

CFD modeling of two-phase pressure drop due to flow of oil-water emulsions through sudden contraction

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Abstract— Pressure drop through sudden contractions were numerically investigated for two-phase oil/water emulsions. Two-phase computational fluid dynamics (CFD) calculations, using Eulerian–Eulerian model were employed to calculate the velocity profiles and pressure drop across sudden contraction. The realizable per-phase $k-\varepsilon$ turbulence model has been used as a closure model for turbulent flow. The pressure loss was determined by extrapolating the computed pressure profiles upstream and downstream of the contraction. From the pressure-loss and velocity data, the loss coefficients were obtained. The oil concentration was varied over a wide range of 0-97.3 % by volume. The loss coefficients for the emulsions were found to be independent of the concentration and type of emulsions. The numerical results were validated against experimental data from the literature and were found to be in good agreement.

Keywords— Two-phase flow, pressure drop, loss coefficient, velocity head, concentration, emulsion, etc

I. INTRODUCTION

Modification of flow due to a sudden change in the pipe diameter gives rise to additional pressure drop along the flow path. Knowledge of the additional pressure drop resulting from an abrupt diameter change is extremely important for a proper assessment of the pumping power required in ducts. Two-phase oil/water emulsions find application in a number of industries, such as petroleum, pharmaceutical, agriculture and food industries etc. In many applications, pumping of emulsions through pipes and pipe fittings is required. Since a detailed physical description of the flow mechanism is still not possible for two-phase flow, a considerable effort is generally needed to calculate the pressure drop along the flow path.

In contrast to the well-known axial pressure profile in the transitional region between the flow separation and reattachment for single-phase liquid flow, the pressure profile and the shape of streamlines in two-phase flow are still unknown; there is no evidence whether the flow contraction occurs at all or not behind the edge of transition. In the recent years, several papers have been published on flow of two-phase gas/liquid and liquid/liquid mixtures through pipe fittings. Hwang & Pal [1] studied experimentally the flow of

oil/water emulsions through sudden expansions and contractions and found that the loss coefficient for emulsions is independent of the concentration and type of emulsions. Schmidt and Friedel [2] studied experimentally two-phase pressure drop across sudden contractions using mixtures of air and liquids, such as water, aqueous glycerol, calcium nitrate solution and refrigerant R₁₂ in dependence of the most relevant physical parameters and concluded that unlike single phase flow a two phase flow does not contract behind the edge of transition. Wadle [3] carried out a theoretical and experimental study on the pressure recovery in abrupt expansions. He proposed a formula for the pressure recovery based on the superficial velocities of the two phases and verified its predictive accuracy with measured experimental steam-water and air-water data. Two-phase flow across sudden contractions is comparably more complicated than sudden enlargements. Many of the published studies have assumed the occurrence of the vena-contracta phenomenon, and in analogy with single-phase flow [4-5] have assumed that dissipation occurs downstream of the vena-contracta point.

II. MATHEMATICAL FORMULATIONS

Here we considered the two-fluid method or Eulerian - Eulerian model, which considers both the phases as interpenetrating continuum, with each computational cell of the domain containing respective fractions of the continuous and dispersed phases. Governing equations for phase q are given by (Drew [6]):

Continuity equation:

$$\frac{\partial}{\partial t}(\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = 0 \quad (1)$$

$$\alpha_q + \alpha_p = 1 \quad (2)$$

Momentum equation:

$$\frac{\partial}{\partial t}(\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = -\alpha_q \nabla p + \nabla \cdot (\bar{\tau}) + \alpha_q \rho_q \vec{g} + M_q \quad (3)$$

$\bar{\tau}_q$, is the qth phase stress tensor

$$\bar{\tau}_q = \alpha_q \mu_q^{eff} (\nabla \vec{v}_q + \nabla \vec{v}_q^T) \quad (4)$$

$$\mu_q^{eff} = \mu_q + \mu_{t,q} \quad (5)$$

Where M_q is the interfacial momentum transfer term, which is given by (neglecting lift force) [7]:

$$M_q = M_q^d + M_q^{VM} \quad (6)$$

$$M_q^d = \frac{3}{4d_p} \alpha_p \rho_q C_D |\vec{v}_q - \vec{v}_p| (\vec{v}_p - \vec{v}_q) \quad (7)$$

The drag coefficient C_D is given by (Wallis [7]):

$$C_D = 24(1+0.15Re^{0.687})/Re; \quad Re \leq 1000 \\ = 0.44; Re > 1000 \quad (8)$$

Relative Reynolds number is given by:

$$Re = \frac{\rho_q |\vec{v}_q - \vec{v}_p| d_p}{\mu_q} \quad (9)$$

M_q^{VM} , is the virtual mass force and is given by

$$M_q^{VM} = -M_p^{VM} = C_{VM} \alpha_p \rho_q \left(\frac{d_q \vec{v}_q}{dt} - \frac{d_p \vec{v}_p}{dt} \right) \quad (10)$$

C_{VM} = Virtual mass coefficient = 0.5

A. Turbulence modeling

Transport Equations for k :

$$\frac{\partial}{\partial t}(\alpha_q \rho_q k_q) + \nabla \cdot (\alpha_q \rho_q \bar{U}_q k_q) = \nabla \cdot \left[\alpha_q \left(\mu_q + \frac{\mu_{t,q}}{\sigma_k} \right) \nabla k_q \right] + (\alpha_q G_{k,q} - \alpha_q \rho_q \varepsilon_q) + K_{pq} (C_{pq} k_p - C_{qp} k_q) - K_{pq} (\bar{U}_p - \bar{U}_q) \cdot \frac{\mu_{t,p}}{\alpha_p \sigma_p} \nabla \alpha_p + K_{pq} (\bar{U}_p - \bar{U}_q) \cdot \frac{\mu_{t,q}}{\alpha_q \sigma_q} \nabla \alpha_q \quad (11)$$

Transport Equations for ε :

$$\frac{\partial}{\partial t}(\alpha_q \rho_q \varepsilon_q) + \nabla \cdot (\alpha_q \rho_q \bar{U}_q \varepsilon_q) = \nabla \cdot \left[\alpha_q \left(\mu_q + \frac{\mu_{t,q}}{\sigma_\varepsilon} \right) \nabla \varepsilon_q \right] + \frac{\varepsilon_q}{k_q + \sqrt{v_{t,q} \varepsilon_q}} [C_{1\varepsilon} \alpha_q G_{k,q} - C_{2\varepsilon} \alpha_q \rho_q \varepsilon_q] + C_{3\varepsilon} (K_{pq} (C_{pq} k_p - C_{qp} k_q) - K_{pq} (\bar{U}_p - \bar{U}_q) \cdot \frac{\mu_{t,p}}{\alpha_p \sigma_p} \nabla \alpha_p + K_{pq} (\bar{U}_p - \bar{U}_q) \cdot \frac{\mu_{t,q}}{\alpha_q \sigma_q} \nabla \alpha_q) \quad (12)$$

Where \bar{U}_q is the phase-weighted velocity. The turbulent viscosity $\mu_{t,q}$ is written in terms of the turbulent kinetic energy of phase q:

$$\mu_{t,q} = \rho_q C_\mu \frac{k_q^2}{\varepsilon_q} \quad (13)$$

The terms C_{pq} and C_{qp} can be approximated as

$$C_{pq} = 2, C_{qp} = 2 \left(\frac{\eta_{pq}}{1 + \eta_{pq}} \right) \quad (14)$$

Unlike standard and RNG $k - \varepsilon$ models, C_μ is not a constant here. It is computed from

$$C_\mu = \frac{1}{A_0 + A_s \frac{kU^*}{\varepsilon}} \quad (15)$$

Where, $U^* = \sqrt{S_{ij} S_{ij} + \bar{\Omega}_{ij} \bar{\Omega}_{ij}}$ (16)

And $\bar{\Omega}_{ij} = \Omega_{ij} - 2\varepsilon_{ijk} \omega_k$ (17)

$$\Omega_{ij} = \bar{\Omega}_{ij} - \varepsilon_{ijk} \omega_k$$

Where $\bar{\Omega}_{ij}$ is the mean rate of rotation tensor, viewed in a rotating reference frame with the angular velocity $\bar{\omega}_k$. The constants A_0 and A_s are given by

$$A_0 = 4.04, A_s = \sqrt{6} \cos \phi$$

Where, $\phi = \frac{1}{3} \cos^{-1}(\sqrt{6}W)$, $W = \frac{S_{ij} S_{jk} S_{ki}}{S^3}$,

$$\tilde{S} = \sqrt{S_{ij} S_{ij}}, S_{ij} = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)$$

The model constants are

$$C_{1\varepsilon} = 1.44; C_{2\varepsilon} = 1.9; C_{3\varepsilon} = 1.3; \sigma_k = 1.0; \sigma_\varepsilon = 1.2.$$

B. Boundary conditions

Velocity inlet boundary condition was employed at the inlet. A no-slip and no-penetrating boundary condition was imposed on the wall of the pipe. At the outlet, the boundary condition was assigned as outflow (diffusion flux for the entire variables in exit direction was set to zero) and axisymmetric boundary condition was considered at the axis.

III. THEORETICAL BACK GROUND

A schematic of the pressure profile for a sudden contraction is shown in Fig. 1. The pressure drop across a pipe contraction (ΔP_c) is defined as a local change of pressure in the inlet and the outlet pipes. In the figure 1(a) the boundary streamlines for the flow through a sudden contraction of a duct area are shown, while fig. 1(b) depicts the path of the static pressure along the flow axis for a steady state flow of an incompressible fluid across a contraction.

The friction loss (h_f) due to a pipe contraction can be calculated from the following equation:

$$h_f = \frac{P_1 - P_2}{\rho} + K_1 \frac{V^2}{2} \quad (18)$$

Where h_f is the friction loss, $(P_1 - P_2)$ is the pressure change at the contraction plane (ΔP_c), ρ is the mean fluid density, K_1 is equal to $[(D_2/D_1)^4 - 1]$, and V is the average velocity in the smaller diameter pipe. Since the flow regime is turbulent, $\Delta P_c/\rho$ versus $V^2/2$ data exhibit a linear relationship that is:

$$\frac{\Delta P_{con}}{\rho} = K_2 \frac{V^2}{2} \quad (19)$$

K_2 is the slope of $\Delta P_c/\rho$ versus $V^2/2$ plot. Substituting these values in equation (17):

$$h_f = (K_1 + K_2) \frac{V^2}{2} = K_c \frac{V^2}{2} \quad (20)$$

Where, the loss coefficient for contraction (K_c) is equal to $(K_1 + K_2)$.

IV. NUMERICAL SOLUTION PROCEDURE

The objective of the present work is to simulate the flow through sudden contraction in pipes numerically by using two phase flow models in an Eulerian scheme. The flow field is assumed to be axisymmetric and solved in two dimensions. The two-dimensional equations of mass, momentum, volume fraction and turbulent quantities along with the boundary conditions have been integrated over a control volume and the subsequent equations have been discretized over the control volume using a finite volume technique to yield algebraic equations which can be solved in an iterative manner for each time step. The conservation equations are solved with the segregate solver using implicit scheme. FLUENT 6.2 double-precision solver is used in all the simulations, to reduce the round-off errors. The discretization form for all the convective variables were taken to be first order up winding initially for better convergence. Slowly as time progressed the discretization forms were switched over to second order up winding and then slowly towards the QUICK scheme for better accuracy. The Phase-Coupled SIMPLE algorithm was used for the pressure-velocity coupling. The velocities were solved coupled by the phases, but in a segregated fashion. The block algebraic multigrid scheme was used to solve a vector equation formed by the velocity components of all phases simultaneously. Pressure and velocities were then corrected so as to satisfy the continuity constraint. The realizable per-phase $k-\epsilon$ model has been used as closure model for turbulent flow. Fine grids were used near the wall as well as near the contraction section.

V. RESULTS AND DISCUSSIONS

The sudden contraction considered in this work was made from two straight pipes having inner diameters of 2.037cm and 4.124cm. Axial static pressure drops were computed both upstream and downstream from the expansion or contraction plane. The pressure differentials were computed with respect

to the reference pressure at $25D_2$ upstream position. The oil used in the present computational work is Bayol-35 (Esso Petroleum, Canada), which is a refined white mineral oil with a density of 780 kg/m^3 and a viscosity of 0.00272 Pa-s at 25°C . Density and viscosity of water were taken as 998.2 kg/m^3 and 0.001003 Pa-s , respectively. The volume fraction of oil was taken as 0, 0.2144, 0.3886, 0.6035, 0.6457, 0.6950, 0.8042, and 0.9728. The emulsions are oil-in-water type up to an oil concentration of 64 % by volume and water-in-oil type beyond it [1].

Fig. 2 shows the domain used to represent the sudden contraction section (half of the section is modelled, with a symmetry boundary at the centreline). For the validation of results, we have referred to the experimental studies conducted by Hwang & Pal [1]. A comparison of computed and experimental results was found to be in good agreement. The pressure profiles for the oil-in-water emulsions are shown in Fig. 3. Each of the graphs differs in the volume fractions of the dispersed phase (oil) in the emulsion, represented by the symbol Φ .

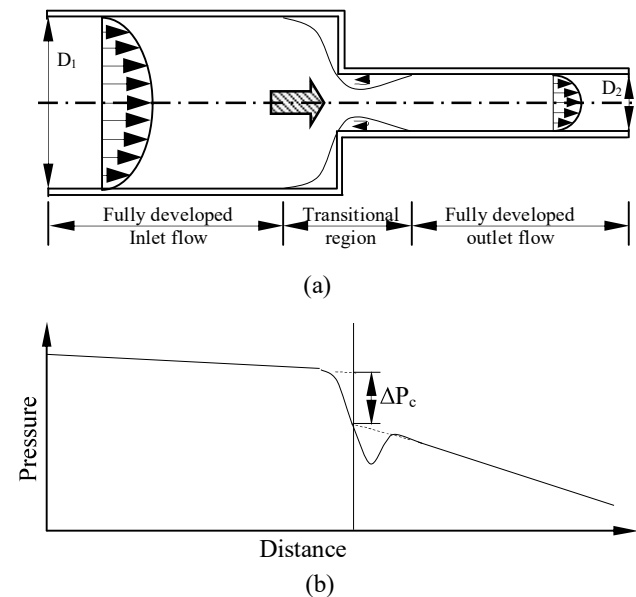


Fig. 1. (a) Idealized course of boundary stream lines and (b) pressure profile for a sudden contraction.

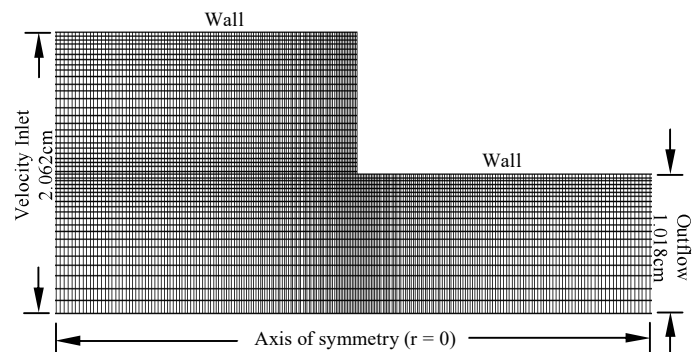
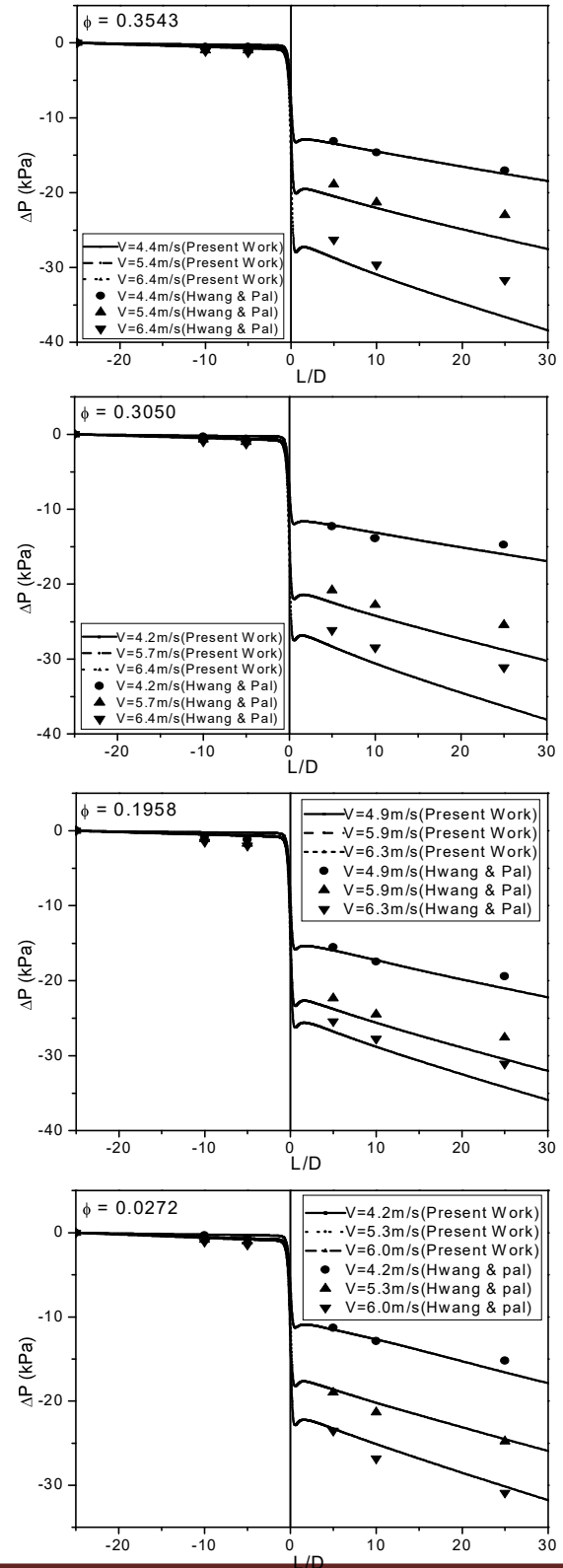
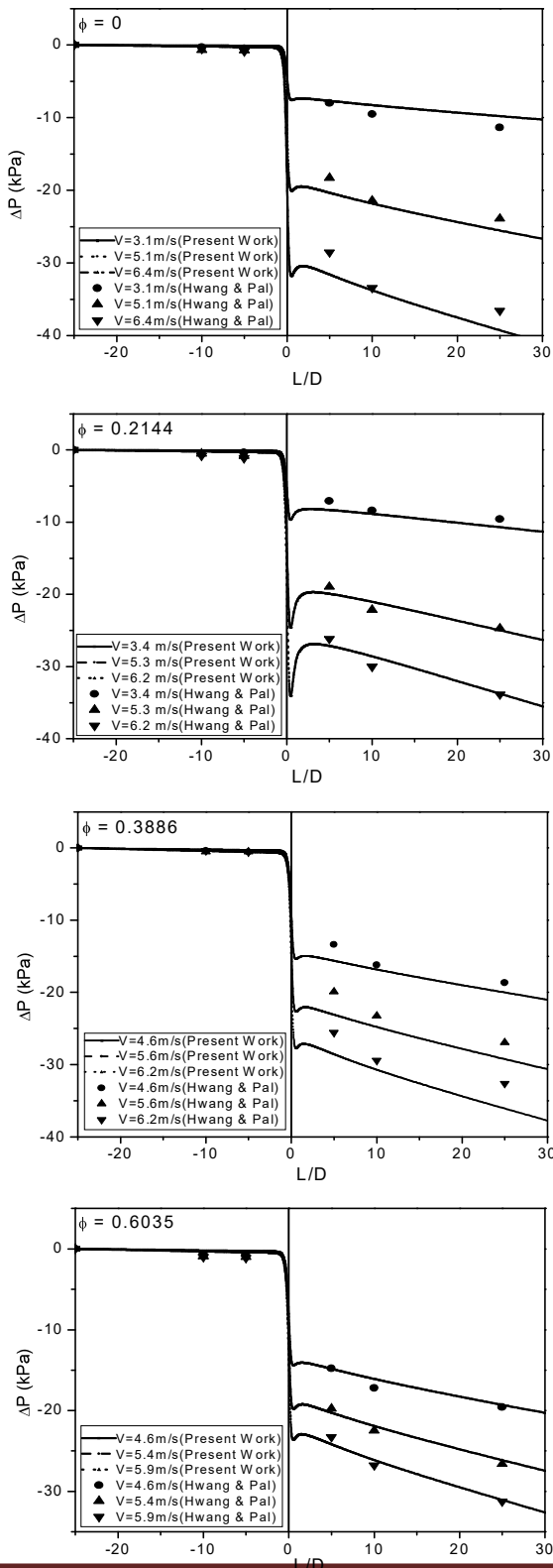


Fig. 2 Computational domain.

Fig. 3 Pressure profile for oil-in-water emulsion flowing through sudden contraction.



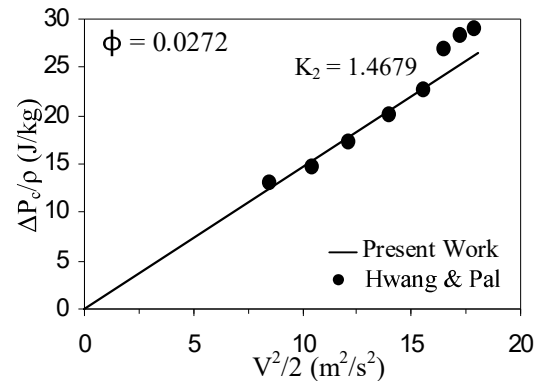
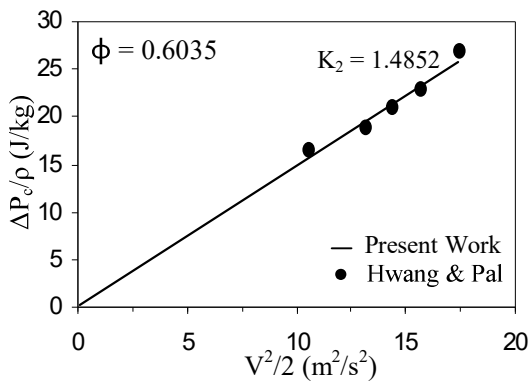
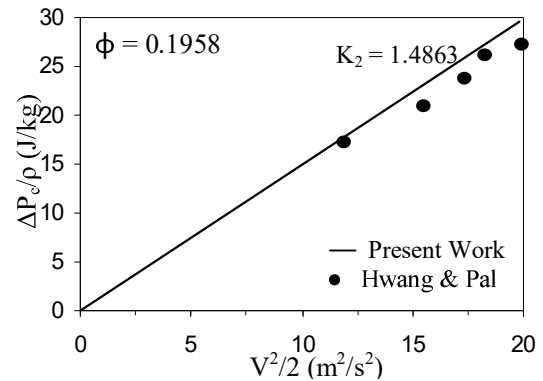
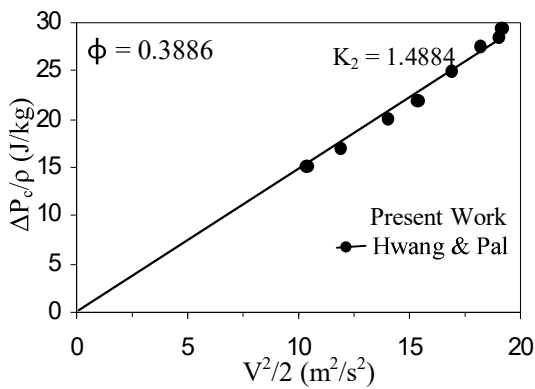
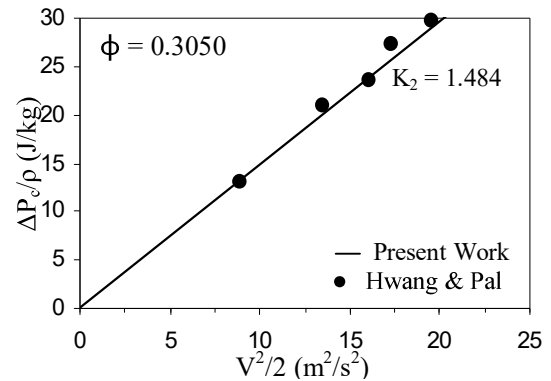
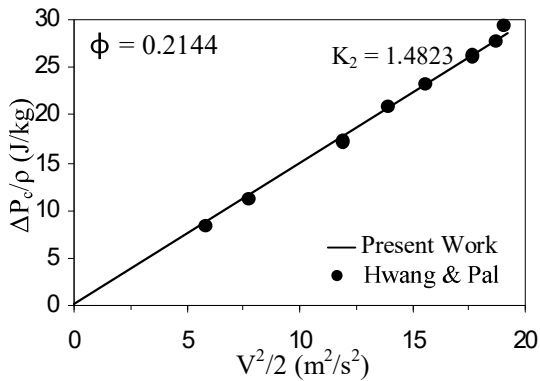
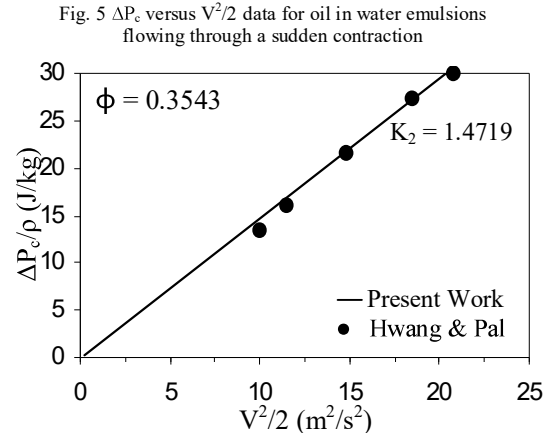
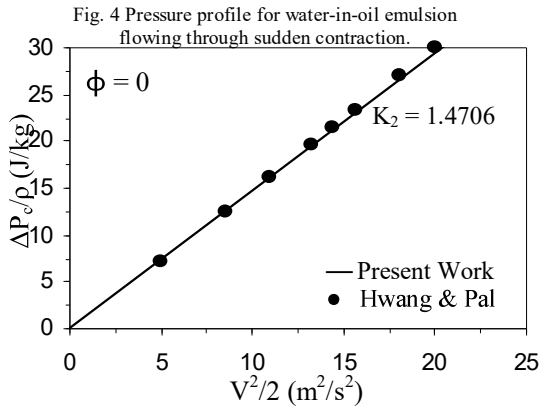


Fig. 6 ΔP_c versus $V^2/2$ data for water in oil emulsions flowing through a sudden contraction

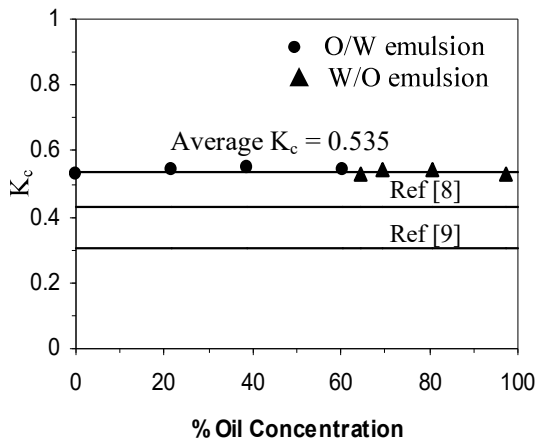


Fig. 7. Contraction loss coefficient as a function of oil concentration

The pressure profiles are nearly linear up to 5 pipe diameters (D_2), both upstream and downstream from the contraction plane. Because there is a change in pipe cross-section and hence a change in mean velocity, the slopes of the pressure profiles before and after the contraction are different. The gradients are greater in the smaller diameter pipe.

The pressure profiles for the water-in-oil emulsions are shown in Fig. 4. The water-in-oil emulsions behave in a manner similar to the oil-in-water emulsions.

Figs. 5 and 6 show $\Delta P_c/\rho$ versus the velocity head ($V^2/2$) data for differently concentrated oil-in-water and water-in-oil emulsions, respectively. Since the flow regime is turbulent, $\Delta P_c/\rho$ versus $V^2/2$ data exhibit a linear relationship, from which K_c (the slope of $\Delta P_c/\rho$ versus $V^2/2$ plot) was found out.

The plot of contraction loss coefficient (K_c) as a function of oil concentration is shown in Fig. 7 for both emulsion types. The loss coefficient is found to be independent of oil concentration and has an average value of 0.54. The value of K_c calculated from the empirical equation, given by McCabe et al. [9]:

$$K_c = 0.4(1 - \beta) = 0.302 \quad (20)$$

Here, β = area ratio = 0.244.

And the value of K_c obtained from empirical equations given by Perry et al. [8] is 0.43.

VI. CONCLUSIONS

The flow through sudden contraction was numerically simulated with oil-water emulsions by using two-phase flow model in an Eulerian scheme. The major observations made

relating to the pressure drop in the process of flow through sudden expansion and contraction can be summarized as follows:

- The contraction loss coefficient is found to be independent of the velocity and hence Reynolds number.
- The loss coefficient is not significantly influenced by the type and concentration of oil-water emulsions flowing through sudden contraction.
- Effect of viscosity is negligible on the pressure drop through sudden contraction.
- The computed contraction loss coefficient is found to be slightly more than the predictions of Perry et al. (1984). Where as the McCabe et al. (1993) correlation under predicts the data significantly.
- The pressure drop increases with higher inlet velocity and hence with higher mass flow rate.
- In the single phase flow of water and two-phase flow of oil-water emulsions vena contracta is always established at a distance of about 0.5D after the contraction section and depends only slightly on the concentration and velocity of flow.
- The satisfactory agreement between the numerical and experimental results indicates that the model may be used as a simple, efficient tool for engineering analysis of two-phase flow through sudden flow area contractions.

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