A New Approach to Monte Carlo Simulation of Operations

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***Abstract*—Monte Carlo simulation are a wide class of computational algorithms that use repeated random sampling to arrive at numerical results. In other words, a simulation is run repeatedly in order to obtain the distribution of an unknown probabilistic entity. In this paper we have proposed a new approach to Monte Carlo simulation of operations thereby optimizing multi -server operations. A case study of a hospital is presented. Besides we have analysed the Monte Carlo methods against the deterministic methods.**

***Keywords*— Monte Carlo, optimization, multi queuing, waiting time, service time**

I. Introduction

Monte Carlo Simulation is a computerized mathematical technique that allows people to account for risk in quantitative analysis and decision making. It furnishes the decision-maker with a range of possible outcomes and the probabilities with which they will occur for any choice of action.

Monte Carlo simulation performs risk analysis by building models of possible results by substituting a range of values—a *probability distribution*—for any factor that has inherent uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending upon the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete. Monte Carlo simulation produces distributions of possible outcome values.

II. LITERATURE REVIEW

Monte Carlo simulations play an important role in computational science and engineering, with applications ranging from materials science to biology to quantum physics [1]. They also play an important role in a variety of other fields, including computer imaging, architecture, and economics. The background of Monte Carlo methods in various domain is given as under:

- In microelectronics engineering, Monte Carlo methods are applied to analyze correlated and uncorrelatedvariations
  in analog and digital integrated circuits.
- In geostatistics and geometallurgy [5], Monte Carlo methods underpin the design of mineral processing flow sheets and contribute to quantitative risk analysis.
- In wind energy yield analysis [6], the predicted energy output of a wind farm during its lifetime is calculated giving different levels of uncertainty.
- Monte Carlo approaches are an attractive option for turbulence simulations [3] due both to their capacity for investigating systems with many degrees of freedom and to their natural generation of a disordered velocity field structure and irregular particle trajectories.
- In aerospace engineering [7], Monte Carlo methods are used to ensure that multiple parts of an assembly will fit into an engine component.
- Path Tracing, occasionally referred to as Monte Carlo Ray Tracing, renders a 3D scene by randomly tracing samples of possible light paths. Repeated sampling of any given pixel will eventually cause the average of the samples to converge on the correct solution of the rendering equation [4], making it one of the most physically accurate 3D graphics rendering methods in existence.
- Monte Carlo methods have been developed into a technique called Monte Carlo tree search [2] that is useful for searching for the best move in a game. Possible moves are organized in a search tree and a large number of random simulations are used to estimate the long-term potential of each move. A black box simulator represents the opponent's moves. Monte Carlo Tree Search has been used successfully to play many games.
- Monte Carlo methods in finance are often used to evaluate investments in projects at a business unit or corporate level, or to evaluate financial derivatives. They can be used to model project schedules, where simulations aggregate estimates for worst-case, best-case, and most likely durations for each task to determine outcomes for the overall project.
- Monte Carlo methods provide a way out of exponential increase in computation time. As long as the function in question is reasonably well-behaved, it can be estimated by

randomly selecting points in 100-dimensional space, and taking some kind of average of the function values at these points. By the central limit theorem, this method displays $1/\sqrt{N}$ convergence—i.e., quadrupling the number of sampled points halves the error, regardless of the number of dimensions [1].

III. METHODOLOGY

By using probability distributions, variables can have different probabilities of different outcomes occurring. Probability distributions are a much more realistic way of describing uncertainty in variables of a risk analysis. Common probability distributions include:

- Normal – Or “bell curve.” The user simply defines the mean or expected value and a standard deviation to describe the variation about the mean. Values in the middle near the mean are most likely to occur. It is symmetric and describes many natural phenomena such as people’s heights.
- Lognormal – Values are positively skewed, not symmetric like a normal distribution. It is used to represent values that don’t go below zero but have unlimited positive potential. Examples of variables described by lognormal distributions include real estate property values, stock prices, and oil reserves.
- Uniform – All values have an equal chance of occurring, and the user simply defines the minimum and maximum. Examples of variables that could be uniformly distributed include manufacturing costs or future sales revenues for a new product.
- Triangular – The user defines the minimum, most likely, and maximum values. Values around the most likely are more likely to occur. Variables that could be described by a triangular distribution include past sales history per unit of time and inventory levels.
- PERT- The user defines the minimum, most likely, and maximum values, just like the triangular distribution. Values around the most likely are more likely to occur. However values between the most likely and extremes are more likely to occur than the triangular; that is, the extremes are not as emphasized.
- Discrete – The user defines specific values that may occur and the likelihood of each. An example might be the results of a lawsuit: 20% chance of positive verdict, 30% change of negative verdict, 40% chance of settlement, and 10% chance of mistrial.

During a Monte Carlo simulation, values are sampled at random from the input probability distributions. Each set of samples is called iteration, and the resulting outcome from that sample is recorded. Monte Carlo simulation does this hundreds or thousands of times, and the result is a probability distribution of possible outcomes. In this way, Monte Carlo simulation provides a much more comprehensive view of what may happen. It tells you not only what could happen, but how likely it is to happen.

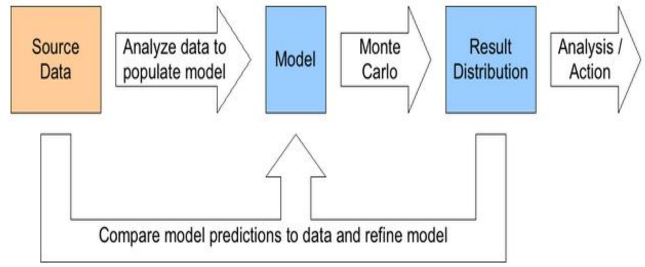

Fig.1. Monte Carlo simulation

The figure above represents Monte Carlo simulation of operations.

IV. ILLUSTRATIVE SCENARIO

The example given under studies the arrival rate of patients in a hospital and performs Monte Carlo simulation to determine the average waiting time for each patient and the average service time for each operation in a two-hour time period.

The following table lists the time required for the various operations in a hospital.

Table.1 Time required for various operations in a Hospital

OPERATION	TIME REQUIRED	PROBABILITY OF OPERATION
Blood Test	5	0.40
Root Canal treatment	30	0.15
Laser Therapy	20	0.20
X Ray	20	0.10
OPD	15	0.10
USG	15	0.05

The Monte- Carlo method is used for simulating the given problem. The solution is given as under:

Random numbers used are:

R = 86, 21, 42, 13, 52, 66, 92, 61, 34, 73, 87, 12, 28.

Table 2 Monte Carlo Simulation of Operations

OPERATION	TIME REQUIRED (in minutes)	PROABABITY OF OPERATION	CUMULATIVE PROBABILITY	RANDOM INTERVAL	RANDOM NUMBER FILLED
Blood Test	5	0.40	0.40	0-29	21(2), 13(4), 12(2), 28(13)
Root Canal treatment	30	0.15	0.55	30-44	42(3), 34(7)
Laser Therapy	20	0.20	0.75	45-59	52(5)
X Ray	20	0.10	0.85	60-74	66(6), 61(8), 73(10)
OPD	15	0.10	0.95	75-89	86(1), 87(11)
USG	15	0.05	1.00	90-100	92(7)

Table.3. Waiting time of each of the operations

ARRIVAL TIME	OPERATION STARTS AT	OPERATION ENDS AT	WAITING TIME (in minutes)
3:00	3:00	3:15	0
3:10	3:15	3:20	5
3:20	3:20	3:50	0
3:30	3:50	3:55	20
3:40	3:55	4:15	15
3:50	4:15	4:35	25
4:00	4:35	4:50	35
4:10	4:50	5:10	40
4:20	5:10	5:40	50
4:30	5:40	6:00	70
4:40	6:00	6:15	80
4:50	6:15	6:20	85
5:00	6:20	6:25	80

The final values are as under:

Average waiting time for each person = 505/13 = **38.846 minutes.**

Average service time for each operation = 205/13 = **15.76 minutes.**

V. RESULTS AND DISCUSSIONS

Monte Carlo simulation is a valuable technique for analyzing risks, specifically those related to time and schedule. The fact that it is based on numeric data gathered by running multiple simulations adds even greater value to this technique. We have taken six operations in a hospital in duration of two hours considering its probability and time required for each operation.

We have observed Average waiting time for each person is 38.846 minutes and Average service time for each operation 15.76 minutes. Though there are numerous benefits of the Monte Carlo simulation, the reliability of the outputs depends on the accuracy of the range values and the correlation patterns.

There are multiple advantages of using Monte Carlo Simulations. They are given as under:

- The results obtained from Monte Carlo simulation not only reveal what could possibly happen but also the extent of possibility for each outcome.
- Due to the data generated by a Monte Carlo simulation, it becomes easier to create graphs of various outcomes as well as their chances of occurrence. This is imperative for communicating findings to other stakeholders.
- With just a few cases, it becomes difficult with deterministic analysis to look for the variables which affect the outcome the most. In Monte Carlo simulation, it is easier to find inputs showing the largest impact on bottom-line results.
- In Monte Carlo simulation, it is possible to form independent relationships between input variables. Moreover, it is important for precision to signify how, actually, when certain factors go up, other go down correspondingly.

VI. APPLICATIONS

Monte Carlo methods are especially useful for simulating phenomena with significant uncertainty in inputs and systems with a large number of coupled degrees of freedom. Areas of application include research, engineering, geophysics, meteorology, and computer applications, public Health studies, and finance. Few explanations are as follows:

- Engineering:

  Monte Carlo methods are widely used in engineering for sensitivity analysis and quantitative probabilistic analysis in process design. The need arises from the interactive, co-linear and non-linear behavior of typical process simulations.

- Computer Graphics:

  Path Tracing, occasionally referred to as Monte Carlo Ray Tracing, renders a 3D scene by randomly tracing samples of possible light paths. Repeated sampling of any given pixel will eventually cause the average of the samples to converge on the correct solution of the rendering equation, making it one of the most physically accurate 3D graphics rendering methods in existence.

- Computational biology:

  Monte Carlo methods are used in computational biology, such for as Bayesian inference in phylogeny. Biological systems such as proteins membranes, images of cancer, are being studied by means of computer simulations.

  The systems can be studied in the coarse-grained or ab initio frameworks depending on the desired accuracy. Computer simulations allow us to monitor the local environment of a particular molecule to see if some chemical reaction is happening for instance. We can also conduct thought experiments when the physical experiments are not feasible, for instance breaking bonds, introducing impurities at specific sites, changing the local/global structure, or introducing external fields.

- Design and visuals:

  Monte Carlo methods are also efficient in solving coupled integral differential equations of radiation fields and energy transport, and thus these methods have been used in global illumination computations that produce photo-realistic images of virtual 3D models, with applications in video games, architecture, design, computer generated films, and cinematic special effects.

- Physical sciences:

  Monte Carlo methods are very important in computational physics, physical chemistry, and related applied fields, and have diverse applications from complicated quantum chromodynamics calculations to designing heat shields and aerodynamic forms. In statistical physics Monte Carlo molecular modeling is an alternative to computational molecular dynamics, and

Monte Carlo methods are used to compute statistical field theories of simple particle and polymer systems. Quantum Monte Carlo methods solve the many-body problem for quantum systems. In experimental particle physics, Monte Carlo methods are used for designing detectors, understanding their behavior and comparing experimental data to theory. In astrophysics, they are used in such diverse manners as to model both the evolution of galaxies and the transmission of microwave radiation through a rough planetary surface. Monte Carlo methods are also used in the ensemble models that form the basis of modern weather forecasting.

VII. COMPARITIVE ANALYSIS

Monte Carlo simulation provides a number of advantages over deterministic, or “single-point estimate” analysis:

- Probabilistic Results. Results show not only what could happen, but how likely each outcome is.
- Graphical Results. Because of the data a Monte Carlo simulation generates, it’s easy to create graphs of different outcomes and their chances of occurrence. This is important for communicating findings to other stakeholders.
- Sensitivity Analysis. With just a few cases, deterministic analysis makes it difficult to see which variables impact the outcome the most. In Monte Carlo simulation, it’s easy to see which inputs had the biggest effect on bottom-line results.
- Scenario Analysis: In deterministic models, it’s very difficult to model different combinations of values for different inputs to see the effects of truly different scenarios. Using Monte Carlo simulation, analysts can see exactly which inputs had which values together when certain outcomes occurred. This is invaluable for pursuing further analysis.
- Correlation of Inputs. In Monte Carlo simulation, it’s possible to model interdependent relationships between input variables. It’s important for accuracy to represent how, in reality, when some factors go up, others go up or down accordingly.

VIII. CONCLUSIONS

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; i.e., by running simulations many times over in order to calculate those same probabilities heuristically just like actually playing and recording your results in a real casino situation: hence the name.

They are often used in physical and mathematical problems and are most suited to be applied when it is impossible to obtain a closed-form expression or infeasible to apply a deterministic algorithm. Monte Carlo methods are mainly used in three distinct problems: optimization, numerical integration and generation of samples from a probability distribution.

Monte Carlo methods are especially useful for simulating systems with many coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures (see cellular Potts model). They are used to model phenomena with significant uncertainty in inputs, such as the calculation of risk in business. They are widely used in mathematics, for example to evaluate multidimensional definite integrals with complicated boundary conditions. When Monte Carlo simulations have been applied in space exploration and oil exploration, their predictions of failures, cost overruns and schedule overruns are routinely better than human intuition or alternative "soft" methods.

ACKNOWLEDGMENT

The author would like to thank Supriya Moond, Devangana Arora and Palak Toshniwal for their contribution in this research work. The author would also like to thank Prof Sayiram G for his constant guidance and support throughout the research work.

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