Characterization and Frequency Analysis of One Day Annual Maximum and Two to Seven Consecutive Days' Maximum Rainfall of Panam Dam, Gujarat, India

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Abstract— **Annual one day maximum rainfall and two, three, four, five, six and seven consecutive day's maximum rainfall corresponding to a return period of 2 to 100 years has been conducted for the Panam dam, Gujarat, India. Three commonly used probability distributions are normal, lognormal and gamma distribution and that have been tested to determine the best fit probability distribution that describes the annual one day maximum and two to seven consecutive days' maximum rainfall series by comparing observed values with tabulated Chi-square value. The results showed that the log-normal distribution was the best fit probability distribution for one day annual maximum as well as two, three and four consecutive days, while for five and seven consecutive days, gamma distribution and for six days normal distribution fits better for the region. Based on the best fit probability distribution, a maximum of 125.56 mm in 1 day, 149.03 mm in 2 days, 168.49 mm in 3 days, 180.36 mm in 4 days, 191.64 mm in 5 days, 216.89 mm in 6 days and 213.61 mm in 7 days is expected to occur at Panam dam every two years. Similarly a maximum rainfall of 413.82 mm, 419.98 mm, 433.82 mm,456.98 mm, 470.87 mm, 473.33 mm and 520.87 mm is expected to occur in 1, 2,3,4,5,6 and 7 days respectively every 100 years. The results from the study could be used as a rough guide by engineers and hydrologists during the design and construction of drainage systems in the catchment area of Panam dam and computation of drainage co-efficient.**

Keywords— **return period, frequency, probability distribution, gamma, log-normal, normal, chi square, consecutive day's maximum rainfall.**

INTRODUCTION

A good understanding of the pattern and distribution of rainfall is vital for water resource management of a country. In particular, analysis of annual one day maximum rainfall and consecutive maximum days rainfall of different return periods (typically 2 to 100 years) is a basic tool for secure and cost-effective planning and design of small dams. Bhakar et al. (2006) observed the frequency analysis of consecutive days peaked rainfall at Banswara, Rajasthan, India, and found gamma distribution as the best fit as compared by other distribution and tested by Chi-square value. Kwaku et al. (2007) showed that the log-normal distribution was the best fit probability distribution for one to five consecutive days' maximum rainfall for Accra, Ghana. Barkotulla et al. (2009) studied several Probability distributions to predict rainfall status of various return period estimating one to seven consecutive days annual maximum rainfall of Boalia, Rajshahi, Bangladesh and found log normal was the best fit. Olophintoye et al.(2009) aimed at the peak daily rainfall distribution characteristic and the results of the frequency analysis suggest log Pearson type III and Pearson type III distributions has the primary distribution pattern for this study site and should be used as a universal distribution model for the prediction of peak daily rainfalls in Nigeria.

In this present study, annual 1 to 7 consecutive days maximum rainfall data of Panam Dam (a raingauge station near panam dam) for 32 years were selected. The area selected for the present study is one of the raingauge stations in the panam catchment area, which is located in panchmahals and godhra district of Gujarat state.

MATERIALS AND METHODS

Study Area and Data collection

The daily rainfall data recorded at the Panam dam (22°03'27"N latitude, 73° 46' 25" E longitude for a period of 32 years (1974-2005 inclusive) were used for this analysis. Annual 1,2,3,4,5,6 and 7 consecutive days rainfall were computed using the method described by Barkotulla *et al.* (2006), by summing up rainfall of corresponding previous days. Maximum amount of annual 1 day to 2 to 7 successive days' rainfall for each year was used for the analysis. Statistical parameters of annual 1 day as well as consecutive days maximum rainfall have been computed and are shown in Table-1. One day to seven days maximum rainfall data were fitted with the three main probability distributions (Table-2).

Modeling the consecutive days Rainfall

Three continuous probability distributions were selected to model the daily rainfall depth. The three models (probability distributions) are presented below along with their probability density functions (PDF). Note that x is the random variable representing the daily maximum rainfall.

Three goodness of fit tests available which are Kolmogrove-smirnov (K-S), Anderson darling (A-D) and chi square. Out of the goodness of fit tests available one of the most commonly used tests for testing the goodness of fit of empirical data to theoretical frequency distribution is the chi-square test. In applying the Chi-square goodness-of-fit test, the data are grouped into suitable frequency classes. The test compares the actual number of observations and the expected number of observations (expected values are calculated based on the distribution under consideration) that fall in the class intervals. The expected numbers are calculated by multiplying the expected relative frequency by the total number of observation. The sample value of the relative frequency of interval *i* is computed in accordance with equation 1 (*Barkotulla et al.*, 2009) as:

(1)

THE EXPECTED RELATIVE FREQUENCY IN A CLASS INTERVAL *I* CAN ALSO BE CALCULATED USING EQUATION 2

$$f_{x_i} = \delta x_i p(\quad (2)$$

The Chi-square test statistic is computed from the relationship

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i} \quad (3)$$

O_i is observed and E_i is the expected (based on the probability distribution being tested) number of observation in the i^{th} class interval. The observed number of observation in the i^{th} interval is computed from equation 1 as: $nf(x_i)=n_i$. Similarly, nf_{x_i} is the corresponding expected numbers of occurrences in interval i.

The distribution of χ_c^2 is the chi-square distribution with $v=k-l-1$ degrees of freedom. In conducting the goodness of fit test using the chi-square test , a confidence level, often expressed as $\alpha-1$, is chosen (where α is referred to as the significance level). Typically, 95% is chosen as the confidence limit. The test poses the null hypothesis (H_o) that the proposed probability distribution is from the specified distribution. H_o is rejected if $\chi^2_{cal} > \chi^2_{Tab}$. The value of χ_{Tab}^2 is determined from published tables with v degrees of freedom at the 5% level of significance.

In this study three commonly used probability distributions were fitted with 1 day and 2 to 7 days consecutive days maximum rainfall. The three distributions are briefly discussed below.

1) Normal distribution

The normal distribution is a two parameter unbounded continuous distribution which has been identified as the most important distribution of variables applied to symmetrically distributed data. The probability density function is given by:

$$N(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] \quad (4)$$

Where, σ is the standard deviation and μ is the mean of the sample.

2) Log normal distribution

A random variable *x* representing the daily maximum rainfall is said to follow a lognormal distribution if the logarithm (usually natural logarithm) of is normally distributed. The probability density functions of such a variable $y=\ln x$

$$:y = \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] \text{ For } 0 \le x \le \infty \quad (5)$$

Where, σ_y is the standard deviation and μ_y is the mean of $y = \ln x$

3) Gamma distribution

The Two parameter gamma distribution with continuous shape parameter (α), continuous scale parameter (β) is represented by the probability density function:

$$F(x) = \frac{1}{\beta^{\alpha} T_{\alpha}} \alpha^{x-1} e \quad for \quad 0 \le x \le \infty \quad (6)$$

Three parameter gamma distributions are commonly used in place of the log-normal distribution function

Frequency analysis

The analysis of the thirty two years rainfall involved sorting of the rainfall amounts against durations of one, two, three, four, five, six and seven consecutive days. Values of consecutive days maximum rainfall can be estimated statistically through the use of the Chow (1951)'s general frequency formula. The formula expresses the frequency of occurrence of an event in terms of a frequency factor, K_T, which depends upon the distribution of particular event investigated. Chow (1951) has shown that many frequencies analyses can be reduced to the form

$X_T = (1+C_V K_T)$ (7)

Where, is the mean, C_V is the coefficient of variation, is the frequency magnitude of a factor and X_T is the event having a return period T. K_T is the frequency factor which depends upon the return period T and the assumed frequency distribution. The expected value of maximum daily rainfall for the same return periods were computed for determining the best probability distributions.

Return period

A return period also known as a recurrence interval is an estimate of the likelihood of an event, such as a rainfall, flood or a river discharge to occur. The probabilities of the rainfall events were obtained using the formula shown in equation (1). Return period was calculated by Weibull's plotting position formula by arranging maximum daily rainfall in descending order giving their respective rank as:

T= (8)

Where, N - the total number of years of record and R- the rank of observed rainfall values arranged in descending order.

RESULTS AND DISCUSSION

For the level of significance(α) as 0.05 and degrees of freedom(v) as 3 the critical chi square value is 7.47.The data as revealed that the computed Chi-square values is for the three probability distributions were less than the critical Chi-square value at the 95% confidence level for one day and as well as other consecutive days maximum rainfall series (except the two and seven consecutive days maximum rainfall for the normal distribution). Results of the Chi-square values for the different distribution as presented in Table-2 indicated that the lognormal distribution gave the minimum value for one day, two day, three day and four day daily rainfall series and gamma has the minimum test statistics for five and seven days and normal distribution for six consecutive days rainfall in the study region. Table-3 gives the annual 1 day and consecutive days maximum rainfall for different return periods as determined by the selected best fit distribution. The result show that a maximum of 125.56 mm in 1 day, 149.03 mm in 2 days, 168.49 mm in 3 days,180.36 mm in 4 days, 191.64 mm in 5 days,216.89 in 6 days and 213.61 mm in 7 days is expected to occur at Panam dam every two years. Similarly a maximum rainfall of 413.82 mm, 419.98 mm, 433.82 mm, 456.98 mm, 470.87 mm, 473.33 mm and 520.87 mm is expected to occur in 1 day, 2, 3, 4, 5, 6 and 7 days respectively every 100 years.

It was recommended by Kwaku (2007) that 2 to 100 years is a sufficient return period for soil and water conservation measures, construction of dams, irrigation and drainage works. The 2 to 100 years return period obtained in this study could be used as a rough guide during the construction of hydraulics structures.

TABLE-1

Summary statistics of annual 1 day as well as consecutive days maximum rainfall.

Statistical parameters	1 day	2 day	3 day	4 day	5 day	6 day	7 day
Maximum(mm)	296	355	355	420	495	562	626
Minimum(mm)	47	52	52	54.2	56.2	57.2	60.2
Mean(mm)	142.3	163.47	163.47	194.08	205.41	216.89	228.43
Standard Deviation(mm)	71.89	75.45	75.45	84.13	91.10	97.34	105.32
Coefficient of Variation	0.50	0.46	0.46	0.43	0.44	0.45	0.46
Coefficient of Skewness	0.65	0.62	0.62	0.79	1.08	1.44	1.74
Excess Kurtosis	-0.59	-0.29	-0.29	0.36	1.88	3.85	5.51

TABLE-2

Chi-square value for the three different distributions.

Consecutive days	gamma (3P)	normal	lognormal(3P)
One day	1.2705	1.9846	0.2750
Two days	4.0870	7.6109	2.2819
Three days	3.1741	4.9546	3.1339
Four days	1.7138	6.1843	0.8712
Five days	1.2798	3.9475	2.5348
Six days	3.9469	2.2203	3.9440
Seven days	0.8668	8.6517	0.8785

TABLE-3

1 day as well as consecutive days' maximum rainfall for various return periods.

Return Period (years)	1 Day	2 day	3 day	4 day	5 day	6 day	7 day
2	125.56	149.03	168.49	180.36	191.64	216.89	213.61
5	193.86	219.18	240.10	255.93	273.36	298.81	304.05
10	242.91	266.62	287.13	305.13	324.53	341.63	360.39
20	292.46	312.73	332.01	351.83	371.38	376.99	411.83
25	300.10	319.70	338.73	358.80	378.23	381.91	419.35
50	360.23	373.48	390.07	411.91	429.23	436.79	475.22
100	413.82	419.98	433.82	456.98	470.87	473.33	520.77

CONCLUSIONS

The log-normal distribution was the best fit probability distribution for one day annual maximum as well as two, three and four consecutive days, while for five and seven consecutive days, gamma distribution and for six days normal distribution fits better for the region. Based on the best fit probability distribution, a maximum of 125.56 mm in 1 day, 149.03 mm in 2 days, 168.49 mm in 3 days, 180.36 mm in 4 days, 191.64 mm in 5 days, 216.89 mm in 6 days and 213.61 mm in 7 days is expected to occur at Panam dam every two years. Similarly a maximum rainfall of 413.82 mm, 419.98 mm, 433.82 mm,456.98 mm, 470.87 mm, 473.33 mm and 520.87 mm is expected to occur in 1, 2,3,4,5,6 and 7 days respectively every 100 years. The results from the study could be used as a rough guide by engineers and hydrologists during the design and construction of drainage systems in the catchment area of Panam dam and computation of drainage co-efficient.

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