Reliability of an (M, M) Machining System with Spares

Rashmita Sharma
Department of Mathematics
D.A.V. (P.G.) College, Dehradun-248001 (India).

Abstract

This paper studies the reliability characteristics of a repairable system with M operating machines, S spare machines. The diffusion approximation technique has been used for multi-repairman problem having spares with Poisson inter-failure time distribution and exponential repair time distribution. The repair is assumed to be state-dependent. We present derivations for the approximate formulae of the average number of failed components and the expected number of components operating in the system.

1. Introduction

As it is known to everyone that spares or standby increase a system`s reliability .The system may consist of one or more units . In many industrial processes where machines work, the problems of providing spare machines may arise frequently.The smooth running of the system depends on the availability of spares . Analytic solution of markovian model for the machine repair problem (MRP) with no spares was first obtained by Feller (1967). Natarajan and Subba Rao (1970) have investigated the reliability characteristics of a single–unit system with spares and several repair facility . Subramnian et.al.(1976) discussed the reliability of a repairable system with standby failure .Bunday and Scraton (1982) solved a machine interference problem (MIP) having N automatic machines are maintained by a team of 'r' repairmen. Kalpakam and Hamed (1984) analyzed the availability and reliability of an n-unit warm standby redundant system .Wang and Sivazlian (1989) studied the reliability of a system with warm standby and repairman. A detailed account of the development of MIP can be found in Bunday and Khorram (1990). Reliability of a repairable system with spare and a removable repairman was studied by Hsieh and Wang (1995). Shawky (1997) discussed single server machine interference model with balking , reneging and an additional server for longer queues .The machine interference model M/M/c/K/N with balking , reneging and spare was analyzed by Shawky (2000). M/M/C repairable system with spare and additional repairman was considered by Jain (1996). Jain and Ghimire (1997) analyzed machine repair queuing system with non-reliable server and hetrogeneous service discipline.

Recently machine repair problems have been discussed by(Jain et.el. (2003), 2004, 2007; Ke, et. al. (2009), Kumar and Jain (2010); Maheshwari, et.al (2010) Masdi, et.al (2012) ; sharma (2012).)

In queuing literature, the diffusion approximation techniques have been played very important role . Sunaga et al. (1982) developed an approximation method based on the diffusion theory for solving multi-server finite queue. Yao and Buzacott (1985) considered a flexible manufacturing system via diffusion approximation. Jain and Sharma (1986) suggested a diffusion approximation method for the GI/G/r MIP with spare machines. Cherian et.el (1988) studied a multi-component system with poisson failure and exponential repair time by using diffusion approximation techniques and obtained an expression for the number of failed component in the system. A diffusion approximation technique for the economic analysis of the G/G/R MRP with warm standby was used by Sivazlian and Wang (1989). Wang and Sivazlian (1992) generlized the work of Sivazlian and Wang (1989) by solving diffusion equation for G^x/G/m MIP by imposing the reflecting boundaries for the solution purpose. G/G/R machine repair problem with warm standby spares was discussed by Sivazlian and Wang (1990) using diffusion approximation. Jain (1997) used diffusion approximation techniques to analyze (m,M) MRP with spares and state- dependent roles . Diffusion models for computer / Communication system were discussed by Kimura (2004)

In this paper, we construct a diffusion equation for a multi-repairman problem having spares. The repair is assumed to be state- dependent. The considered model has Poisson inter-failure time distribution and exponential repair time distribution. The discrete flow of failed machines in the system is approximated by a continuous one, and reflecting boundaries are taken into considerations to obtain system size distribution. The approximate formulae for the average number of failed components and the expected number of components operating in the system have been derived.

2. The model and steady state diffusion equation

We consider a multi-component system having M identical machines which are continuously operating in a system with failure rate λ under the care of R repairman and a maximum of S spare machines are in hand. The failure rate of S spare machines is a ($0 \leq a \leq \lambda$). The system is in operating state when at least m out of M machines are working. We suppose that the time to failure and the time for repair are independent and identically distributed (i. i. d.). Each repairman serves at the rate μ_1 until there are n (n < R) failed machines in the system. When all the repairman are busy, he switches to the faster rate μ ($\mu_1 \leq \mu$). We assume that the switchover time from standby to operating state, from failure to repair, or from repair to standby stand is instantaneous. It is assumed that at any time a machine is either in good condition for production or in breakdown condition. Each of the operating spare machine fails sent for repairing at once and the spare is put into operation. The failed machines are repaired by the repairman on the first-come first-served (FCFS) discipline. Each repairman can repair only one failed machine at a time. When a machine is repaired, it is as good as new and goes into standby or operating state. If it happens that all spares are being used and a machine fails, then we say that the system become short in which case there are less than M operating machines.

For considered (m,M) machining system, as soon as M+S-m+1 machined fail the system is in failed condition. The state dependent mean arrival rate λ_n and failure rate μ_n are given by

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha & ;\ 0 \leq n \leq S \\ CM + (S-n)\lambda & ;\ S < n \leq M+S-m \end{cases} \quad \ldots (1)$$

and

$$\mu_n = \begin{cases} n\mu_1 & ;\ 1 \leq n \leq R \\ R\mu & ;\ R \leq n \leq M+S-m+1 \end{cases} \quad \ldots (2)$$

Let L(t) be the number of failed machined at time t and $P_n(t)$ be the probability that there are n failed machined at time t in the system. We approximate the discrete variable L(t) by a continuous variable X(t), the diffusion process in one dimension. The diffusion equation foe steady state is given by

$$0 = \frac{1}{2}\frac{d^2}{dx^2}(b(x)f(x)) - \frac{d}{dx}(a(x)f(x)) \quad \ldots (3)$$

where f(x) is the steady state p.d.f. , and a(x) and b(x) are the infinitesimal mean and variance respectively for diffusion process X(t).

For our model a(x) and b(x) are proposed as follows :

Case A : R ≤ R :

(i) For 0≤x≤ R :

$$a(x) = M\lambda + (S-x)\,\alpha - x\mu_1$$

and $$b(x) = M\lambda + (S-x)\,\alpha + x\mu_1$$

The solution f(x) of equation (3) in this interval is given by,

$$f_1(x) = \frac{Ce^{2A_1x}[M\lambda + (S-x)a + x\mu_1]^{2A_1B_1-1}}{(a-\mu_1)^{2A_1B_1}}$$

$$= C\, g_1(x) \qquad \text{. . . (4)}$$

where $A_1 = \frac{\alpha+\mu_1}{\alpha-\mu_1}$, $B_1 = \frac{2\mu_1(M\lambda+S\alpha)}{\alpha^2\mu_1{}^2}$

and C is constant

(ii) For R≤x≤ N :

$$a(x) = M\lambda + (S-x)\,\alpha - R\mu$$

and $$b(x) = M\lambda + (S-x)\,\alpha + R\mu$$

The solution f(x) of equation (3) in this interval is given by,

$$f_2(x) = \frac{C_1[M\lambda + (S-x)a + R\mu]^{2A_2-1}}{(\alpha)^{2A_2}} \qquad \text{. . . (5)}$$

where $A_2 = \frac{2R\mu}{\alpha}$ and C_1 is constant.

From the continuity at x = R, C_1 becomes

$$C_1 = C\left[\frac{g_1(R)}{g_2(R)}\right]$$

(iii) For R≤x≤ N :

$$a(x) = CM + (S-m)\lambda - R\mu$$

and $$b(x) = CM + (S-x)\lambda + R\mu$$

The solution f(x) is given by

$$f_3(x) = \frac{C_1[CM + (S-x)\lambda + R\mu]^{2A_3-1}}{(\lambda)^{2A_3}} \; e^{2x} \qquad \ldots (6)$$

where $A_3 = \frac{2R\mu}{\lambda}$

and C_2 is constant.

Again using continuity at x = S, we get

$$C_2 = C\left[\frac{g_1(R)}{g_2(R)}\right]\left[\frac{g_1(R)}{g_2(R)}\right]$$

The constant C can be obtained by using normalizing condition,

i.e.,

$$\int_0^R f_1(x)dx + \int_R^S 2(x)dx + \int_S^{M+S-m} f_3(x)dx = 1 \qquad \ldots (7)$$

Now we can approximate the value of P_n by using any one of the following methods of discretization

(i) $P_n = \int_n^{n+1} f(x)dx$

(ii) $P_n = \int_{n-1}^{n} f(x)dx$

(iii) $P_n = P(n)$

(iv) $P_n = \int_{n-0.5}^{n+0.5} f(x)dx$

For our model, the best suited approximation is (iv), By using it, the approximate formula for the mean number of failed machined in the system is given by

$$L_1 = \sum_{n=1}^{M+S-m} n\,P_n$$

$$= \sum_{n=1}^{r-1} \int_{n-0.5}^{n+0.5} f_1(x)dx + R\left[\int_{R-0.5}^{R} f_1(x)dx + \int_{R}^{R+0.5} f_2(x)dx\right] + \sum_{n=S+1}^{S-1} n \int_{n-0.5}^{R+0.5} f_2(x)dx + \left[\int_{S-0.5}^{S} f_3(x)dx + S\right] + \sum_{n=S+1}^{m+S-m-1} n \int_{n-0.5}^{n+0.5} f_3(x)dx + (M+S-1)\int_{M+S-m}^{M+S-m} f(x)dx \qquad \ldots (8)$$

where

$$P_{M+S-m} = \int_{M+S-m-0.5}^{M+S-m} f(x)dx \qquad \dots(9)$$

An approximate formula for the expected number of machines in operation is given by

$$E_A = M\sum_{n=0}^{S} P_n + \sum_{n=S+1}^{M+S-m} (M+S-n)$$

$$= M\left[\sum_{n=1}^{R-1}\int_{n-0.5}^{n+0.5} f_1(x)\,dx + \int_{R-0.5}^{R} f_1(x)\,dx + \sum_{n=S+1}^{S-1}(M+S-n)\int_{n-0.5}^{n+0.5} f_3(x)\,dx + m\int_{M+S-m-0.5}^{M+S-m} f_3(x)\,dx\right] \quad \dots(10)$$

Case B : S < R

(i) For $0 \le x \le S$:

$$a(x) = M\lambda + (S-x)a - x\mu_1$$

and $$b(x) = M\lambda + (S-x)a + x\mu_1$$

The solution of f(x) in this interval is $f_1(x)$ as given in equation (4)

(ii) For $R \le x \le N$:

$$a(x) = CM + (S-x)\lambda - x\mu_1$$

and $$b(x) = CM + (M+S-x)\lambda + x\mu_1$$

From the continuity of the solution, f(x) in this interval is given by

$$f_4(x) = C\left[\frac{g_1(S)}{g_4(S)}\right] g_4(x) \qquad \dots(11)$$

where $f_4(x) = \dfrac{[CM+S-x)\lambda + x\mu_1]^{2A_4B_4-1}}{(a-\mu_1)^{2A_1B_1}} \exp(2A_4x)$,

$A_4 = \dfrac{\lambda+\mu_1}{\lambda-\mu_1}$, and $B_4 = \dfrac{2\lambda\mu_1(M+S)}{\lambda^2-\mu_1^2}$

(iii) For $R \le x \le M+S-m$:

In this interval, the values of a(x) and b(x) are same as those described in Case A for interval $S \le x \le M+S-m$. From the continuity of the solution, f(x) in this interval is given by,

$$f_4(x) = C_4\, g_3(x) \qquad \dots(12)$$

where

$$C_4 = C\left[\frac{g_1(S)}{g_2(R)}\right]\left[\frac{g_4(R)}{g_4(R)}\right]$$

The normalizing condition is given by

$$\int_0^S f_1(x)dx + \int_S^R 4f_4(x)dx + \int_R^{M+S-m} f_5(x)dx = 1 \qquad \dots(13)$$

In this case, the approximate formula for the mean number of failed components is given by

$$L_2 = \sum_{n=1}^{M+S-n} nP_n$$

$$= \sum_{n=1}^{S-1} n \int_{n-0.5}^{n+0.5} f_1(x)\,dx + S\left[\int_{S-0.5}^{S\rightarrow} f_1(x)\,dx + \int_S^{S+0.5} f_4(x)dx\right]$$
$$+ \sum_{n=S+1}^{R-1} n \int_{n-0.5}^{n+0.5} f_4(x)dx + R\left[\int_{R-0.5}^{R} f_4(x)dx + \int_R^{\leftarrow R+0.5} f_5(x)dx\right]$$
$$+ \sum_{n=S+1}^{m+S-m-1} n \int_{n-0.5}^{\leftarrow n+0.5} f_5(x)dx$$
$$+ (M+S-m)\int_{M+S-m-0.5}^{M+S-m} f_5(x)dx \qquad \dots(14)$$

The expected number of components operating in the system can be approximates as

$$E_8 = M\sum_{n=0}^{S} P_n \sum_{n-S+1}^{M=s-m} (M+S-n)P_n$$

$$= M\left[\sum_{n=0}^{S-1} \int_{n-0.5}^{n+0.5} f_1(x)\,dx + \int_{n-0.5}^{S} f_1(x)\,dx\right]\left[\sum_{n=S+1}^{R-1} (M+S-n)\int_{n-0.5}^{n+0.5} f_4(x)dx\right.$$
$$+ (M+S-x)\left[\int_{R-0.5}^{R} f_4(x)dx + \int_R^{R+0.5} f_5(x)dx\right]$$
$$\left.+ \sum_{n=R+1}^{m+S-m-1} (M+S-n)\int_{n-0.5}^{n+0.5} f_4(x)dx + m\int_{M+S-m-0.5}^{M+S-m} f_5(x)dx\right] \qquad \dots(15)$$

3. SOME MORE RESULTS

The variance of failed machines in the system is

$$V(n) = \sum_{n=1}^{M+S-m} n^{2P_n - L^2} \qquad \dots(16)$$

An approximate formula for the mean number of repaired machines per unit time is

$$D = \mu \sum_{n=1}^{R-1} nP_n + R\mu \sum_{n=R}^{M+S-m} P_n \quad \text{... (17)}$$

The system reliability is

$$R = \sum_{n=0}^{M+S-m} P_n \quad \text{... (17)}$$

The results obtained by equations (16)-(18) are valid for both cases 'A' and 'B' may be written in the form similar to equation (8) for case A and equation (14) for case B.

4. Modified Boundary Conditions

The diffusion equation for machine interference problem has two reflecting boundaries at x = 0

and x = M+S-m. But equation (3) cannot be solved at these boundaries. Therefore, we shift the boundaries at x = 0 to x = -0.5 and at x = M+S-m to M+S-m+0.5.

(i) For $-0.5 \leq x \leq 0.5$:

$a(x) = M\lambda + (N-x)\alpha - 0$

and $b(x) = M\lambda + (N-x)\alpha + 0$

(ii) For $M+S-m \leq x \leq M+S-m+0.5$:

$a(x) = 0 - R\mu$

and $b(x) = 0 + R\mu$

Now, f(x) can be obtained by equation (3) as

$$f_0(x) = \frac{C_0}{M\lambda + (N-x)\alpha} \exp(2x) \equiv C_0 g_0(x) \quad \text{... (19)}$$

$(0.5 \leq x \leq 0)$

and $f_6(x) = C_6 \exp(-2x) \equiv C_6 g_6(x)$ ... (20)

where C_0 *and* C_6 are the constants.

The values of C_0 *and* C_6 can be determined as

$$C_0 = \frac{g_1(0)}{g_0(0)}$$

For Case A

$$C_6 = C_0 \left(\frac{g_0(0)}{g_1(0)}\right)\left(\frac{g_1(R)}{g_2(R)}\right)\left(\frac{g_2(S)}{g_3(S)}\right)\left(\frac{g_3(M+S-m)}{g_6(M+S-m)}\right) \quad \text{... (18)}$$

For Case B

$$C_6 = C_0 \left(\frac{g_0(0)}{g_1(0)}\right)\left(\frac{g_1(S)}{g_2(S)}\right)\left(\frac{g_2(R)}{g_3(R)}\right)\left(\frac{g_3(M+S-m)}{g_6(M+S-m)}\right) \quad \text{... (19)}$$

In this case, the mean number of failed components is given by

$$L = \sum_{n=1}^{M+S-n} \int_{n-0.5}^{n+0.5} f(x)\,dx$$

5. Discussion

In this section, we have proposed a diffusion approximation technique for solving the steady state machine interference problem with sparing. Numerical results can be easily obtained by using Gauss formula. Modified conditions are also discussed to improve the approximation.

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