On Nano Regular Generalized and Nano Generalized Regular Closed Sets in Nano Topological Spaces

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ABSTRACT-The purpose of this paper is to define and study a new class of sets called Nano regular generalized and Nano generalized regular closed sets in nano topological spaces. Basic properties of nano regular generalized closed sets and nano generalized regular closed sets are analysed. We also used them to introduce the new notions like nano regular generalized closure and nano generalized regular closure and their relation with already existing well known sets are also investigated.

Keyword- Nano closed set, Nano regular closed, Nano Regular generalized closed set, Nano generalized regular closed set.

1. INTRODUCTION

In 1970, Levine [1] introduced the concept of generalized closed sets as a generalization of closed sets in Topological space. Later on N.Palaniappan [4] studied the concept of regular generalized closed set in a topological space. In 2011, Sharmistha Bhattacharya [5] have introduced the notion of generalized regular closed sets in topological space. The notion of nano topology was introduced by Lellis Thivagar [2] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. In this paper, we have introduced a new class of sets on nano topological spaces called nano regular generalized closed sets and nano generalized regular closed sets and the relation of these new sets with the existing sets.

2. PRELIMINARIES

Let us recall the following definitions and results which are used in the sequel.

Definition 2.1: A subset A of a space (X,τ) is called

(i) Pre - open set if $A \subseteq int(cl(A))$ and a Pre - closed set if $cl(int(A)) \subseteq A$

(ii) Semi – open if $A \subseteq cl(int(A))$ and a Semi – closed $int(cl(A)) \subseteq A$

(iii) α – open if $A \subseteq int(cl(int(A)))$ and a α – closed if $int(cl(int(A))) \subseteq A$

(iv) Regular open if $A = int(cl(A))$ and a Regular closed is $A = cl(int(A))$.

Definition 2.2[2]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation of U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = U\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.

(ii) The upper approximation of X with respect to R is the set of all objects. Which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$U_R(X) = U\{R(x): R(x) \cap X \neq \Phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither X nor as not X with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) – L_R(X)$.

Property 2.3[2]: If (U, R) is an approximation space and X, Y $\subseteq$ U, then

(i) $L_R(X) \subseteq x \subseteq U_R(X)$

(ii) $L_R(\Phi) = U_R(\Phi) = \Phi$ & $L_R(U) = U_R(U) = U$

(iii) $U_R(XUY) = U_R(X)\ U\ U_R(Y)$

(iv) $L_R(XUY) \supseteq L_R(X)\ U\ L_R(Y)$

(v) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$

(vi) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_R\ U_R(X) = L_R\ U_R(X) = U_R(X)$

(x) $L_R\ L_R(X) = U_R\ L_R(X) = L_R(X)$

Definition 2.4[2]: Let U be the universe, R be an equivalence relation on U and

$\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\Phi \in \tau_R(X)$.

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call

(U, $\tau_R(X)$) as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. The elements of $(\tau_R(X))^C$ are called as nano closed sets.

Remark 2.5[2]: If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set

B={U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition 2.6[2]: If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the nano interior of the set A is defined as the union of all Nano open subsets contained in A and it is denoted by NInt(A). That is NInt(A) is the largest Nano open subset of A. The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is NCl(A) is the smallest Nano closed set containing A.

Definition 2.7[2]: A Nano topological space

(U, $\tau_R(X)$) is said to be extremely disconnected, if the Nano closure of each Nano open set is Nano open.

Definition 2.8[2]: Let (U, $\tau_R(X)$) be a Nano topological space and A $\subseteq$ U. Then A is said to be

(i) Nano Semi open if A $\subseteq$ NCl(NInt (A))

(ii) Nano Pre-open if A $\subseteq$ NInt(NCl (A))

(iii) Nano α-open if A $\subseteq$ NInt(NCl(NInt (A)))

(iv) Nano Regular open if A $\subseteq$ NInt(NCl (A))

(v) Nano Regular closed if NCl(NInt (A)) $\subseteq$ A

NSO(U,X), NPO(U,X), NRO(U,X) and $\tau_R^{\alpha}(X)$ respectively, denote the families of all Nano semi-open, Nano Pre-open, Nano Regular open, Nano Regular closed and Nano α-open subsets of U.

Definition 2.9[1]: A subset A of a Nano topological space

(U, $\tau_R(X)$) is called;

Nano Generalized closed set if

NCl(A) $\subseteq$ V whenever A $\subseteq$ V and V is Nano open in $\tau_R(X)$.

Result 2.10[1]: Every Nano closed set is nano generalized closed set.

Definition 2.11[1]: A subset A of a Nano topological space

(U, $\tau_R(X)$) is called;

(i) Nano semi generalized closed set if NSCl(A) $\subseteq$ V whenever A $\subseteq$ V and Vis Nano open in $\tau_R(X)$.

(ii) Nano generalized semi closed set if

NSCl(A) $\subseteq$ V whenever A $\subseteq$ V and Vis Nano open in $\tau_R(X)$.

(iii) Nano generalized α-closed set if

NαCl(A) $\subseteq$ V whenever A $\subseteq$ Vand V is Nano open in $\tau_R(X)$.

(iv) Nano α generalized closed set if

NαCl(A) $\subseteq$ V whenever A $\subseteq$ V and V is Nano α open in $\tau_R(X)$.

3. FORMS OF NANO REGULAR GENERALIZED CLOSED SETS AND NANO GENRALIZED REGULAR CLOSED SETS

In this section, we introduce Nano regular generalized closed set and Nano generalized regular closed set and investigate some of their properties**.**

*Definition 3.1***:** A subset A of a Nano topological space (U, $\tau_R(X)$) is called Nano regular generalized closed set if NrCl(A) $\subseteq$ V whenever A $\subseteq$ V and V is Nano regular open.

Definition 3.2: A subset A of a Nano topological space (U, $\tau_R(X)$) is called Nano generalized regular closed set if NrCl(A) $\subseteq$ V whenever A $\subseteq$ V and Vis nano open.

Definition 3.3: If (U, $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U then (i) The nano regular closure of A is defined as the intersection of all nano regular closed sets containing A and it is denoted by NrCl(A). NrCl(A) is the smallest nano regular closed set containing A.

(ii) The nano regular – interior of A is defined as the union of all nano regular open subsets of A contained in A and it is denoted by NrInt(A). NrInt(A) is the largest nano regular open subset of A.

Theorem 3.4: Every Nano closed set is a Nano regular closed.

Proof: Let A be a nano closed set in X such that A $\subseteq$ U; whenever U is regular open. That is NCl(A) =A. We have to prove that NCl(NInt(A)) $\subseteq$ A. Since A is Nano regular open in U; NInt(A) = A. $\Rightarrow$NCl(NInt(A)) = NCl(A). Since A is nano regular closed NCl(A)=A $\Rightarrow$NCl(NInt(A))=A. Hence every nano closed set is nano regular closed set.

The converse of the above theorem is true the following example.

Example 3.5: = {a,b,c,d} with U/R = { {a}, {c}, {b,d} and X = {a,b}. Then the nano topology $\tau_R(X)$ = {U, Φ,{a}, {a,b,d}, {b,d}}. The set {b,c,d}, {c} and {a,c} is Nano regular closed.

Theorem 3.6: Every nano regular closed set is nano regular generalized closed.

Proof: Let A be a nano regular closed set in X such that A $\subseteq$ V, V is Nano regular open.

That is NCl(NInt (A)) = A. Since A is nano regular open, NInt(A) = A. Every Nano open set is nano regular open. Therefore NCl(A) = A $\subseteq$ V $\Rightarrow$NCl(A) $\subseteq$ V.

Since A $\subseteq$ V then NCl(A) $\subseteq$ V whenever V is nano regular open. Hence A is nano regular generalized closed.

The converse of the above theorem is not true from the following example.

Example 3.7: Let U={a,b,c,d} with U/R = {{a}, {c}, {b,d}}. Then the nano topology $\tau_R(X)$ = {U, Φ, {a}, {a,b,d}, {b,d}}. Here {a,c,d} is nano regular generalized closed sets but it is not nano regular closed.

Theorem 3.8: Every nano regular closed set is nano generalized regular closed set.

Proof: Let A be a nano regular closed set in X such that A $\subseteq$ V, V is Nano open. That is NCl(NInt (A)) = A. Since A is nano open, NInt(A) = A. Every Nano open set is nano regular open. Therefore NCl(A) = A ⊆ V ⇒NCl(A) ⊆ V. Since A ⊆ V then NCl(A) ⊆ V whenever V is nano open. Hence every nano regular closed set is nano generalized regular closed.

The converse of the above theorem is not true from the following example.

Example 3.9: Let U={a,b,c,d}with
U/R = {{a}, {c}, {b,d}}. Then the nano topology
τ_R(X) = {U,Φ, {a}, {a,b,d}, {b,d}}. Here {a,b,c} is nano generalized regular closed sets but it is not nano regular closed.

Example 3.10: Let U={a,b,c,d} with
U/R= {{a}, {c}, {b,d}} and X={a,b}
Then τ_R(X)={U, Φ, {a}, {a,b,d}, {b,d}} which are open sets.
The nano regular closed sets =
{Φ,U,{b,c,d},{a,c}, {c}}
The nano regular open sets =
{U, Φ, {a}, {b,d}, {a,b,d}}
The nano regular – generalized closed sets =
{U,Φ,{c},{b,c},{c,d},{a,c}, {a,b,c}, {b,c,d}, {a,c,d}}
The nano regular – generalized open sets =
{Φ, U, {a}, {b}, {d}, {a,b}, {a,d},{b,d}, {a,b,d}}
The nano generalized regular open sets =
{U,Φ,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}}
The nano generalized regular closed sets =
{Φ,U,{c},{b,c},{c,d}, {a,c},{a,b,c},{b,c,d}, {a,c,d}}

Note 3.11: Every nano regular generalized closed set is a nano generalized regular closed. In the above example (3.10), all nano regular generalized closed sets are nano genrealized regular closed sets. The converse of the theorem is true.

Note 3.12: In the Example (3.10) A = {a},⊆ V,
V = {a,b,c,d}, V is nano open
NCl(A) = {a,c} ⊆ V, Now NrCl(A) = {a} ⊆ NCl(A),
If NCl(A) ⊆ V, then NrCl(A) ⊆ NCl(A).

Theorem 3.13: Every nano generalized closed set is nano generalized regular closed set.

Proof: let V be any nano generalized closed set. Then NCl(A) ⊆ V whenever A⊆ V and V is nano open in U.But NrCl(A) ⊆NCl(A) whenever A ⊆ V,V is nano open in U.Now we have NrCl(A) ⊆ V,
A⊆V, V is nano open in U. Hence A is nano generalized regular closed set.

Remark 3.14: The converse of the above theorem is not true in general. In the example (3.10), let
A = {b}, V = {a,b,d} whenever A ⊆ V, V is nano open. Now NrCl(A) = {b} ⊆ V. Hence A = {b}is nano generalized regular closed set. But
NCl(A) = {b,c} ⊈ V. Hence the subset A={b} is not nano generalized closed set. Hence every nano generalized regular closed set need not be a nano generalized closed set.

Theorem 3.15: The union of two nano regular generalized closed sets in (U, τ_R(X)) is also a nano regular generalized closed set in (U, τ_R(X)).

Proof: Let A and B be two Nano regular generalized closed sets in (U, τ_R(X)) and V be any nano regular open set in U such that A ⊆ V and B ⊆ V. Then we have AUB ⊆ V. As A and B are nano regular generalized closed sets in (U, τ_R(X)).

Therefore NrCl(A) ⊆ V; NrCl(B) ⊆ V.Now NrCl(AUB)=NrCl(A) U NrCl(B) ⊆ V.Thus we have
NrCl(AUB)) ⊆ V whenever AUB⊆ V, V is nano regular open set in U. AUB is a nano regular generalized closed set in U.

Theorem 3.16: The intersection of any two subsets of nano regular generalized closed sets in (U, τ_R(X)) is nano regular generalized closed set in (U, τ_R(X)).

Proof: Let A and B are any two nano regular generalized closed sets. A $\subseteq$ V; V is an nano regular open and B $\subseteq$ V; V is nano regular open. Then NrCl(A) $\subseteq$ V; NrCl(B) $\subseteq$ V. Therefore NrCl(A∩B) $\subseteq$ V; V is nano regular open in X. Since A and B nano regular generalized closed sets. Hence A∩B is a nano regular generalized closed set.

Theorem 3.17: If a set A is nano regular generalized closed set iff NrCl(A) – A contains no non empty, nano regular closed set.

Proof: Necessity: Let F be a nano regular closed set in $(U, \tau_R(X))$ such that

F $\subseteq$ NrCl(A) - A. Then A $\subseteq$ X –F. Since A is nano regular generalized closed set and X-F is nano regular open then NrCl(A) $\subseteq$ X – F. That is

F $\subseteq$ X – NrCl(A). so

F $\subseteq$ (X-NrCl(A)) ∩ (NrCl(A)-A). Therefore F=φ.

Sufficiency: Let us assure that NrCl(A)-A contains no non empty nano regular closed set. Let A $\subseteq$ V; V is nano regular open.

Suppose that NrCl(A) is not contained in V, NrCl(A) ∩ V^C is non empty, nano regular closed set of NrCl(A)-A which is contradiction therefore NrCl(A) $\subseteq$ V.

Hence A is nano regular generalized closed.

Theorem 3.18: If A is both Nano regular open and nano regular generalized closed set in X, then A is nano regular closed set.

Proof: Since A is nano regular open and nano regular generalized closed in X, NrCl(A) $\subseteq$ V But A $\subseteq$ NrCl(A). Therefore A = NrCl(A). Since A is nano closed NInt(A)=A. $\Rightarrow$ NrCl(A)) = A. Hence A is nano regular closed.

Theorem 3.19: Φ and U are nano generalized regular closed subset of U.

Remark 3.19: The finite union or intersection of nano generalized regular closed set needs not be a nano generalized regular closed set. It follows from the following two examples Let U = {a,b,c,d,e} and the corresponding nano topological space be $\tau_R(X)$ = {Φ, U {a,b}, {c}, {a,b,c}}. Let A = {a,c,d}. Obviously A is a nano generalized regular closed subset of U. Again let B = {b,c,d}. B is also a nano generalized regular closed subset of U. But A∩B = {c} is not a nano generalized regular closed subset of U. Since A∩B = {c}$\subseteq$ the open set {{c}, {a,b,c}, U}. But NrCl(A) = {c}. So finite intersection of nano generalized regular closed sets need not be a nano generalized regular closed subset of U. Again let U = {a,b,c,d,e} and the corresponding topological space be

$\tau_R(X)$ = {Φ, U {a,b}, {c}, {a,b,c}, {d,e}}. Let

A = {d} and B ={e}. Here A and B are nano generalized regular closed subset of U. But

AUB = {d,e} is not a nano generalized regular closed subset of U. Since NrCl(AUB) $\subseteq$ {d,e} an open subset of U. Hence finite union of nano generalized regular open set need not be nano generalized regular open subset of U.

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