Quantized Coefficient F.I.R. Filter for the Design of Filter Bank

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***Abstract* - This paper presents a very simple and efficient Quantized coefficient finite impulse response (FIR) low pass filter design procedure. This involves approximation of a quantized coefficient FIR filter by rounding operation to design a filter bank. The prototype filter is designed using rounding technique to provide quantized coefficient FIR filter which is computationally efficient. The rounding factors, requiring minimum to maximum number of multipliers, are used to show the performance of the designed filter bank. In that way the filter is based on combining one simple filter with integer coefficients. Our analysis indicates that utilizing this approach the required numbers of total non-zero bits become quite low and less multiplier and adders will be employed in the design of filter bank to make it computationally efficient**.

***Keywords*-FIR filter, filter bank, rounding.**

I. INTRODUCTION

Finite impulse response (FIR) filters, find widespread application in various fields since they can be designed with exact linear phase and exhibit no stability problems. Finite impulse response (FIR) filters are often preferred to infinite Impulse response (IIR) filters because of their low phase distortion. But FIR filters have more computational complexity compared to infinite impulse response (IIR) filters with equivalent magnitude responses. Many design methods have been proposed to reduce the complexity of the FIR filters.

In most applications, the required number of multipliers is excessively large compared to an equivalent IIR filter. An approach to the reduction of computational complexity is to reduce the number of non-zero bits in every multiplier coefficient to a very small number so that the multiplication can be implemented by a few shifts and add operations. However, their application generally requires more computation. Several techniques have been developed to improve FIR filter efficiency in terms of computational requirements [7]-[12]. As suggested by A. Mehrnia and A. Willson [13], IFIR technique reduces number of multipliers and adders at the cost of increased system delay known as cascaded structure design and its generalisation is FRM technique [14]. Recently Gordana et al. [1] has suggested a technique to design a computationally efficient multiplier-free FIR filter based on rounding operation. By FIR filter coefficient rounding we may design multiplier-less filters which can be utilized in many signal processing applications in the field of uniform and non-uniform filter bank.

In this work, Cosine Modulated filter bank is used for the design of filter bank and then by rounding operation we have tried to reduce the computational complexity for the designed filter bank. Among the different classes of multi-channel filter banks, cosine-modulated filter banks are the most frequently used filter banks due to simpler design, where analysis and synthesis filter banks are derived by cosine modulating a low-pass prototype filter. The design of whole filter bank thus reduces to that of a single low-pass prototype filter. Cosine modulated filter bank (CMFB) finds wide application in different areas of digital signal processing such as equalization of wireless communication channel, sub band coding, spectral analysis, adaptive signal processing, denoising, feature detection and extraction. Several design techniques have been developed for these filter banks in the past [1], [2], [3]. In CMFB, analysis and synthesis filters are cosine modulated versions of low pass prototype filter. Thus, the design of whole filter bank reduces to that of the prototype filter and the cost of overall filter bank is almost equal to the cost of one filter with modulation overheads.

II. DESIGN OF PROTOTYPE FILTER USING WINDOW FUNCTION

For this work, Blackman window is used for the design of the prototype filter for CM filter bank. The impulse response coefficients of a causal *N*th-order linear phase FIR filter *h*(*n*) using window technique is given by Eq.

$$h(n) = h_i(n).w(n) \tag{1}$$

Where

$$h_i(n) = \frac{\sin(w_c(n-N/2)}{\pi(n-N/2)} \tag{2}$$

is an ideal impulse response filter with cut-off frequency (w_c), while *w*(*n*) is a window function of order *N*. The filter order *N* and transition width $\Delta\omega$ is estimated as

$$N = 5.5/\Delta w \tag{3}$$

$$\Delta w = (w_s - w_p)/2\pi \quad (4)$$

For a given value of pass band (w_p) and stop band (w_s), the required number of adders and multipliers are equal to (N) and (N+1)/2, respectively. In case of FIR filters the filter order is inversely proportional to the transition bandwidth. Thus, for narrow transition bandwidth, filter order becomes very high, and hence number of adders and multipliers are also high. Therefore, researchers are putting efforts on developing computationally efficient design techniques. In this proposed work, computationally efficient and multiplier-less design technique of FIR filters is used to design the prototype low pass filter as suggested by Mitra *et al*. [1].

III. ROUNDING TECHNIQUE

The rounding technique is applied on window based FIR filter to satisfy the given specifications. The technique is briefly described in this section.

The impulse response rounding is given by

$$h(n) = \alpha \times g_i(n) = \alpha \times round\,(h\,(n)/\alpha) \quad (5)$$

Where, $h(n)$ is an impulse response of the FIR filter which satisfies the given specifications, *round* (.) means the rounding operation, $g_i(n)$ is the new impulse response derived by rounding all the coefficients of $h(n)$ to the nearest integer. The rounded impulse response $gi\ (n)$ is scaled by a factor α which determines the precision of the approximation of $g(n)$ to $h(n)$. The rounding constant is chosen in the form of $\alpha = 2^{-N}$. Where N is an integer. The process of rounding introduces some null coefficients in the rounded impulse response. The number of nonzero integer coefficients corresponds to the number of the sums and the number of integer multiplications corresponds to the number of a different positive integer coefficients. Computational complexity is expressed in terms of number of integer multiplications, which itself depends on rounding constant.

IV. COSINE MODULATED FILTER BANK

Cosine modulation is the cost effective technique for M-band filter bank [4], [5], [6] as shown in Fig 1.Cosine Modulated Filter banks (CMFB) are widely used in different multirate applications. The main advantages of the Cosine Modulated filter banks are that, they have computationally efficient design and all the coefficients are real. It is sufficient to design only the prototype filter. All the analysis and synthesis filters are derived from this filter by cosine modulation. Since all the analysis and synthesis filters are modulated versions of the prototype filter, the shapes of their amplitude responses are the same as those of the prototype filter. Let $H\,(z)$ denote the transfer function of the prototype filter, then

$$H(z) = \sum_{n=0}^{N-1} h(n).z^{-n} \quad \text{With } h(n)=h(n\text{-N}) \quad (6)$$

Where, N is order of the prototype filter for the analysis and synthesis sections. The analysis and synthesis filters are determined by

$$H_k(n) = 2h(n)\cos(\tfrac{\pi}{M}(k+0.5)\left(n-\tfrac{N}{2}\right)+(-1)^k\tfrac{\pi}{4}) \quad (7)$$

$$F_k(n) = 2h(n)\cos(\tfrac{\pi}{M}(k+0.5)\left(n-\tfrac{N}{2}\right)-\left(-1\right)^k\tfrac{\pi}{4}) \quad (8)$$

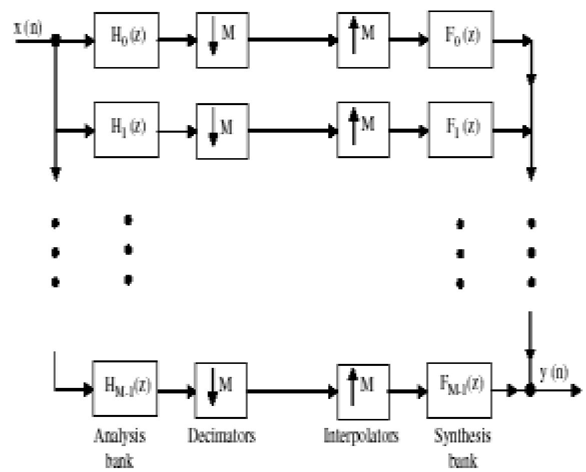

Fig. 1. M-Channel Uniform Filter Bank.

The conventional M channel maximally decimated filter bank is shown in Fig. 1. Based on input /output relationship of filter bank, the reconstructed output is expressed as,

$$Y(z) = T_O(z)X(z) + \sum_{l=0}^{M-1} T_l(z)X(ze^{-j2\pi l/M}) \quad (9)$$

Where,

$$T_O(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k(z) \quad (10)$$

And

$$T_l(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k(ze^{-j2\pi l/M})$$

for l=0,1,2…………..(M-1) (11)

Here, $T_O(z)$ is the distortion transfer function and determine the distortion caused by the overall system for the unaliased component $X\,(z)$ of the input signal. $T_l(z)$ for l =1, 2,...(M –1) are called the alias transfer function, which determine how well the aliased components $X\left(ze^{-\frac{j2\pi l}{M}}\right)$ of the input signal are attenuated.

Amplitude distortion

Amplitude distortion error is given by,

$$E_r = max\left|\left[\left|MT_0(e^{jw})\right| - 1\right]\right| \quad (12)$$

Aliasing distortion

The worst case aliasing distortion is given by,

$$E_a = max(T_{alias}(w)) \tag{13}$$

Where

$$T_{alias}(w) = \frac{1}{M}\left[\sum_{l=1}^{M-1}\left|T_l(e^{jw})\right|^2\right]^{\frac{1}{2}} \tag{14}$$

Cosine modulation results in a class of filters with only real coefficients. Adjacent channel aliasing cancellation is inherent in the filter bank design itself. If the initial filter chosen is a linear phase filter, then the overall response will have linear phase, thereby eliminating the phase distortion.

V. EXPERIMENTAL RESULTS

Two examples are taken in which computational complexity and respective amplitude and aliasing distortion is shown.

Example 1.

An eight-band CMFB is designed. The design specifications of the filter are: pass band frequency $w_p = 0.063\pi$, Stop band frequency, $w_s = 0.080\pi$, sampling frequency F = 6000 (in Hertz), N = 101. The rounding constant α is varied from 2^{-4} to 2^{-24}. Figure 2 (a)-(h) shows the magnitude responses of optimized prototype filter, analysis filter bank, analysis bank at $\alpha = 2^{-12}$, amplitude distortion function and the aliasing error without rounding and with rounding and computational complexity graph for the designed filter bank in terms of no. of adders and multipliers. At $\alpha = 2^{-12}$, the obtained values of amplitude distortion and aliasing error are Er = 0.0612, Ea=0.0603 respectively with minimum computational reduction.

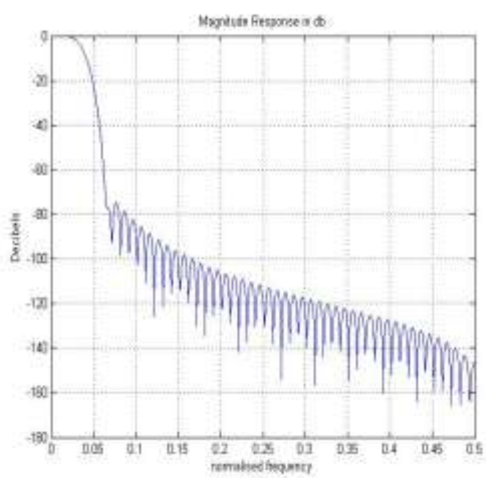

Fig.2 (a) Magnitude responses of optimized prototype filter

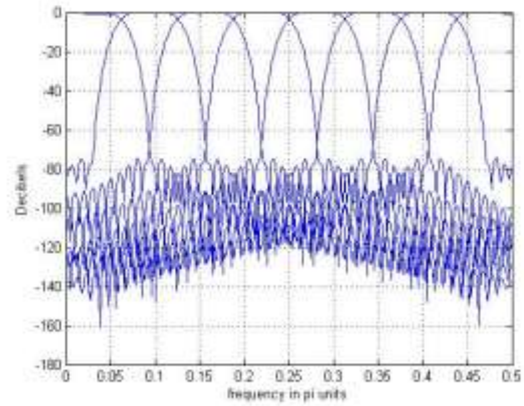

Fig.2(b) Magnitude response of 8-band analysis filter bank

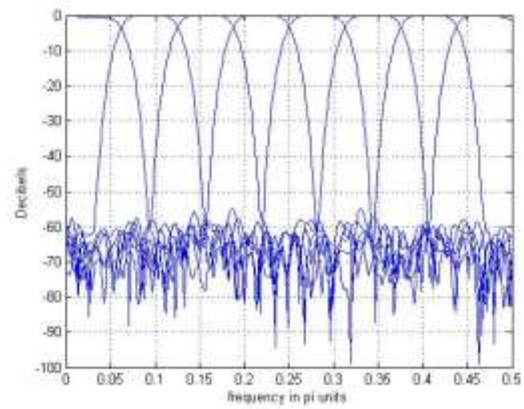

Fig.2 (c) Magnitude response of 8-band analysis filter bank at $\alpha = 2^{-12}$

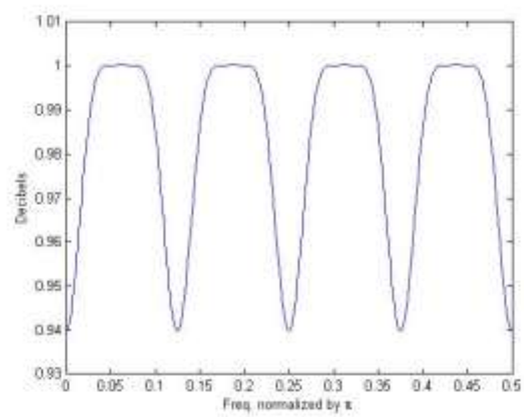

Fig.2 (d) Magnitude response of Amplitude distortion function

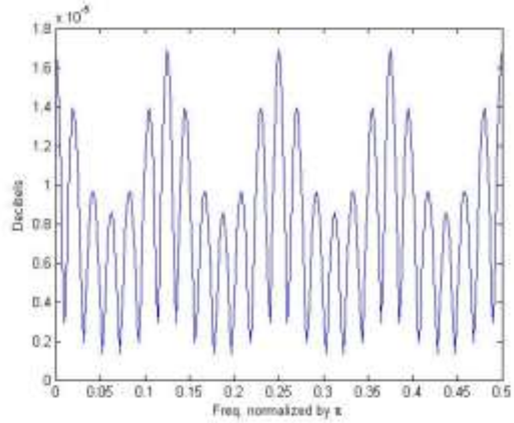

Fig.2 (e) Magnitude response of Aliasing distortion function

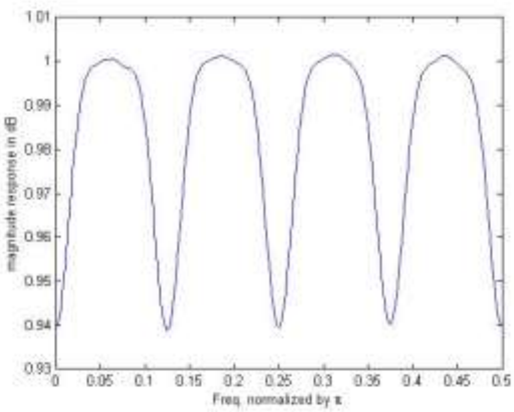

Fig.2 (f) Magnitude response of Amplitude distortion function at $\alpha = 2^{-12}$

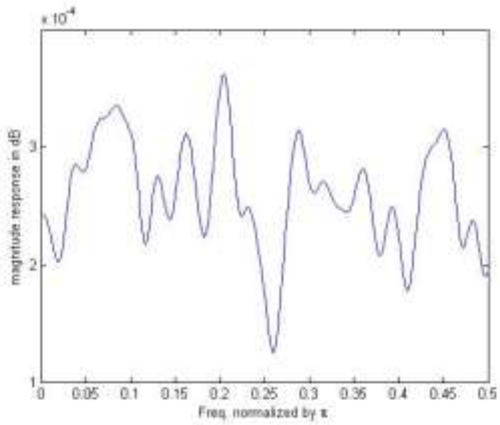

Fig.2 (g) Magnitude response of Aliasing distortion function at $\alpha = 2^{-12}$

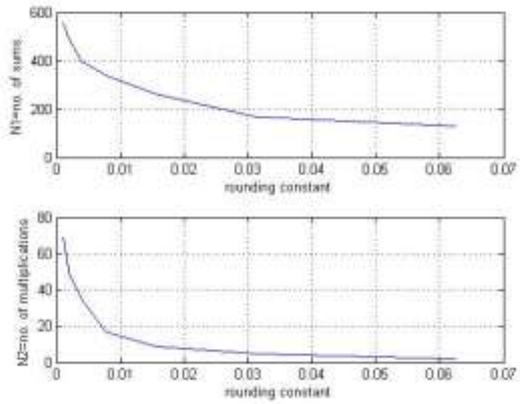

Fig.2 (h) Computational complexity graph for the designed filter bank according to Table.1

Rounding constant, α	**No. of integer sums, N1**	**No. of integer multiplications, N2**
2^{-4}	131	2
2^{-5}	169	5
2^{-6}	262	9
2^{-7}	340	17
2^{-8}	400	35
2^{-9}	489	49
2^{-10}	555	69

Table.1 Computational complexity of designed 8 band filter bank in terms of no. of multipliers (N2) and no. of adders (N1) for the values of rounding constant $\alpha=2^{-4}$ to $\alpha=2^{-10}$.

Example 2.

A 16-band CMFB is designed. The design specifications of the filter are: pass band frequency $w_p = 0.0325\pi$, Stop band frequency, $w_s = 0.0431\pi$, sampling frequency F = 7500 (in Hertz), filter length, N =164. The rounding constant α is varied from 2^{-4} to 2^{-24}. Figure 3 (a)-(h) shows the magnitude responses of optimized prototype filter, analysis filter bank, analysis bank at $\alpha = 2^{-12}$, amplitude distortion function and the aliasing error without rounding and with rounding and computational complexity graph for the designed filter bank in terms of no. of multipliers. At $\alpha = 2^{-12}$, the obtained values of amplitude distortion and aliasing error are *Er*=0.0475 and *E*a=0.0258 respectively with minimum computational reduction.

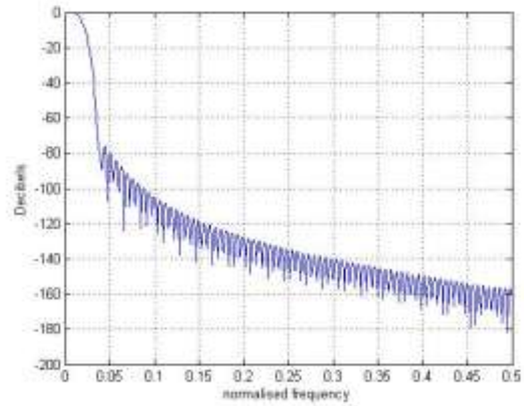

Fig.3 (a) Magnitude responses of optimized prototype filter

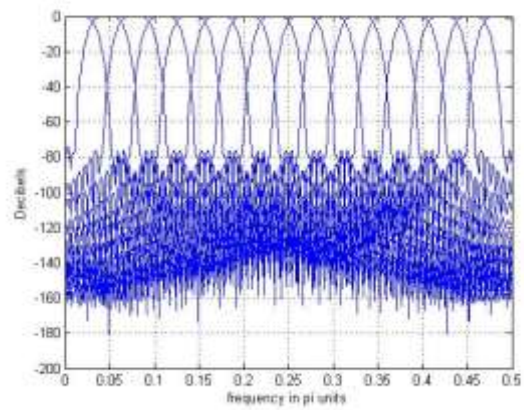

Fig.3 (b) Magnitude response of 16-band analysis filter bank

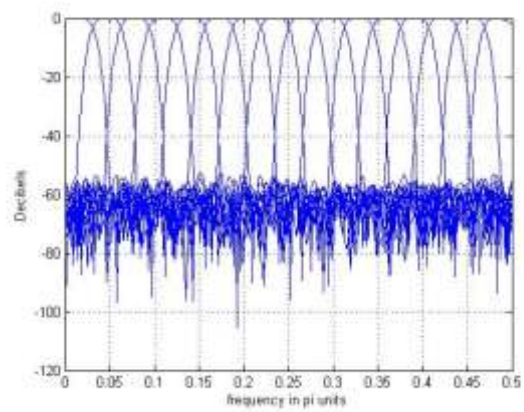

Fig.3 (c) Magnitude response of 16-band analysis filter bank at $\alpha = 2^{-12}$

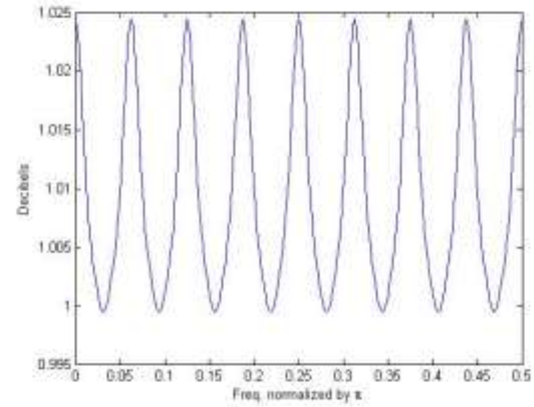

Fig.3 (d) Magnitude response of Amplitude distortion function

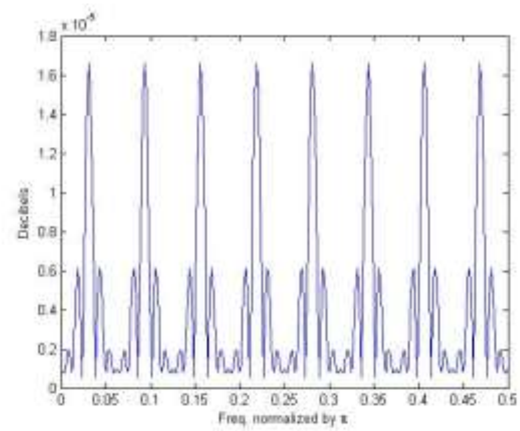

Fig.3 (e) Magnitude response of Aliasing distortion function

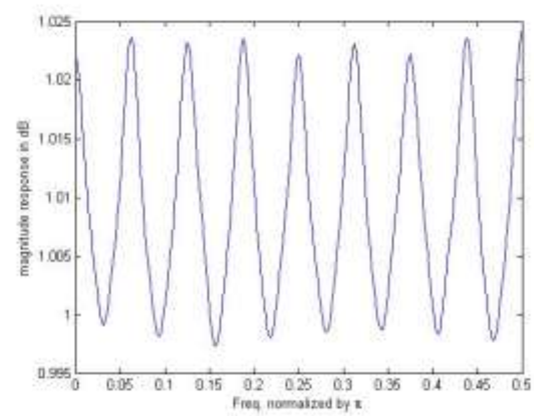

Fig.3 (f) Magnitude response of Amplitude distortion function at $\alpha = 2^{-12}$

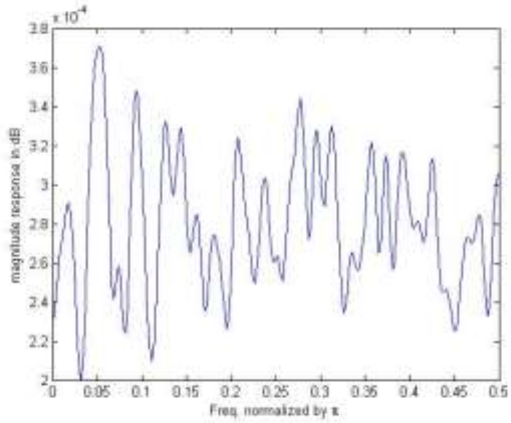

Fig.3 (g) Magnitude response of Aliasing distortion function at $\alpha = 2^{-12}$

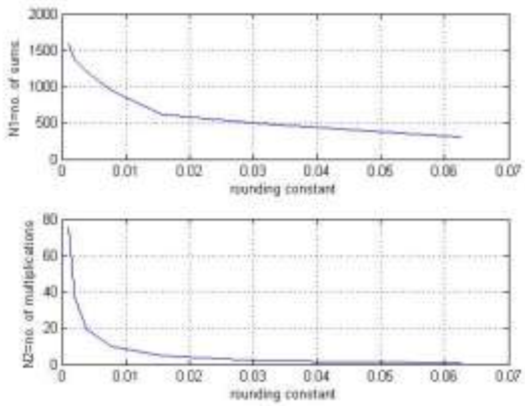

Fig.3 (h) Computational complexity graph for the designed 16 band filter bank according to Table.2

Rounding constant, α	**No. of integer sums, N1**	**No. of integer multiplications, N2**
2^{-4}	304	1
2^{-5}	496	2
2^{-6}	623	5
2^{-7}	950	10
2^{-8}	1195	19
2^{-9}	1366	37
2^{-10}	1585	75

Table.2. Computational complexity of designed filter bank in terms of no. of multipliers (N2) and no. of adders (N1) for the values of rounding constant $\alpha=2^{-4}$ to $\alpha=2^{-10}$.

VI. DISCUSSION

In the case study, two examples are taken to show the performance of the designed CMFB using quantized coefficient FIR filter by the rounding technique. The complexity of rounded filter depends on choice of rounding constant. At higher values of rounding constant null coefficients are more, number of adders & multipliers are less, distortion in gain response & aliasing error is more, which leads to less computational complexity and high computational saving. At the lower value of α complexity is more and distortion in gain response is less. In the field of filter bank design where distortion parameters play very important role, compromise with these cannot be agreed upon at the cost of computational reduction hence without compromising with these parameters, significant reduction in computation is achieved. This approach can provide better computational reduction at the cost of other performance parameters like aliasing error and amplitude distortion.

VII. CONCLUSION

In this proposed work quantized coefficient FIR filter for the design of filter bank have been presented which is computationally efficient. Different rounding factors are used to improve the computational efficiency of the parent filter. This approach yields linear phase FIR filters that can meet the given specifications with a reduced number of multiplier. Since the design is in a single stage, without introducing additional delay, it significantly reduces the computational complexity of the prototype filter. The obtained values of performance parameters are smaller and there is significant reduction in computation. The simulation results show that the rounding can provide the filter banks with higher performance as compared to conventional method at the cost of other performance parameters like aliasing error and amplitude distortion.

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