will occur when  $2\theta = -90^{\circ}$  or  $B_n = -T_{xy}$  (compressive). In other words a case of a pure shear is equivalent to a direct tensile stress and a direct compressive stress acting perpendicular to each other.

Resolving parallel to AA:  $T LN = T_{xy} MN \sin \theta - T_{xy} LM \cos \theta$   $T = T_{xy} \frac{MN}{LN} \sin \theta - T_{xy} \frac{LM}{LN} \cos \theta$   $= T_{xy} \sin \theta \sin \theta - T_{xy} \cos \theta \cos \theta$   $= T_{xy} (\sin^2 \theta - \cos^2 \theta)$   $= (-)T_{xy} \cos 2 \theta$ 

Therefore,  $\uparrow$  will be zero when  $\theta = \pm 45^{\circ}$ 

Two Mutually Perpendicular Direct Stresses

Refer to Fig: 3.3. At any point in a material where stress is acting, it is possible to assume that the point consists of a very small triangular block, such that the stresses act across the faces of the block. Consider that direct stresses  $\mathbf{B}_x$  and  $\mathbf{b}_y$  act across the faces LM and

MN and that the block has unit depth perpendicular to LMN. Let the stresses T and  $B_n$  act on the same plane at an angle  $\theta$  to LM.

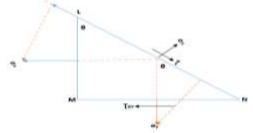


Figure 3: Two Mutually Perpendicular Direct Stress

Resolving normal to LN: Resolving normal to Liv.  $\sigma_{n} LN = \sigma_{x} LM \cos\theta + \sigma_{x}MN \sin\theta$   $\sigma_{n} = \sigma_{x} \frac{LM}{LN} \cos\theta + \sigma_{y} \frac{MN}{LN} \sin\theta$   $= \sigma_{x} \cos^{2}\theta + \sigma_{y} \sin^{2}\theta = \frac{\sigma_{x}}{2} 2 \cos^{2}\theta + \frac{\sigma_{y}}{2} 2 \sin^{2}\theta$   $= \frac{\sigma_{x}}{2} (1 - \sin^{2}\theta + \cos^{2}\theta) + \frac{\sigma_{y}}{2} (1 - \cos^{2}\theta + \cos^{2}\theta)$  $\sin^2\theta$ ) 
$$\begin{split} &= \frac{\sigma x + \sigma y}{2} + \sigma_x \; (\frac{\cos 2 \square - \sin 2 \square}{2}) - \sigma_y \; (\frac{\cos 2 \square - \sin 2 \square}{2}) \\ &= \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos \; \square \end{split}$$
When  $\theta = 0$ , then  $\sigma_n = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} = \sigma_x$ And, when  $\theta = \frac{\pi}{2}$ , then  $\sigma_n = \frac{\sigma x + \sigma y}{2} - \frac{\sigma x - \sigma y}{2} = \sigma_y$ Resolving parallel to LN:  $\begin{array}{l}
\text{T} \quad \text{LN} = \sigma_{x} \text{ LM } \sin\theta - \sigma_{y} \text{ } MN \cos\theta \\
\text{T} = \sigma_{x} \frac{LM}{LN} \sin\theta - \sigma_{x} \frac{MN}{LN} \cos\theta \\
= \sigma_{x} \cos\theta \sin\theta - \sigma_{y} \sin\theta \cos\theta = (\sigma_{x} - \sigma_{y} \cos\theta) \cos\theta
\end{array}$ 

$$\sigma_{y}$$
)  $\sin\theta\cos\theta$ 

$$= \frac{\sigma x - \sigma y}{2}\sin \Box$$

(7)

The maximum value of T occurs when  $2\theta = \frac{\pi}{2}$  or  $\theta$  $= \frac{\pi}{4} \text{ and then,}$   $T_{\text{max}} = \frac{\sigma x - \sigma y}{2}$ 

The resultant stress,

$$\sigma_{\rm r} = \sqrt{(\sigma_{\rm n}^2 + T^2)}$$

 $\tan^{\emptyset} = \frac{1}{\sigma n}$  where  $^{\emptyset}$  is the angle which the resultant stress makes with the normal to the plane and is called obliquity.

Two- Dimensional Stress System (General)

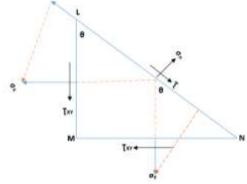


Figure 4: Two-Dimensional Stress System

When the stresses at some point are considered to be acting on a small triangular block at a point, then a general stress system will consist of direct and shearing stresses acting across the faces of the block. Consider (Figure 4) some plane LN at angle  $\theta$  to the plane of the stress.

Resolving normal to LN:

 $\sigma_n \ LN = T_{xy} \ LM \ sin \ \theta \ + \sigma_x \ LM \ cos \ \theta \ + T_{xy} \ MN$ 

 $\cos \theta + \sigma_{y} MN \sin \theta.$ Therefore,  $\sigma_{n} = T_{xy} \frac{LM}{LN} \sin \theta + \sigma_{x} \frac{LM}{LN} \cos \theta + T_{xy} \frac{MN}{LN}$  $\cos\theta + \sigma_{y} \frac{MN}{LN} \sin\theta$ 

 $= \int_{xy}^{L/v} \cos \theta \sin \theta + \sigma_x \cos^2 \theta + \int_{xy}^{L/v} \sin \theta \cos \theta + \sigma_y \sin^2 \theta$ 

$$= 2 \prod_{xy} \sin \theta \cos \theta + \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta$$

$$= \prod_{xy} \sin 2\theta + \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta$$
(8)

Resolving along LN:

Therefore,  $T = T_{xy} \frac{MN}{LN} \sin \theta + \sigma_x \frac{LM}{LN} \sin \theta - T_{xy} \frac{LM}{LN} \cos \theta - \sigma_y \frac{MN}{LN} \cos \theta$ 

 $\begin{array}{c} & = \prod_{xy} \sin \theta \, \sin \theta \, + \sigma_x \! \cos \theta \, \sin \theta \, - \\ \prod_{xy} \! \cos \theta \, \cos \theta \, - \sigma_y \sin \theta \, \cos \theta \\ & = \prod_{xy} \sin^2 \theta \, + \sigma_x \, \sin \theta \, \cos \theta \, - \prod_{xy} \\ \cos^2 \! \theta \, - \sigma_y \sin \theta \, \cos \theta \end{array}$ 

$$= \int_{xy} (\sin^2 \theta - \cos^2 \theta) + (\frac{\sigma x - \sigma y}{2})$$

$$= \left(\frac{\sigma x - \sigma y}{2}\right) \sin 2\theta - T_{xy} \cos 2\theta$$

(i) In order to find out the principal stresses, the maximum and minimum values of  $\mathbf{b}_n$  must be obtained.

Differentiating  $\sigma_n$  w.r.t. $\theta$  in equation (6),

$$\frac{d(\sigma n)}{d\Box} = 2 T_{xy} \cos 2\theta - \frac{2(\sigma x - \sigma y)}{2} \sin 2\theta$$

Equating this to zero for maximum  $B_x$ , we get  $0 = (\frac{\sigma x - \sigma y}{2}) \sin 2\theta - T_{xy} \cos 2\theta$ 

Comparing with equation (7), we have T = 0

Hence for a principal plane there may be no shear

stress acting. Also, 
$$(\frac{\sigma x - \sigma y}{2}) \sin 2\theta = T_{xy} \cos 2\theta$$

Or, 
$$\tan 2\theta = \frac{2xy}{\sigma x - \sigma y}$$
 (10a)

It follows that for any particular system, the principal stresses may be calculated by considering the planes which carry shearing stress (Fig 2.5).

> To get maximum value differentiating T w.r.t.  $\theta$  in eqn. (7) and equating to zero, we get

and equating to zero, we get
$$\frac{d()}{d\Box} = \left(\frac{\sigma x - \sigma y}{2}\right) 2\cos 2\theta + 2 T_{xy} \sin 2\theta = 0$$
Or,  $(\sigma x - \sigma y) \cos 2\theta = -2 T_{xy} \sin 2\theta$ 
Or,  $\tan 2\theta = -\left(\frac{\sigma x - \sigma y}{2xy}\right)$ 

Also,  $\cot (180-2\theta_1) = -\cot 2\theta_1$ 

And,  $\cot (360-2\theta_2) = -\cot 2\theta_2$ 

Where  $\theta_1$  and  $\theta_2$  are the inclination of maximum shear stress with the plane of tensile stress  $\sigma_x$ 

Therefore, 
$$\cot (180-2\theta_1) = (\frac{2xy}{\sigma x - \sigma y})$$

(10b)

And, cot 
$$(360-2\theta_2) = (\frac{2xy}{\sigma x - \sigma y})$$

Resolving normal to LM:

$$\sigma_n LM + T_{xy} x MN = \sigma_n LM \cos \theta$$

Dividing both side by LN, we get

$$\frac{\sigma_{x}\cos\theta + T_{xy}\sin\theta}{LN} = \frac{\sigma_{n}\cos\theta}{LN} = \frac{\sigma_{n}\cos\theta}{LN}$$

$$\frac{LM}{LN} = \cos\theta, \frac{MN}{LN} = \sin\theta$$

 $\sigma_x + T_{xy} \tan \theta = \sigma_n$ 

Resolving parallel to LM:

$$\sigma_{y} MN + T_{xy} LM = \sigma_{n} LN \sin\theta$$

Dividing both sides by LN, we get

$$\sigma_y \sin \theta + T_{xy} \cos \theta = \sigma_n \sin \theta$$

$$\sigma_{y} + T_{xy} \cot \theta = \sigma_{n}$$

(2.11a)

$$\begin{array}{l} T_{xy}^2 = (\sigma_n - \sigma_x)(\sigma_n - \sigma_y) \\ \text{Or,} \end{array}$$

$$T_{xy}^2 = \sigma_n^2 - (\sigma x + \sigma y) \sigma_n + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y + \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y^2 = \sigma_y^2 - (\sigma x + \sigma y) \sigma_y^2 +$$

Solving, 
$$\sigma_n$$
 (= $\sigma$ ) say
$$\frac{\sigma x + \sigma y}{2} \pm \frac{1}{2} \sqrt{(\sigma x + \sigma y)^2 - 4\sigma x \sigma y + 4^2 x y}$$
That is,  $\sigma = \frac{\sigma x + \sigma y}{2} \pm \frac{1}{2} \sqrt{(\sigma x + \sigma y)^2 + 4^2 x y}$ 

That is, 
$$\sigma = \frac{\sigma x + \sigma y}{2} \pm \frac{1}{2} \sqrt{(\sigma x + \sigma y)^2 + 4^2 x^2}$$
(12)

Therefore, Major Principal Stress,  

$$\sigma_1 = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + 2xy}$$
(12a)

$$\sigma_2 = \frac{\sigma x + \sigma y}{2} - \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2} + \frac{2}{2}xy$$
(12b)

Also, 
$$T_{\text{max}} = \frac{\sigma x - \sigma y}{2}$$
 or  $\tan 2\theta = \frac{2 xy}{\sigma x - \sigma y}$   
(Also  $\sigma_1 + \sigma_2 = \sigma x + \sigma y$ )

More Circle Construction for Like stresses

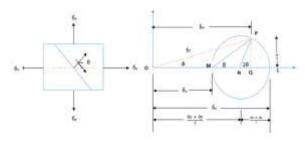


Figure 5: Mohr's Circle Construction for 'Likes Stress'

Now, from, stress diagram  $NP = NL = \frac{x-y}{2}$  $PQ = NP\sin^2\theta = \frac{x-y}{2}\sin^2\theta = T$ Similarly,  $OQ = ON + NQ = \frac{x+y}{2} + \frac{x-y}{2} \cos 2\theta = B_n$ Also, from stress circle, T is maximum when  $2\theta = 90^{\circ}$ , or  $45^{\circ}$ And,  $T_{\text{max}} = \frac{x-y}{2}$ 

In case  $B_x$  and  $B_y$  are not like, the same procedure will be followed except that  $\mathbf{b}_x$  and  $\mathbf{b}_y$  will be measured to the opposite sides of the origin. The construction is given in Fig: 3.6. it may be noted that the direction of  $\mathbf{b}_n$  will depend upon its position with respect to the point O. if it is to the right of O, the direction of  $\mathbf{B}_n$  will be the same as that of  $\mathbf{B}_x$ .

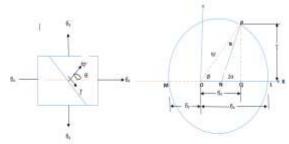


Figure 6: Mohr's Circle Construction for 'Unlike Stress'

Considering the difficulty for manual computation, Java programming language was used to program the mathematical formulations. With the program, The desired result can be determined by clicking the buttons. If the initial stresses and angle of inclination are provided, different results like normal, tangential, resultant and maximum stresses are determined including the obliquity of the resultant stress. The main window of the software is shown in figure 7 and figure 8.

The program is divided into two parts, namely; stress computation and Mohr's circle.

The stress computation is sub-divided into two. Type I and type II. Type I involves stress computation where parameters like cross sectional area, axial compressive load and angle of inclination are needed to compute the values of various stresses and obliquity of resultant stress, while Type II solves similar problems, but with different parameters like principal stresses, shear stress and angle of inclination, these can be achieve by clicking on the input value button. Type II gives the values of all the stresses at the same type while type I calculate for the stresses separately. Also these results can be obtained graphically by using Mohr's circle. It consists of two axes namely; x-axis with give the value of the normal stresses and the y-axis the give the value of the shear stress including the maximum shear stress.



Figure 7: The Main Window of the Software for Stress Computation

Resultant stress and obliquity of resultant stress can also be obtained by drawing a line and an arc through the curve in the circle. In the main window (stress computation and Mohr's circle), some spaces are provided to input parameters such as principal stresses and angle of inclination, these stresses are sign sensitive.

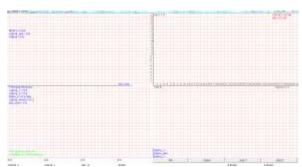


Figure 8: The Main Window of the Software for Mohr Circle

After inputting the parameters in the space provided in the main window, one can find the value of normal stress, tangential/shear stress, resultant stress, maximum shear stress and obliquity of resultant stress by a click of button as shown in figure 9 and 10.



Figure 9: Normal Stress, Tangential/Shear Stress, Resultant Stress, Maximum Shear Stress and Obliquity of Resultant

72.845	
Tangential Stress (metre	Newton per meter square)
46.09	
Resultant Stress (metre	Newton per meter square)
86.202	
Obliquity of Resultant Str	ess (in degrees)
32 32221718959704	

Figure 10: Normal Stress, Tangential/Shear Stress, Resultant Stress, Maximum Shear Stress and Obliquity of Resultant Stress

Finding the Mohr's Circle of Stresses

To see the Mohr's circle of stresses for the two dimensional stresses, it is needed to click the button. A new form will be opened just like figure 8.

Then it is needed to press the this new form. Then the Mohr's circle of stresses will be found in the new form. Result of the Mohr's circle he obtained clicking can by Draw button and the Mohr's circle can be made more visible by pressing the Fill This Mohr's button. magnified circle can be (+) by clicking the size 2 and reduced by size 1 pressing the button.

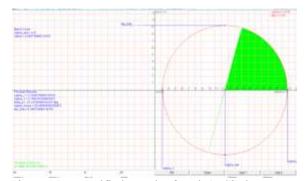


Figure 11: Magnified Result of Mohr's Circle

# **Results and Discussion**

Sensitivity analysis of a model can help determine relative effects of model parameters on model results (Okonkwo, 2009). In other words, the purpose of sensitivity testing of a model is to investigate whether a slight perturbation of the parameter values will result in a significant perturbation of the model results, that is, the internal dynamics of the model (Okonkwo et. al,

2014). With the use of the software, sensitivity analysis was carried out to determine the impact of variations of the input model on the output. In this way the understanding of the relationships between input and output variables of the system can be determined. Also, it enables the identification of the input models that cause significant uncertainty in the output models.

Figure 12 is an exponential curve, whose gradients vary from point to point, depending on the point of interest. The force which acts perpendicularly to the plane to which a force has been applied (normal stress) increases as the angle of inclination increases until maximum stress is reached at about 65°. Any further increase in stress will cause the material to experience deformations which will not be completely reversed upon removal of the loading and as a result the member will have a permanent deflection.

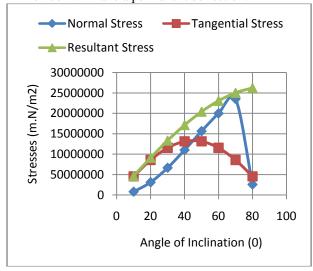


Figure 12: Relationship between Angle of inclination and stresses subjected to axial compressive load

This stress starts declining as the angle of inclination increases until it becomes zero at 90° due to continuous increase in the angle of inclination while the axial loading is kept constant. The component of force acting along the plane of area or stress state where the stress is parallel to the surface of the material increases as the angle of inclination of the material increase until it get to 45° where maximum stress is obtained. As the angle increases, that is above 45°, this stress starts reducing until tangential/shear stress is no longer excerted on the material. The material possesses a resultant stress which continues to increase as the angle of inclination of the material increases. At 10° the material possesses equal tangential and resultant stress. At about 45°, the material has equal normal and tangential stress. At about 70°, the material has equal normal and resultant stress. The increase in internal forces (stresses) within the member is due to continuous increase in load applied to the mechanical member and angle of

inclination. The stress acting on the material causes deformation of the material, while the declination of the stress is due to constant axial compressive loading of the material and further increase in the angle of its inclination

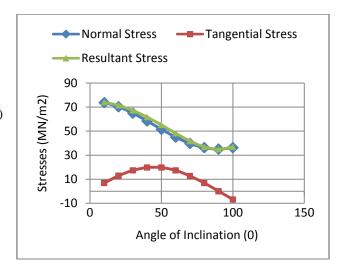


Figure 13: Relationship between angle of inclination and stresses perpendicular planes subjected to principal stresses and shear stress

Figure 13 shows the relationship between angle of inclination and stresses. The plot is an exponential graph whose gradients differ from point to point along the curve. Increase in the angle of inclination decreases both in normal and resultant stresses. At 90° the material possesses the same normal and resultant stress. For the tangential stress, it continues to increase until the angle of inclination is 50°. At this point, slight increase in stress, makes the material to exceed its ultimate strength. Further increase in the angle, causes decrease in the stress and become zero at 90°, due to steady shear stress. Maximum stress is exhibited in the material at 10°. Thus, increase in the angle of inclination of a material led to decrease in normal and resultant stress and increase in the tangential stress vice versa. The negative values of tangential stress are due to the increase in the angle of inclination beyond 90°, while the Principal stresses and Shear stresses are kept stationary. The material experiences equal normal and resultant stresses between (10°-20°) and (80°-100°).

# Conclusion

Software for stress and Mohr's circle computation and transformation has been developed. The authors hope that the person who has engineering, or physics or applied physics background will be able to use this software very easily understanding each of the parts of the software with the theoretical background. The software can showcases to a student or researcher how stresses exerted on a material can undergo changes as the angle of inclination changes. The normal and resultant stress exerted on a material is equal when the

angle of its inclination is 90°. The normal and resultant stresses exerted on a material are always greater than the tangential stress. A material will not experience tangential stress when the angle of inclination is 90°, even when subjected to axial or compressive loading. A material can undergo failure when stresses exerted on it exceed its ultimate strength. Also, the Mohr Circle can be drawn within seconds.

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