

# Inverse Kinematic Solution for Five bar Parallel Linkage Planar Manipulator using PYTHON and Optimization by Taguchi Method

Dr. Reddy Sreenivasulu<sup>#1</sup>, Dr. Goteti Chaitanya<sup>2</sup>, Dr. Gangolu Vijay Kumar<sup>3</sup>, Matti Radha Devi<sup>4</sup>

<sup>#1</sup>Assistant Professor, Department of Mechanical Engineering, R.V.R & J.C.College of Engineering, Chowdavaram, Guntur, Andhra Pradesh, INDIA

<sup>2</sup>Associate Professor, Department of Mechanical Engineering, R.V.R & J.C.College of Engineering, Chowdavaram, Guntur, Andhra Pradesh, INDIA

<sup>3</sup>Professor, Department of Mechanical Engineering, P.V.P. Siddhartha Institute of Technology, Vijayawada, Andhra Pradesh, INDIA

<sup>4</sup>Assistant Professor, Department of Mechanical Engineering, P.V.P. Siddhartha Institute of Technology, Vijayawada, Andhra Pradesh, INDIA

rslu1431@gmail.com

**Abstract** - The kinematic design of robotic manipulator is a vital role while designing a robot for particular application by the mechanical engineers. It is a basic thing for further automation to work with a robot in a shop floor. In this connection, five bar parallel linkages provided in the advanced kinematics is an important design aspect towards robotic field where ever complex applications are involved not only in manufacturing, in other areas also. So, in the present work design of a manipulator which operated by five bar parallel linkage mechanism is described by considering geometric approach to solve inverse kinematics. In geometric approach, whenever number of links or degrees of freedom increases then solution becomes critical and almost laborious process to attain the solution. PYTHON, simple software presently numerous engineers are applied for solution of variety of problems occurred in all fields including engineering. The developed geometric solution is converted into coded form using PYTHON to reduce the complexity and save the design time. Once, the solution is generated by giving the inputs randomly and then infinite solution schemes are obtained. From that solution scheme, to get feasible solution for optimum set of kinematic structural solution taguchi method is utilized in the present study. The hypothesis presented in this paper is valid or not is checked with analysis of variance and found good agreement.

**Keywords** — Inverse kinematics, 5- bar parallel linkage, PYTHON, Taguchi method, S/N ratio

## I. INTRODUCTION

Increasing the demand in numerous applications of robotic manipulators, for professional purposes (robots in the area of surveillance, defense, environmental, bio medical and various manufacturing industries) and personnel use (robots

in home service and entertainment) with technological developments in the automation. For these reasons numerous researchers currently work on this area and developed new techniques towards optimal design of manipulators. While designing a manipulator the important issues are what type of factors affect the execution. The suitable alternative for a manipulator mechanism is influenced by the task or application to be executed as result link dimensions and configurations are determined. A five bar linkage mechanism is a two degree of freedom system that is from five links that are assembled together in a closed kinematic chain. All links are joined by five joints in sequence to form a loop. One of the links is in the ground which may be connected to prismatic type whenever wants variable movement or may be fixed if fixed off set length is considered. The direct and towards the back kinematics of this robotic configuration can be found in closed chain equations through geometric correlations. This type of linkage find in different areas of applications like prosthetic to hepatic feedback devices, automatic drawing toy, Ackermann steering gear mechanism and ankle foot prosthetic device.

Andrew Liou (1993) was proposed by orthogonal arrays of Taguchi design of experiments method and validated results with Monte Carlo simulations to recognize the noteworthy parameters for selection of best tolerance range to two link manipulator. Hyunseop Lim et al (2010) adopted grey coupled Taguchi technique to optimize the link geometry of the robot manipulator with superior global conditioning, overall manipulability and structural length index. The influence of link parameters on multi performance indices found by using ANOVA. Hyunpyo Shin et al (2011) presented a parameter optimization towards maximize the work space of a two degrees of freedom planer parallel manipulator using Taguchi method and discussed the relationships between the link length, workspace and



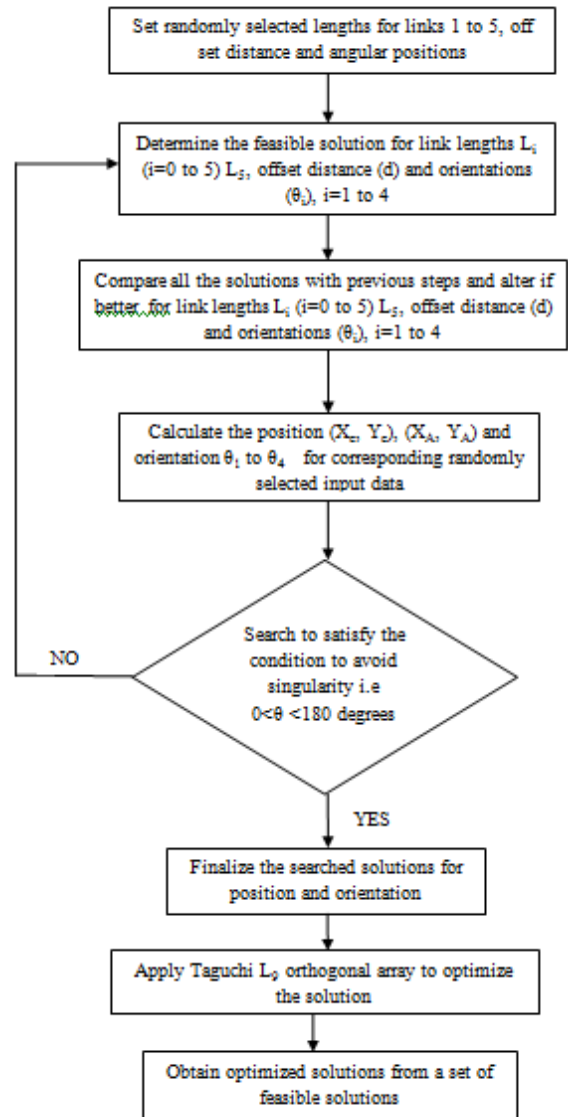
mean stiffness inside the workspace. Xingxing Feng et al (2018) focused on the kinematic investigation of a PPPR spatial serial manipulator with various geometric errors using computational algorithms especially for positioning accuracy. Thuy Le Thi Thu and Long Pham Thanh (2019) are proposed generalized reduced gradient method to determine the link and joint tolerances from the requirement of the precision of the given end effector of 6 DoF industrial robot. H. P. Jawale and H. T. Thorat et al (2013) studied a technique for examining the outcome of errors in joint parameter on accuracy needed in the end effector position for a serial chain 2R planar manipulator; in this connection preparation for manipulator link characteristics under subjected to the joint clearances are analyzed. Fu-Chen et al (2010) proposed a combination approach of Taguchi method coupled with PCA and fuzzy logic approach for the tolerance design of a multipurpose 6 bar mechanism to evaluate positional errors as a function of angular error for each quality control factors of the mechanism.

Hadi Kalani and Alireza Akbarzadeh (2012) investigations performed on a 16 link snake robot in serpentine locomotion and identified important kinematics and dynamics parameters to reduce average power utilization per unit distance travelled, for this Taguchi orthogonal arrays and ANOVA are utilized to fine best parameter settings. Hyeon Guk Kim et al (2014) also proposed Taguchi method integrated with grey relational analysis to minimize the time of optimization of the whole work space by considering condition number (CN), structural length index (SLI) and global conditioning index (GCI). James Nielsen and Bernard Roth et al (1999) reviewed on solutions for both inverse kinematics of continuous chain manipulators and direct kinematics of parallel linkages serial chain manipulators and parallel linkages using elimination methods and also provided the information regarding, how to formulate direct kinematics solutions. Pengzhan Liu et al (2018) also utilized Taguchi method to identify the optimal selection of parameters for wheel flip type robot to enhance its climbing capacity as well as adaptability for various types of staircases. S.S. Rao and P.K. Bhatti (2001) studied the effects of tolerances in the joint actuators and controllers; in this connection Markov probabilistic approach was adapted to review the performance of manipulator kinematics by taking various robotic configurations. T. Mathavaraj Ravikumar and R. Saravanan (2015) determined the optimal localization of a mobile robot using odometry technique with design of experiments based on Taguchi technique and proved that good agreement. B.K. Rout and R.K. Mittal (2006 & 2007), illustrated elaborately on manipulator parameter tolerance with a new offline strategy to get optimal performance using Taguchi based design of experiments.

**A. Methodology**

The algorithm of proposed approach is shown in figure.1. In the present study, optimal solution for structural dimensions

and orientations of robotic manipulator having kinematic mechanism of five bar parallel linkage to reach desired end effectors position in the work space obtained with analytical method, PYTHON and Taguchi method.



**Fig.1 Algorithm for proposed methodology**

**II. FIVE BAR PARALLEL LINKAGE TYPE OF PLANER ROBOTIC MANIPULATOR**

In the present analysis, considered three link parallel planer manipulator as shown in figure.2 and labelled various joint and link positions properly as per DH notations. The main motive of this study is to identify the suitable dimensions of robot arm structure and orientation with respect to previous axes. In present work, link lengths

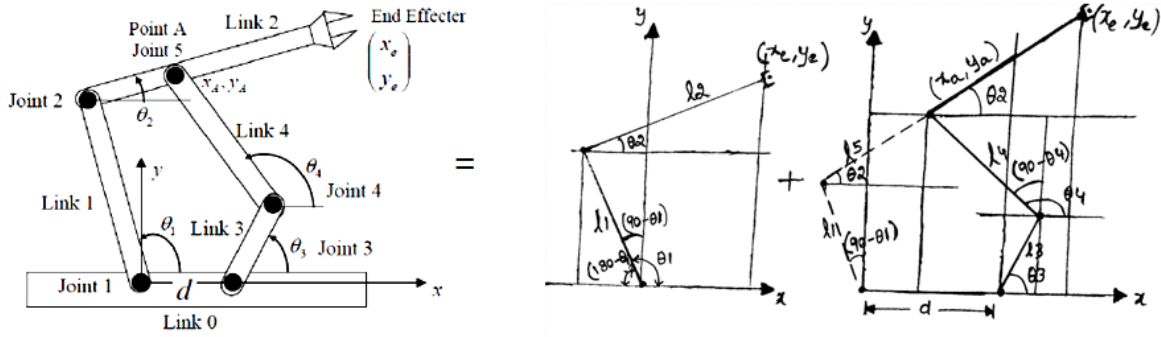


Fig.2 Five bar parallel linkage planer manipulator

are randomly selected and found their corresponding orientations using robot kinematics by geometric method. The application of this mechanism found in automatic dog structures which is generally used in defense areas especially for searching of bombs. Taguchi method is one of the design of experiments technique applied by numerous authors in optimization of machining parameters, there researchers are selected the factors that influence the responses by brain storming methods and fix the range of values of input factors and certain calibrated apparatus the record the required output responses. So, in the current work also follow the same procedure to identify best combinations of dimensions to avoid singularities while movement of robotic manipulator in its work envelope.

**III. INVERSE KINEMATICS BY GEOMETRIC APPROACH**

Robot kinematics deals with position, velocity and acceleration of an end effector to reach accurately in the working environment. For a given position of end effector, solve the joint positions (both linear and angular) depends on type of actuators the manipulator contains to drive the arms called as inverse kinematics [16-20].

$$X_e = l_1 \cos \theta_1 + l_2 \cos \theta_2 \text{ ----- [1]}$$

$$Y_e = l_1 \sin \theta_1 + l_2 \sin \theta_2 \text{ ----- [2]}$$

$$X_A = l_1 \cos \theta_1 + l_5 \cos \theta_2 = d + l_3 \cos \theta_3 + l_4 \cos \theta_4 \text{ ---- [3]}$$

$$Y_A = l_1 \sin \theta_1 + l_5 \sin \theta_2 = l_3 \sin \theta_3 + l_4 \sin \theta_4 \text{ ---- [4]}$$

In step.1 calculate  $\theta_1$  and  $\theta_2$  angular positions of corresponding links 1 and 2 by solving equation 1 and equation 2.

In step.2 find the  $X_A$  and  $Y_A$  positions by substituting the obtained values from step.1 in equation 3 and 4.

In step.3 calculate  $\theta_3$  and  $\theta_4$  with above solutions and input data.

The analytical solution of inverse kinematics is laborious, once the robotic manipulator consist more number of links.

To solve such a complexity, apply different software packages are available to get solutions or write a code in MATLAB or in JAVA or in PYTHON. In the present work, PYTHON (current burning topic adopted by engineering professionals globally) is a general purpose programming language to solve inverse kinematic formulations in coded form in simple and easy to understand by budding engineers obtained by geometric approach in the coded form. The following analytical equations are derived based on geometry of figure1 and solutions are determined in three steps.

**IV. INVERSE KINEMATIC SOLUTION CODE GENERATED FROM PYTHON**

For the above analytical equations in PYTHON code is developed in simple manner and then solutions are generated by giving the randomly selected input data and obtain the desired orientations of links in closed form to reach desired predefined end effector position. Further Taguchi method is applied to optimize the obtained set of solutions generated from PYTHON.

The following is the code written in PYTHON environment for inverse solution of parallel linkage of 3 link planer robotic manipulator.

```
import math
def calcAngle(reqcos):
    res1 = math.atan2(math.sqrt(1-math.pow(reqcos, 2)), reqcos)
    res2 = math.atan2(math.sqrt(1-math.pow(reqcos, 2))*(-1), reqcos)
    return (res1, res2)
def Calc1(px, py, l1, l2):
    ctheta2 = (px**2 + py**2 - l1**2 - l2**2) / (2 * l1 * l2)
    stheta2 = math.sqrt(1-math.pow(ctheta2, 2))
    ctheta1 = (px * (l1 + l2 * ctheta2) + py * l2 * stheta2) / (px**2 + py**2)
    theta1a, theta1b = calcAngle(ctheta1)
    theta2a, theta2b = calcAngle(ctheta2)
    return (theta1a, theta1b, theta2a, theta2b)
```

```

def Calc2():
px = float (input("xe:"))
py = float(input("ye:"))
l1 = float (input("l1:"))
l2 = float (input("l2:"))
l3 = float (input("l3:"))
l4 = float (input("l4:"))
l5 = float (input("l5:"))
d = float (input("d:"))
t1a, t1b, t2a, t2b = Calc1(px, py, l1, l2)
for i in (t1a, t1b):
    for j in (t2a, t2b):
        xa = l5 * math.cos(j) + l1 * math.cos(i)
        ya = l5 * math.sin(j) + l1 * math.sin(i)
        t3a, t3b, t4a, t4b = Calc1(xa-d, ya, l3, l4)
        print
        ("theta1:{ } theta2:{ }".
format(math.degrees(i), math.degrees(j)))
        print ("xa:{ } and ya:{ }".
format(xa, ya))
        print ("\ttheta3:{ } and { }".
format(math.degrees(t3a), math.degrees(t3b)))
        Print ("\ttheta4:{ } and { }".
format(math.degrees(t4a), math.degrees(t4b)))
Calc2()

```

In this article, chosen randomly one end effector position i.e  $X_e = 20$  mm and  $Y_e = 15$  mm, to reach this assumed goal position the optimum arrangement of input parameters such as link lengths and offset distance are obtained via Taguchi method to satisfy the angular (orientation) positions of successive input links with respect to preceding structural positions. The position ( $X_a$ ,  $Y_a$ ) chosen along the link 2 which is pinned and labeled by link 5. The input values chosen at the beginning of the iterations are illustrated table.I

**Table.I Input data given to PYTHON and tabulated as per Taguchi  $L_9$  orthogonal array**

Trial	Input data					
	L1, mm	L2, mm	L3, mm	L4, mm	L5, mm	d, mm
1	19	21	13	12	10	8
2	18	22	11	10	12	7
3	17	17	8	9	7	6
4	18	16	8	10	9	7
5	13	17	9	12	8	8
6	10	18	10	12	6	8
7	14	18	13	9	7	5
8	19	11	8	14	11	9
9	17	18	15	14	8	5

The solutions obtained with respect to number of iterations, for each iteration four inverse solutions obtained from those only nine best solutions found by eliminating singular positions and depicted the final values of outputs in table.II

**Table.II Output data obtain from PYTHON**

Trial	Output data					
	$X_a$	$Y_a$	$\Theta_1$	$\Theta_2$	$\Theta_3$	$\Theta_4$
1	15.84	15.66	18.12	102.81	20.17	91.15
2	13.91	18.42	22.02	103.36	50.00	40.91
3	17.48	8.69	5.8	85.33	2.86	64.31
4	18.68	9.84	2.77	85.52	3.97	64.30
5	16.01	7.89	2.15	67.8	27.13	117.11
6	13.34	5.01	0.19	56.06	43.06	142.44
7	15.33	8.77	7.89	77.97	0.77	106.50
8	22.31	14.43	12.45	69.99	11.12	55.89
9	16.94	10.71	9.17	88.87	11.61	112.95

**V. RESULTS AND DISCUSSIONS**

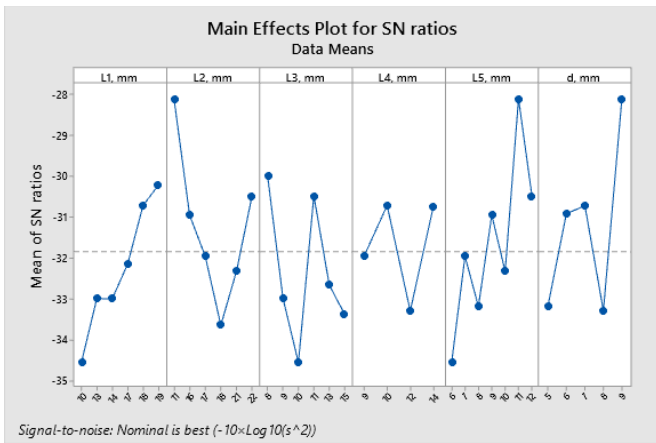
Signal to noise ratio symbolically represented by S/N is a statistical measure used in the field of science and engineering that compare the level of output response signal to the level of noise obtained in the experimentation. In general, sometimes it is defined as the ratio of useful information to false or irrelevant data in a conversation and it conveys information about the behaviour of some phenomenon / hypothesis. In Taguchi design of experiments, S/N ratio indicates appraise of robustness to recognize control factors (called as process parameters) to diminish variability in a process by reducing the effects of uncontrollable parameters (called as noise factors)[23-24]. According to Taguchi method, higher the S/N ratio shows the setting of process parameters that lower the effect of noise factors. From minitab@19 design of experiments software S/N ratios are obtain as per  $L_9$  orthogonal array of Taguchi method (from table.1) and depicted in table.2. The setting of level wise input parameters to get closed form inverse kinematic solutions for a five bar parallel linkage achieved from Taguchi method as  $L_5$ ,  $L_2$ ,  $d$ ,  $L_3$ ,  $L_1$  and  $L_4$  influences more on reaching a goal of end effector in a rank wise order respectively. So while synthesize the five bar parallel linkage mechanism put more emphasis on control factors  $L_5$ ,  $L_2$  and offset distance.

**Table.III S/Noise ratios for link dimensions verses output responses (Nominal is the best)**

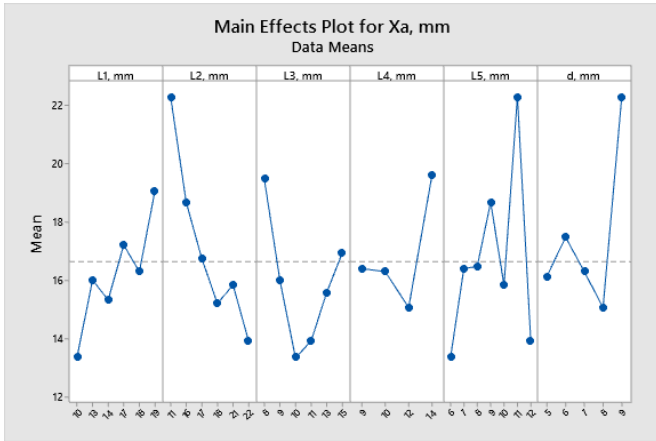
Level	L1, mm	L2, mm	L3, mm	L4, mm	L5, mm	d, mm
1	-34.54	-28.10	-29.98	-31.95	-34.54	-33.17
2	-32.98	-30.94	-32.98	-30.71	-31.95	-30.91
3	-32.98	-31.94	-34.54	-33.28	-33.17	-30.71
4	-32.14	-33.63	-30.47	-30.73	-30.94	-33.28
5	-30.71	-32.31	-32.65	-30.52	-32.31	-28.10
6	-30.21	-30.47	-33.36	-30.92	-28.10	-29.37
Delta	4.33	5.52	4.55	2.57	6.44	5.17
Rank	5	2	4	6	1	3

Main effects plots are drawn for values S/N ratio for data means of responses measured (in this article, numerical

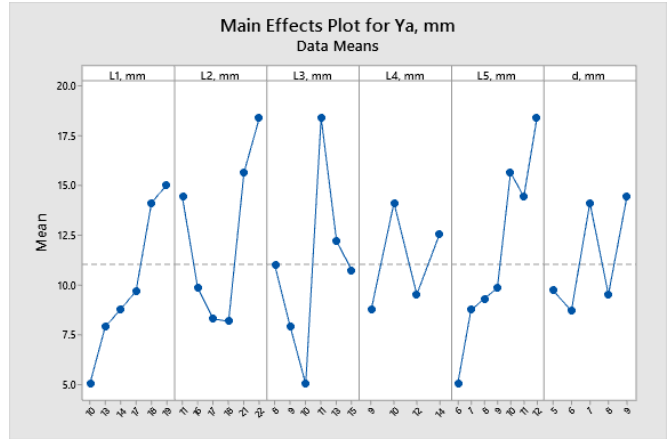
outputs generated from PYTHON by giving randomly selected inputs). From this graph, it is observed that  $L_2$ ,  $L_5$  and  $d$  values are high peaks rather than remaining parameters, so their influence is more while synthesize the mechanism. Similarly, remaining main effects plots obtained from Minitab@19 and shown in below corresponding Fig. 3 to Fig. 8 to analyze the effect of randomly selected parameters on position of point A on link 5 (represented as  $X_a$ ,  $Y_a$ ) selected along link 2 as shown in previous Fig.2 and depicted in Fig.e 4 & Fig. 5. Also, effect of input parameters on orientations of link 1 to 5 symbolically represented in Fig.2 as  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$  and  $\theta_4$ . Fig 5, 6 & 7 shows that  $L_2$ ,  $L_3$  and  $L_5$  influenced more on orientations of link1, link2 and link3 but for  $\theta_4$  only link3 and link5 influence more which is found from Fig.9.



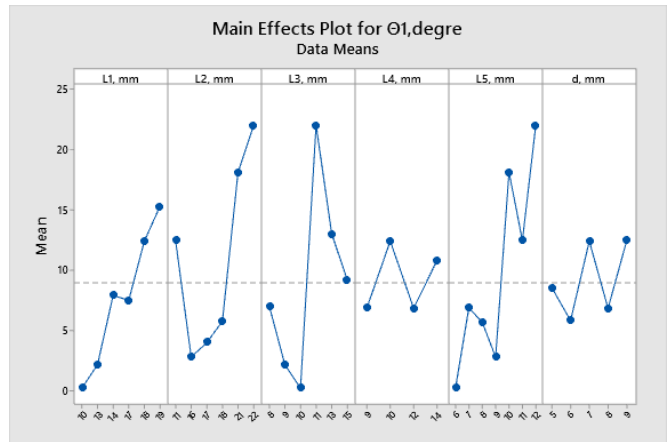
**Fig. 3** Graph shows main effects of S/N ratios of calculated responses



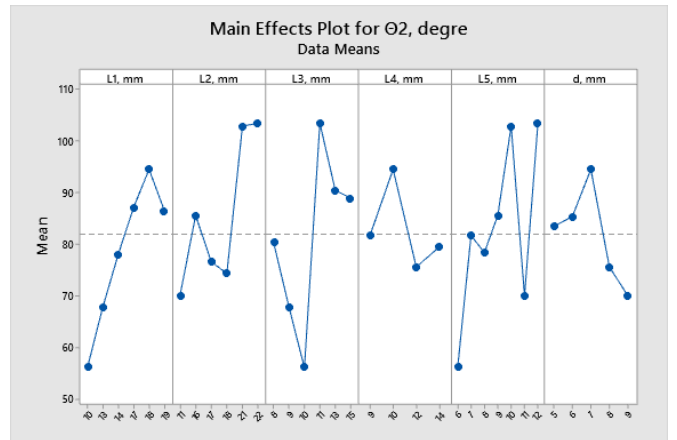
**Fig. 4** Graph shows main effects of data means on position of  $X_a$  on link 5



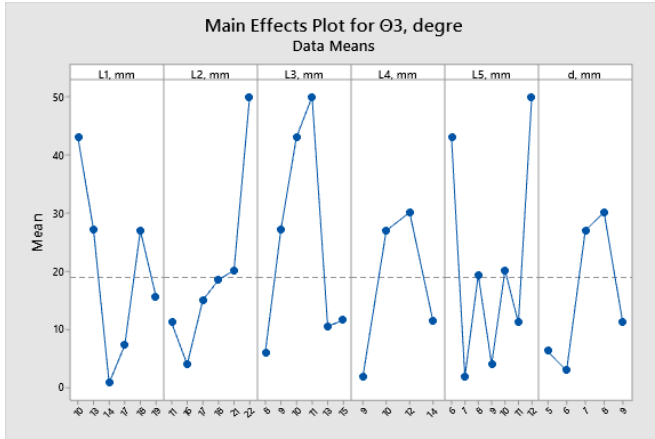
**Fig. 5** Graph shows main effects of data means on position of  $Y_a$  on link 5



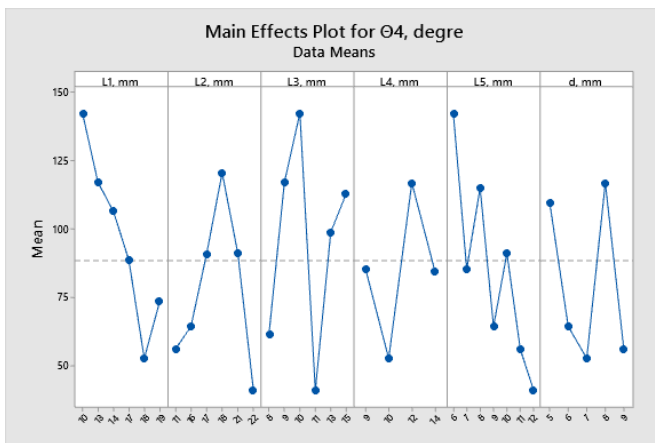
**Fig. 6** Graph shows main effects of data means on orientation of link 1



**Fig. 7** Graph shows main effects of data means on orientation of link 2



**Fig. 8 Graph shows main effects of data means on orientation of link 3**



**Fig. 9 Graph shows main effects of data means on orientation of link 4**

**VI. CONCLUSIONS**

The kinematic design of 5-bar parallel linkage type of planar robotic manipulator performed in this article initially using geometric approach and then a programming code is developed from PYTHON a programming language and output values generated by feeding input data randomly see such that to avoid singular positions of links in a closed form manipulator. After that using Taguchi orthogonal array and S/N ratio found optimum combination of set of values from assumed iterations and drawn the following conclusions.

1. The setting of level wise input parameters to get closed form inverse kinematic solutions for a five bar parallel linkage achieved from taguchi method as  $L_5$ ,  $L_2$ ,  $d$ ,  $L_3$ ,  $L_1$  and  $L_4$  influences more on reaching a goal of end effector in a rank wise order respectively.

2. To avoid singularities the final optimum dimensions of links generated from PYTHON to reach assumed end effector position in particular case as  $L_1 = 19\text{mm}$ ,  $L_2 = 22\text{ mm}$ ,  $L_3 = 10.5\text{mm}$ ,  $L_4 = 11\text{mm}$ ,  $L_5 = 9\text{mm}$  and offset distance,  $d = 9\text{mm}$  to satisfy all the conditions as per algorithm.

**REFERENCES**

- [1] Y. H. Andrew Liou, Paul P. Lin, Richard R. Lindeke and H. D. Chiang, Tolerance specification of robot kinematic parameters using experimental design technique- The Taguchi method, *Robotics & Computer Integrated Manufacturing*, 10(3)(1993) 199-207.
- [2] Hyunseop Lim, Soonwoong Hwang, Kyoosik Shin, and Changsoo Han, The Application of the Grey-based Taguchi Method to Optimize the Global Performances of the Robot Manipulator, *The 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems* October (2010) 18-22, Taipei, Taiwan
- [3] Hyunpyo Shin, Sungchul Lee, Woosung In, Jay I. Jeong and Jongwon Kim, Kinematic Optimization of a Redundantly Actuated Parallel Mechanism for Maximizing Stiffness and Workspace Using Taguchi Method, *Journal of Computational and Nonlinear Dynamics*, JANUARY 6(2011) / 011017-1
- [4] Xingxing Feng, Haihua Sun, Tianqi Lv and Yunqing Zhang, Kinematic analysis of a PPPR spatial serial mechanism with geometric errors, *Proc IMechE Part C: J Mechanical Engineering Science*, 0(0)(2018) 1–16.
- [5] Thuy Le Thi Thu and Long Pham Thanh, Tolerance Calculation for Robot Kinematic Parameters to Ensure End-Effector Errors within a Predetermined Limit Area, *International Journal of Engineering Research and Technology*, 12(9)(2019) 1584-1597.
- [6] H. P. Jawale and H. T. Thorat, Positional Error Estimation in Serial Link Manipulator Under Joint Clearances and Backlash, *Journal of Mechanisms and Robotics*, MAY 5(2013) / 021003-1.
- [7] Fu-Chen, C, Yih-Fong, T, Meng-Hui, H, and Wei-Ren, C, Combining Taguchi Method, Principal Component Analysis and Fuzzy Logic to the Tolerance Design of a Dual Purpose Six Bar Mechanism, *Trans. Can. Soc. Mech. Eng.*, 34(2)(2010) 277–293.
- [8] Hadi Kalani and Alireza Akbarzadeh, Parameter Optimization of a Snake Robot Using Taguchi Method, *Applied Mechanics and Materials*, (2012) 110-116 4220-4226.
- [9] Hyeon-Guk Kim, Kyoo-Sik Shin, Soon-Woong Hwang, and Chang-Soo Han, Link Length Determination Method for the Reduction of the Performance Deviation of the Manipulator: Extension of the Valid Workspace, *International journal of precision engineering and manufacturing* 15(9)(2014) 1831-1838.
- [10] James Nielsen and Bernard Roth, On the Kinematic Analysis of Robotic Mechanisms, *The International Journal of Robotics Research*, 18(12)(1999) 1147-1160.
- [11] Pengzhan Liu, Jianzhong Wang, Xin Wang and Peng Zhao, Optimal design of a stair-climbing mobile robot with flip mechanism, *32(6)(2018) 325-336*.
- [12] S.S. Rao and P.K. Bhatti, Probabilistic approach to manipulator kinematics and dynamics, *Reliability Engineering and System Safety*, 72(2001) 47-58.
- [13] T. Mathavaraj Ravikumar and R. Saravanan, Experimental investigation for better relative localization of a mobile robot using Taguchi method, *Robotica*, 33(7)(2015) 1415-1423.
- [14] B.K. Rout, R.K. Mittal, Tolerance design of robot parameters using Taguchi method, *Mechanical Systems and Signal Processing*, 20(8) (2006) 1832–1852.
- [15] Rout, B.K., Mittal, R.K. Tolerance .01 using design of experiment approach, *Structural and Multidisciplinary Optimization*, 34(2007) 445- 462.
- [16] Tringali, A., & Cocuzza, S., globally optimal inverse kinematics method for a redundant robot manipulator with linear and nonlinear constraints, *Robotics*, 9(3)(2020) 61.
- [17] Craig, J. J., *Introduction to robotics: mechanics and control*, 3/E. Pearson Education India.
- [18] Wang, L. C., & Chen, C. C., A combined optimization method for solving the inverse kinematics problems of mechanical manipulators, *IEEE Transactions on Robotics and Automation*, 7(4)(1991) 489-499.
- [19] Rokbani, N., & Alimi, A. M. (2013), Inverse kinematics using particle swarm optimization, a statistical analysis, *Procedia Engineering*, 64 1602-1611.
- [20] Karna, S. K., & Sahai, R., an overview on Taguchi method. *International Journal of Engineering and Mathematical*

- Sciences, 1(1)(2012) 1-7.
- [21] Sreenivasulu, R., Simulation of desired end point trajectory for a 2-dof planar manipulator. *International Journal of Advanced Scientific and Technical Research (IJAST)*, 5(2)(2012) 688-696.
- [22] Chaitanyaa, G., & Reddy, S., Genetic algorithm based optimization of a two Link planar robot manipulator, *International Journal of Lean Thinking*, 7(2)(2016) 1-13.
- [23] Yogesh Raut, Dr. Amit Bahekar. Optimization of Parameters to Reduce the Surface Roughness in Face Milling Operation on Cast Iron Work piece Applying Taguchi Method, *International Journal of Engineering Trends and Technology*, 68 (1)(2021) 9-13.
- [24] Santosh Kumar Patod, Dr. Suman Sharma. Optimization of CNC Turning Cutting Parameter for Geometrical Dimensional Accuracy with Surface roughness on the non-ferrous Material Applying Taguchi Technique *International Journal of Engineering Trends and Technology*, 67(12)(2019) 56-66.