International Journal of Engineering Trends and Technology
ISSN: 2231 – 5381 /doi:10.14445/22315381/IJETT-V69I3P215

Volume 69 Issue 3, 85-95, March 2021

Geometrically Non-Linear Free In-Plane Vibration of Circular Arch Elastically Restrained Against Rotation At The Two Ends

Omar Outassafte[#1], Ahmed Adri[#2], Yassine El khouddar[#3], Said Rifai [#4] and Rhali Benamar[*5]

[#]*Hassan II University of Casablanca, EST, LMPGI, B.P.8012, Oasis Casablanca, Morocco.*
[*]*University Mohammed V in Rabat, E.M.I. BP 765, Rabat, Morocco*

[1]omar.outassafte@ensem.ac.ma ,[2] ahmedadri@gmail.com, [3] yassine.elkhouddar@ensem.ac.ma,

Abstract - *In this present paper, the geometrical non-linearity in free In-plane vibration of inextensible circular arch with uniform cross section and elastically restrained against rotation at the two ends has been investigated. Using the ends conditions and the transfer matrix, the eigen values of problem are determined iteratively using the Newton-Raphson algorithm. The kinetic and potential energy are discretized into a series of a finite spatial functions which are a combination of linear modes and basic function contribution coefficients. The use of Hamilton's principle energy reduces the problem into a set of non-linear algebraic system that solved numerically using an approximate explicit method developed previously the so-called second formulation. Considering the multi-mode approach, the effect of the dimensionless rotational stiffness of springs at the two ends on non-linear frequency in the neighborhood of the first mode shape of the arch has been presented with their corresponding non-linear deflections and curvatures.*

Keywords - *Geometrical non-linearity, circular arch, elastically restrained, second formulation, the dimensionless rotational stiffness of springs*

I. INTRODUCTION

The Arches are one of the important structural element on many fields such as mechanical, aerospace, and civil engineering, the authors classified the arches by their shallowness ratio into two classes; shallow arch and deep or no-shallow arch.

The vibration analysis of the arches is different from the analogous problems of rectilinear beam, because they depend on two variables: radial displacement, and tangential displacement. The present study will be mainly focused on the nonlinear free in-plane vibration of non-shallow circular arch. Many researchers have investigated dynamic behaviour and vibration of arches, such as a review papers of P. Chidamparam and A.W. Leissa [1]. N. M. Auciello and M. A. De Rosa [2], Laura et al [3], Also the books about the Theory of Arched Structure by Igor A. Karnovsky , and The Dynamics of Arches and Frames by J. Henrych [4]. The early investigators into the in-plan theory of rings and arches were Hope (1871) and Love (1944). Robert R. Archer [5] employed the equations of motion as given in Love with the addition of terms to represent damping effects to find the first four natural frequencies clamped incomplete ring with a constant symmetrical cross section. The general dynamic slope-deflection equations, which include the effect of dynamic load for a continuous circular arch frames are presented by T. M. Wang and J. M. Lee [6]. F Yang, R. Sedaghati and E. Esmailzadeh [7] investigated the free vibration of curved beam based on different hypotheses including and excluding the axial extensity, rotary inertia and the shear deformation. Chu. Chengyi et al [8] developed a new theory for nonlinear buckling and nonlinear analysis of circular arch including the shear deformation, the theory of a Cosserat point was used by M.B. Rubin, E. Tufekci [9] to study a three-dimensional free vibrations of a circular arch in small deformations, the problem of the base-excited motions of a circular arch is solved by Mau and Williams [10]. Concerning the geometrical nonlinearity of circular arch, J.D. Yau, Y.B. Yang [11] proposed a nonconventional structural approach for deriving the planar curved beam element. C.A. Dimopoulos, C.J. Gantes [12] investigated the effect of other behaviour factors, such as the geometrical and material nonlinearities and the initial imperfections on the strength of the arches . Recently, A semi analytical solution for free in-plane vibration of inextensible circular arches with a uniform cross-section and added point masses is presented by Ahmed Babahammou and Rhali Benamar [13].

In this present paper, the geometrically non-linear free in plane vibration of circular arch elastically restrained at the two ends against the rotation will be presented following the analogous of references [14] and [15]. For the simplification, the complicating effects such as rotary inertia and shear deformation will be ignored. The arch is assumed to be inextensible. The theoretical model is based on the Euler–Bernoulli beam theory in polar coordinates and the von Karman geometrical non-linearity assumptions. Harmonic motion is assumed and expended into a series of finites spatial functions. The use of Hamilton's principle energy, reduces the problem into a set of non-linear algebraic equations solved numerically using an approximate method the so-called second formulation developed in [16]. The effect of the dimensionless rotational stiffness of spring at the two ends on non-linear behaviour of arch will be presented.

II. REVIEW OF THE MATHEMATICAL APPROACH

A. Linear problem

Considering the circular arch elastically restrained at the two ends by a rotational springs K_R and K_L in Fig. 1, with radius R and opening angle θ, young's modulus E, cross section A, moment of inertia I and distributed mass μ. The shear deformation and rotary inertia are neglected because the arch is supposed to be thin. u and w are respectively radial and tangential displacement of the arch, when The arch axis is supposed to inextensible radial and tangential displacement are related by [17] :

$$\frac{\partial w}{\partial \varphi} = u \tag{1}$$

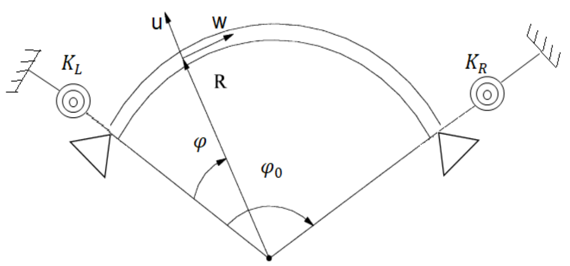

Fig.1: **Schematic of system**

In this case the planar equation of motion can expressed as:

$$\frac{\partial^5 u}{\partial \varphi^5} + 2\frac{\partial^3 u}{\partial \varphi^3} + \frac{\partial u}{\partial \varphi} + \frac{\mu R^4}{EI}\frac{\partial^3 u}{\partial t^2 \partial \varphi} = 0 \tag{2}$$

Considering the arch in a harmonic motion, we put:

$$u(\varphi,t) = U(\varphi)\cos(\omega t) \tag{3}$$

By writing this form of solution:

$$U(\varphi) = e^{\Omega\varphi} \tag{4}$$

By replacing the Equation (3) and (4) into equation (2) the characteristic equation can be written as:

$$\Omega^5 + 2\Omega^3 + (1-\lambda^2)\Omega = 0 \tag{5}$$

Where $\lambda^2 = \frac{\mu R^4 \omega^2}{EI}$ is the non-dimensional frequency.

The general solution of the differential equation (2) in term of radial displacement can be written as:

$$U(\varphi) = C_1 + C_2\cosh(\alpha\varphi) + C_3\sinh(\alpha\varphi) + C_4\cos(\beta\varphi) + C_5\sin(\beta\varphi) \tag{6}$$

Where: $\alpha = \sqrt{\lambda - 1}$, $\beta = \sqrt{\lambda + 1}$ and $\lambda \geq 1$

The boundary condition circular arch may be written as:

$$U(\varphi = 0) = U(\varphi_0 = 0) \tag{7}$$

$$W(\varphi = 0) = W(\varphi_0 = 0) \tag{8}$$

$$M(\varphi = 0) - k_L\delta(\varphi = 0) = 0$$
$$M(\varphi_0 = 0) + k_R\delta(\varphi_0 =) = 0 \tag{9}$$

Where the bending moment expression may be written as:

$$M(\varphi) = \frac{-EI}{R^3}\left(\frac{\partial^4 w}{\partial \varphi^4} + \frac{\partial^2 w}{\partial \varphi^2}\right) \tag{10}$$

The eigenvalues of system 6×6 are determined iteratively using the Newton-Raphson algorithm in Matlab . The corresponding frequency parameter λ are presented in Table 1 and 2 for various values of the dimensionless rotational stiffness of spring $K_R^* = \frac{K_R R}{EI}$ and $K_L^* = \frac{K_L R}{EI}$. Using the Cartesian coordinate the first and second mode shape of arch are plotted respectively in Fig.3.

Table1: Frequency parameter λ for circular arch elastically restrained against the rotation at the ends

φ_0	$K^*=0$			$K^*=6$			$K^*=12$		
	Present	Galerkin	Ref [19]	Present	Galerkin	Ref [19]	Present	Galerkin	Ref [19]
80°	19,25	17,932	17.964	23,3875	23,345	22.788	25,3507	25,489	24.711
120°	8	6,9168	6.9268	9,93484	9,8534	9.5407	10,8348	10,719	10.337
180°	3	2.2646	2.2646	3,97311	3.7656	3.6196	4,40242	4.068	3.9151

	$K^*=24$			$K^*=100$			$K^*=10^7$		
	Present	Galerkin	Ref [19]	Present	Galerkin	Ref [19]	Present	Galerkin	Ref [19]
80°	27,2562	26,275	26.399	29,7513	29,268	28.383	30,8942	30,061	29.218
120°	11,6994	10,9	10.954	12,8193	11,778	11.599	13,3281	12,225	11.848
180°	4,80443	4.2825	4.1194	5,31211	4.4710	4.3140	5,53832	4.5387	4.3844

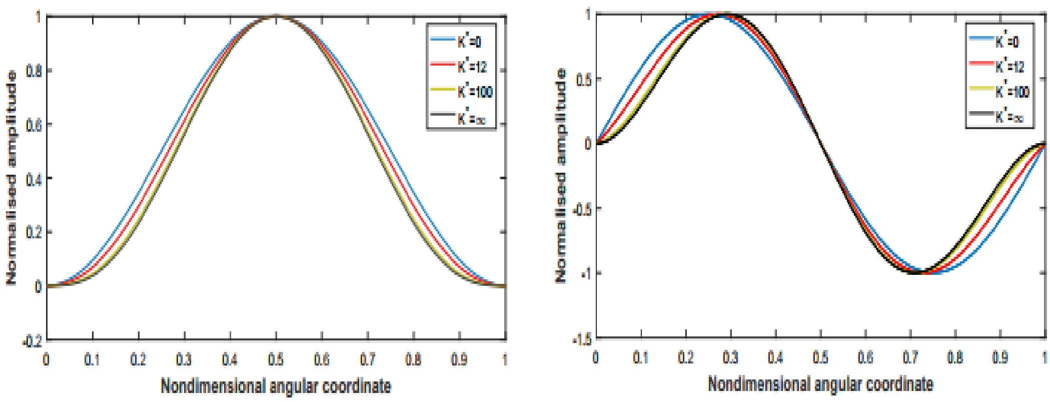

Fig.2: Representation of the first longitudinal and transversal displacement of circular arch elastically Restrained at the ends by a rotational springs at the two ends

Table2: Frequency parameter λ for circular arch elastically restrained against the rotation at the ends $K_L^*=\infty$ and K_R^* variable.

		First frequency parameter λ_1					
		40	60	80	100	120	180
0	Present	101.203	44.497	24.652	15.468	21.311	4.195
	Ref [18]	99,582	42,940	23,178	14,091	9,210	3,254
6	Present	109.9715	48.435	26.899	16.934	11.523	4.706
	Ref [18]	106,407	46,892	25,826	16,010	10,672	4,027
12	Present	114.201	50.328	27.975	17.631	12.015	4.940
	Ref [18]	110,360	48,752	26,888	16,684	11,129	4,205
24	Present	118.274	52.147	29.005	18.297	12.482	5.157
	Ref [18]	114,705	50,500	27,771	17,189	11,440	4,298
100	Present	123.479	54.467	30.3152	19.138	13.070	5.423
	Ref [18]	119,970	52,473	28,720	17,718	11,766	4,411
∞	Present	125.792	55.495	30.894	19.509	13.328	5.538
	Ref [18]	123,977	53,740	29,218	17,926	11,848	4,384

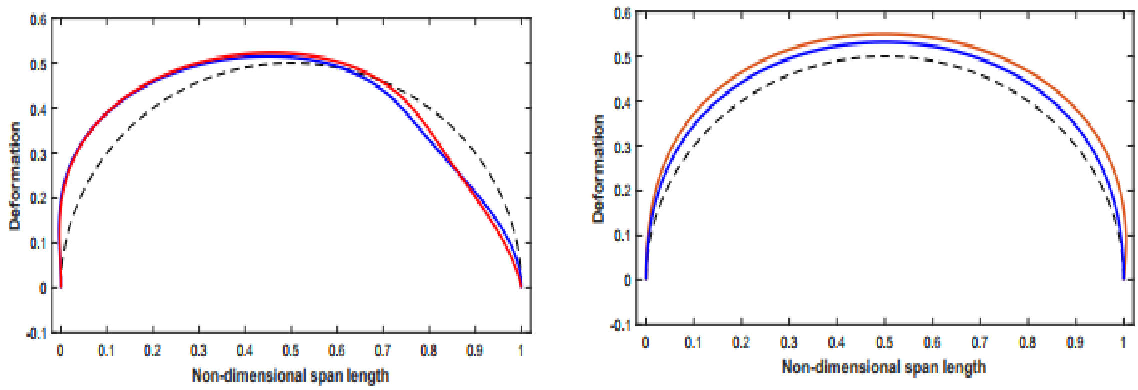

Fig.3: Representation of the first longitudinal and transversal displacement of circular arch elastically

At two ends. In blue $K_R^* = \infty$ **,** $K_L^* = \infty$ **, in red** K_R^* **=100 and** $K_L^* = \infty$

B. *Formulation of the Nonlinear Problem*

Kinetic energy T potential energies V for the system in Fig.1 can be written as [20]:

$$T = \frac{R\mu}{2}\int_0^{\varphi_0}\left[\left(\frac{\partial w^{(1)}}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right]dt \qquad (11)$$

$$V_a = \frac{EI}{2R^3}\int_0^{\varphi_0}\left(\frac{\partial w}{\partial \varphi} + \frac{\partial^3 w}{\partial \varphi^3}\right)^2 d\varphi \qquad (12)$$

$$+\frac{K_L}{2R^4}\left(\frac{\partial^2 w}{\partial \varphi^2}\right)^2\Bigg|_{\varphi=0} + \frac{K_R}{2R^4}\left(\frac{\partial^2 w}{\partial \varphi^2}\right)^2\Bigg|_{\varphi=\varphi_0}$$

$$V_b = \frac{EA}{8R^3}\int_0^{\varphi_0}\left(w + \frac{\partial^2 w}{\partial \varphi^2}\right)^4 d\varphi \qquad (13)$$

To develop the non-linear theory, the displacement function is expanded as a series of N basic spatial functions:

$$w(\varphi,t) = \sum_{i=1}^{N} a_i W_i \sin(\omega t) \qquad (14)$$

Using a generalized parameterization and the usual summation convention defined in [10] and [9] The kinetic energy T

and potential energy V due to axial, bending and elastic foundation energy of the arch can be expressed as:

$$T = \frac{1}{2} a_i a_j \omega^2 m_{ij} \cos^2(\omega t) \qquad (15)$$

$$V_a = \frac{1}{2} a_i a_j k_{ij} \sin^2(\omega t) \qquad (16)$$

$$V_b = \frac{1}{2} a_i a_j a_k a_l b_{ijkl} \sin^4(\omega t) \qquad (17)$$

m_{ij}, k_{ij} and b_{ijkl} are respectively, the mass tensor, linear and quadratic nonlinear rigidity tensors of the arch in which their expressions are defined as:

$$m_{ij} = R\mu\int_0^{\varphi_0}\left(\frac{\partial W_i}{\partial \varphi}\frac{\partial W_j}{\partial \varphi} + W_i W_j\right)d\varphi \qquad (18)$$

$$k_{ij} = \frac{EI}{R^3}\int_0^{\varphi_0}\left(\frac{\partial W_i}{\partial \varphi} + \frac{\partial^3 W_i}{\partial \varphi^3}\right)\left(\frac{\partial W_j}{\partial \varphi} + \frac{\partial^3 W_j}{\partial \varphi^3}\right)d\varphi \qquad (19)$$

$$+\frac{K_R}{2R^4}\left(\frac{\partial^2 W_i}{\partial \varphi^2}\frac{\partial^2 W_j}{\partial \varphi^2}\right)\Bigg|_{\varphi=0} + \frac{K_L}{2R^4}\left(\frac{\partial^2 W_i}{\partial \varphi^2}\frac{\partial^2 W_j}{\partial \varphi^2}\right)\Bigg|_{\varphi=\varphi_0}$$

$$b_{ijkl}=\frac{EI}{R^3}\int_0^{\varphi_0}\left(W_i+\frac{\partial^2 W_i}{\partial\varphi^2}\right)\left(W_j+\frac{\partial^2 W_j}{\partial\varphi^2}\right)d\varphi \int_0^{\varphi_0}\left(W_k+\frac{\partial^2 W_k}{\partial\varphi^2}\right)\left(W_l+\frac{\partial^2 W_l}{\partial\varphi^2}\right)d\varphi \quad (20)$$

The coefficients a_i are unknowns as well as the frequency ω.

The dynamic behavior of a conservative system may be obtained using Hamilton's principle by taking into account the forcing term, may be written as follow:

$$\delta\int_0^{2\pi/\omega}(V-T)\,dt \quad (21)$$

The process leads to non-linear algebraic equations for the unknown coefficients a_i that can be written in a matrix form as follows:

$$\left([K]-\omega^2[M]\right)\{A\}+\frac{3}{2}[B(A)]\{A\}=\{0\} \quad (22)$$

Where {A} is a column vector of a basic functions contribution a_i .To evaluate the non-dimensional parameter, ones put:

$$W_i(\varphi)=hW_i^*\left(\frac{\varphi}{\varphi_0}\right)=hW_i^*\left(\varphi^*\right) \quad (23)$$

$$\frac{m_{ij}}{m_{ij}^*}=\mu R^3h^2,\ \frac{k_{ij}}{k_{ij}^*}=\frac{b_{ijkl}}{b_{ijkl}^*}=\frac{EIh^2}{R^3} \quad (24)$$

Which leads to:

$$\omega^{*2}=\frac{EI}{\mu R^3} \quad (25)$$

Where m_{ij}^*, k_{ij}^* and b_{ijkl}^* are the non-dimensional

generalized parameter given by:

$$m_{ij}^*=\int_0^1\left(\frac{1}{\varphi_0^2}\frac{\partial W_i^*}{\partial\varphi^*}\frac{\partial W_j^*}{\partial\varphi^*}+W_i^*W_j^*\right)\varphi_0 d\varphi^* \quad (26)$$

$$k_{ij}^*=\int_0^1\left(\frac{1}{\varphi_0}\frac{\partial W_i^*}{\partial\varphi^*}+\frac{1}{\varphi_0^3}\frac{\partial^3 W_i^*}{\partial\varphi^{3*}}\right)\left(\frac{1}{\varphi_0}\frac{\partial W_j^*}{\partial\varphi^*}+\frac{1}{\varphi_0^3}\frac{\partial^3 W_j^*}{\partial\varphi^{3*}}\right)\varphi_0 d\varphi^* \quad (28)$$

$$b_{ijkl}^*=\alpha\int_0^1\left(W_i^*+\frac{\partial^2 W_l^*}{\varphi^2\partial\varphi^{2*}}\right)\left(W_j^*+\frac{\partial^2 W_j^*}{\varphi^2\partial\varphi^{2*}}\right)\varphi d\varphi^* \int_0^1\left(W_k^*+\frac{\partial^2 W_k^*}{\varphi^2\partial\varphi^{2*}}\right)\left(W_l^*+\frac{\partial^2 W_l^*}{\varphi^2\partial\varphi^{2*}}\right)\varphi d\varphi^* \quad (29)$$

For uniform rectangular section α = 3.

Equation cans be rewritten in non-dimensional form as:

$$\left(\left[K^*\right]-\omega^2\left[M^*\right]\right)\{A\}+\frac{3}{2}\left[B^*(A)\right]\{A\}=\{0\} \quad (30)$$

Using the tonsorial notation, one put:

$$a_ik_{ir}^*+\frac{3}{2}a_ia_ja_kb_{ijkr}^*-a_i\omega^{*2}m_{ir}^*=0 \quad (31)$$

with $r=1,\dots,n$

ω^{*2} is given in by:

$$\omega^{*2}=\frac{\{A\}^T[K]\{A\}+\frac{3}{2}\{A\}^T\left[B\left(\{A\}\right)\right]\{A\}}{\{A\}^T[M]\{A\}} \quad (32)$$

For higher amplitudes, a second formulation has been considered in which only second order terms of the type $\varepsilon_i\varepsilon_ja_1b^*_{ij1r}$ are neglected when considering the first non-linear mode. By separating in the non-linear expression $a_ia_ja_kb^*_{ijkr}$ terms proportional to a_1^3, terms proportional to $\varepsilon_ia_1^2$ and neglecting terms proportional to $a_1\varepsilon_i\varepsilon_j$ leads to:

$$a_ia_ja_kb^*_{ijkr}=a_1^3b_{111r}^*+a_1^2\varepsilon_ib_{11ir}^* \quad (33)$$

After substituting and rearranging, equation can be written in matrix form as:

$$\left(\left[K_r^*\right]_R-\omega^2\left[M_r^*\right]_R\right)\{A_r\}_R+\frac{3}{2}\left[\alpha^*_r\right]_R\{A_r\}_R=\left\{-\frac{3}{2}a_r^3b_{111r}^*\right\} \quad (34)$$

Where the term $\left[\alpha^*_r\right]_R=\left[a_r^{*2}b_{ijrr}^*\right]_R$ is a $r\times r$ matrix depending on a_r.

III. NUMERICAL RESULTS AND DISCUSSION

The first eight linear frequencies of an arch elastically restrained against the rotation at the two ends, are summarized in Table 3 for $K^* = K_L^* = K_R^*$ and in Table 4 for $K_L^* = \infty$ and various values of K_R^* with an opening angle $\varphi_0 = 180^\circ$.

The linear analysis was performed in order to use the linear modes shapes as a basics functions on non-linear analysis.

For a given value of a_1, Table 5,6 and 7 present the frequency ratio $\omega_{nl}^* / \omega_l^*$, the dimensionless non-linear amplitude $W_{\max}^* / h$ and the basic Contribution coefficient functions ε_i obtained by this method in case of SS circular arch ($K_L^* = K_R^* = 0$), CS circular arch ($K_L^* = 0, K_R^* = \infty$) and CC circular arch ($K_L^* = K_R^* = \infty$) respectively .

The Fig.4 and 5 give the backbone curves corresponding to various values of dimensionless rotational stiffness at the two ends K_L^* and K_R^*.

The corresponding normalized first non-linear mode shape and curvatures are shown in Fig.6 to 11 respectively.

The hardening behaviour type of geometrical non-linearity can be clearly observed in Fig. 4 and 5.

The effect of the dimensionless rotational stiffness at the two ends on the curvatures and frequency ratio associated to the first non-linear deflection can be clearly observed in Table 8 by the difference between linear and non-linear theory for an estimate non-linear normalized amplitude $W_{\max}^* / h = 1.6$.

Table 3: The first fundamental frequencies of circular elastically restrained at two ends with $K_L^* = \infty$ and various level of K_R^* and $\varphi_0 = 180^\circ$.

K_R^*	0	6	12	100	∞
λ_1	4,1957	4,7068	4,9402	5,4237	5,5383
λ_2	8,9916	9,369	9,5976	10,242	10,439
λ_3	17,119	17,647	17,988	19,047	19,398
λ_4	25,982	26,431	26,753	27,964	28,449
λ_5	38,095	38,641	39,043	40,66	41,35
λ_6	50,978	51,465	51,844	53,583	54,449
λ_7	67,082	67,64	68,082	70,21	71,326
λ_8	83,976	84,475	84,903	87,124	88,45

Table 4 : The first eight fundamental frequency of circular arch elastically restrained at two ends with various values of $K^* = K_L^* = K_R^*$ and $\varphi_0 = 180^\circ$

K^*	**0**	**6**	**12**	**100**	∞
λ_1	3	3,9731	4,4024	5,3121	5,5383
λ_2	7,5875	8,3637	8,8096	10,05	10,439
λ_3	15	16,02	16,671	18,705	19,398
λ_4	23,582	24,494	25,131	27,489	28,449
λ_5	35	36,066	36,847	39,984	41,35
λ_6	47,583	48,567	49,322	52,735	54,449
λ_7	63	64,095	64,96	69,116	71,326
λ_8	79,584	80,613	81,448	85,822	88,45

Table 5: Contribution coefficient functions corresponding to the first non-linear mode shape of circular arch for $K_L^* = K_R^* = 0$ and $\varphi_0 = 180^\circ$

ω_{nl}^*/ω_l^*	W_{max}^*/h	a_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8
1,002004	0,08165	0,05	3,70E-10	-7,82E-07	2,91E-13	-4,91E-08	7,57E-14	-8,25E-09	7,54E-14
1,064259	0,46947	0,2875	4,86E-08	-0,00015	1,72E-11	-9,28E-06	1,03E-11	-1,56E-06	1,34E-11
1,200969	0,857231	0,525	1,70E-07	-0,0009	-1,10E-10	-5,56E-05	3,95E-11	-9,46E-06	7,57E-11
1,390341	1,244886	0,7625	3,13E-07	-0,00274	-6,59E-10	-0,000166	8,25E-11	-2,87E-05	2,18E-10
1,613943	1,632399	1	4,57E-07	-0,00613	-1,78E-09	-0,000364	1,41E-10	-6,38E-05	4,64E-10

Table 6: Contribution coefficient functions corresponding to the first non-linear mode shape of circular arch for $K_L^* = 0, K_R^* = \infty$ and $\varphi_0 = 180^\circ$

ω_{nl}^*/ω_l^*	W_{max}^*/h	a_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8
1,000552	0,084803	0,05	9,47E-06	5,16E-07	1,96E-07	5,21E-08	2,71E-08	1,06E-08	6,30E-09
1,018089	0,487909	0,2875	0,001797	9,88E-05	3,77E-05	1,01E-05	5,25E-06	2,05E-06	1,23E-06
1,059116	0,892589	0,525	0,010914	0,000613	0,000237	6,36E-05	3,34E-05	1,31E-05	7,87E-06
1,121058	1,301556	0,7625	0,033317	0,001932	0,0007599	0,000206	0,00011	4,30E-05	2,60E-05
1,200683	1,719943	1	0,074879	0,004508	0,001812	0,000497	0,00027	0,000106	6,47E-05

Table 7 : Contribution coefficient functions corresponding to the first non-linear mode shape of circular arch for $K_L^* = K_R^* = \infty$ and $\varphi_0 = 180^\circ$

ω_{nl}^*/ω_l^*	W_{max}^*/h	a_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8
1,000595	0,087679	0,05	-4,26E-11	3,36E-06	3,12E-13	2,95E-07	-3,78E-14	6,05E-08	4,52E-12
1,019487	0,504433	0,2875	-7,82E-09	0,000634	6,03E-11	5,62E-05	-8,61E-12	1,16E-05	8,76E-10
1,063593	0,922336	0,525	-4,41E-08	0,003789	3,93E-10	0,000344	-6,99E-11	7,19E-05	5,57E-09
1,129963	1,342267	0,7625	-1,22E-07	0,01129	1,41E-09	0,001061	-2,74E-10	0,000226	1,82E-08
1,214955	1,764991	1	-2,47E-07	0,024557	4,00E-09	0,002406	-7,19E-10	0,000524	4,43E-08

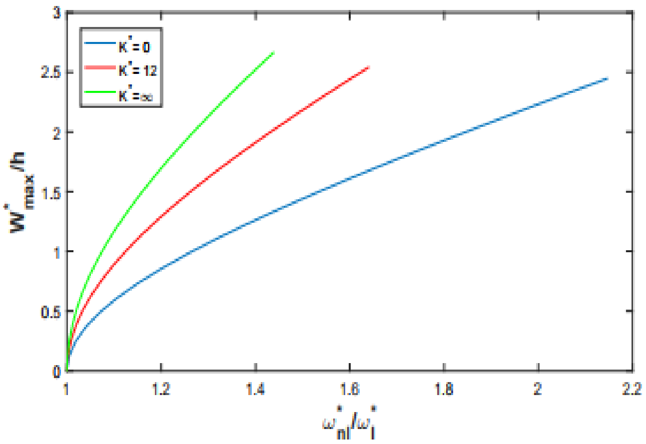

Fig.4: Backbone curve of circular arch elastically restrained at the end , in the vicinity of the first mode, for various values of K^* **, with** $K^* = K_L^* = K^*$

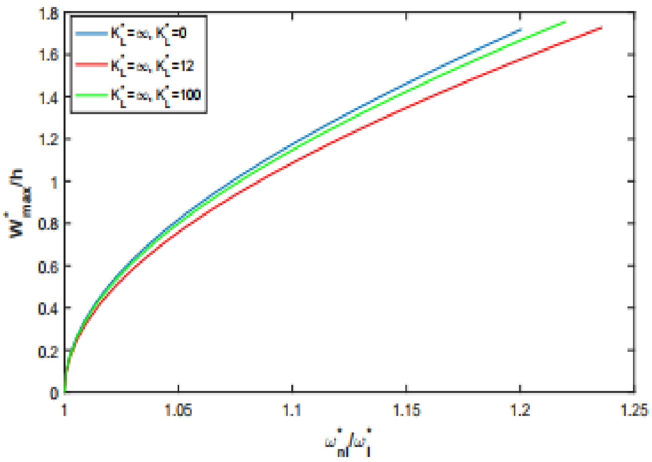

Fig.5: Backbone curve of circular arch elastically restrained at the end , in the vicinity of the first mode, $K_L^* = \infty$ **and various values of** K_R^*

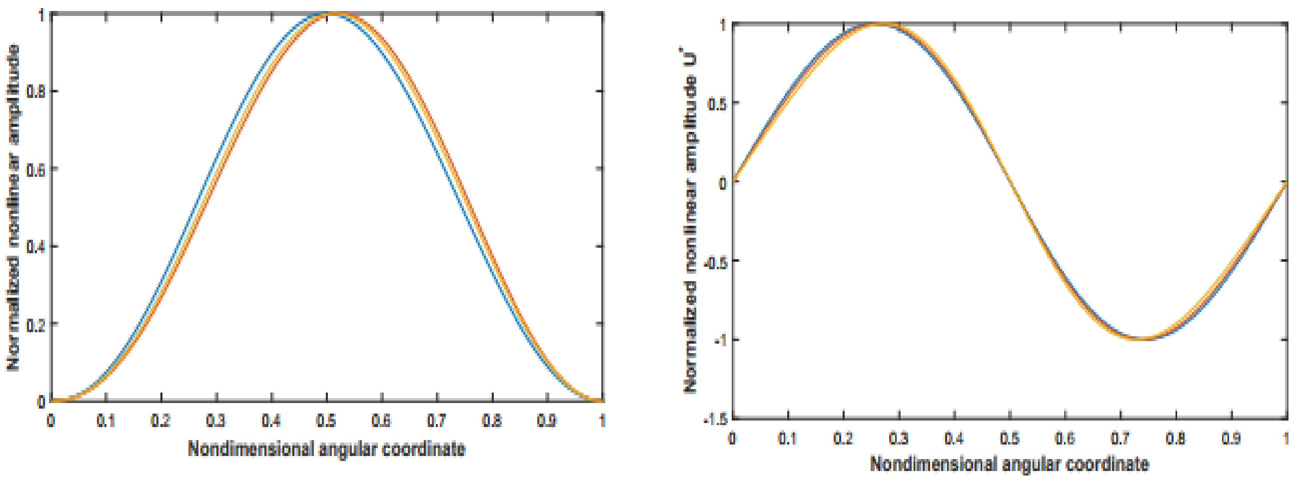

Fig.6:Representation of the first normalized longitudinal transversal non-linear amplitude of circular arch elastically restrained at the end with $K_L^* = K_R^* = 0$

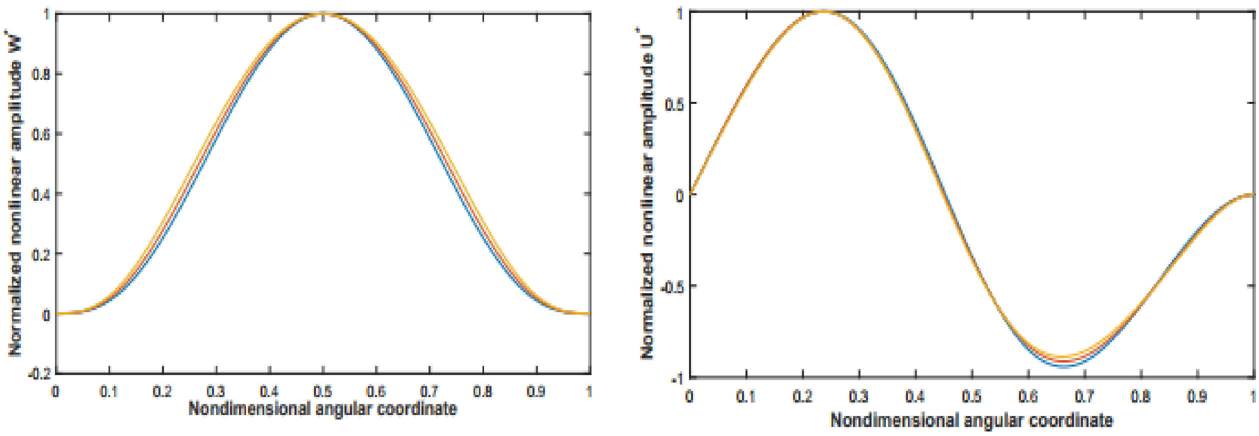

Fig.7:**Representation of the first normalized longitudinal transversal non-linear amplitude of circular arch elastically restrained at the end with** $K_L^* = 0$ **and** $K_R^* = \infty$ **.**

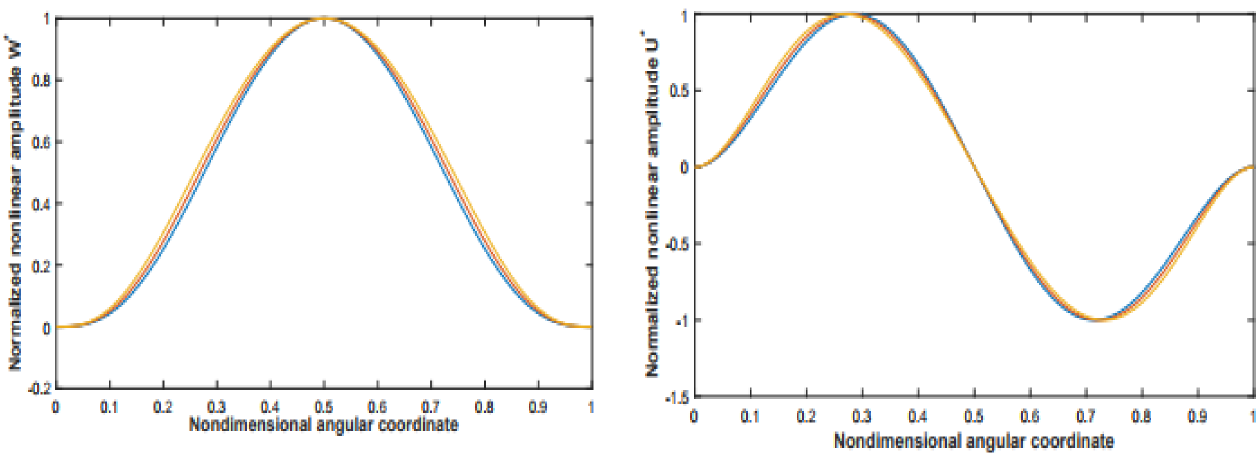

Fig.8:**Representation of the first normalized longitudinal transversal non-linear amplitude of circular arch elastically restrained at the end with** $K_L^* = \infty$ **and** $K_R^* = \infty$

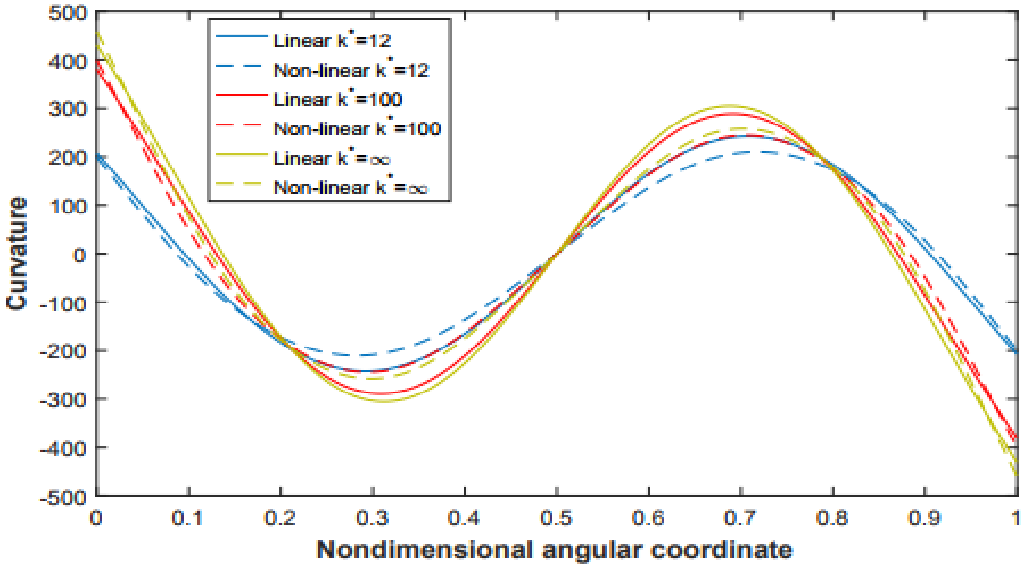

Fig.9: **The curvature associated to the first non-linear modes shape for various values of** K^*

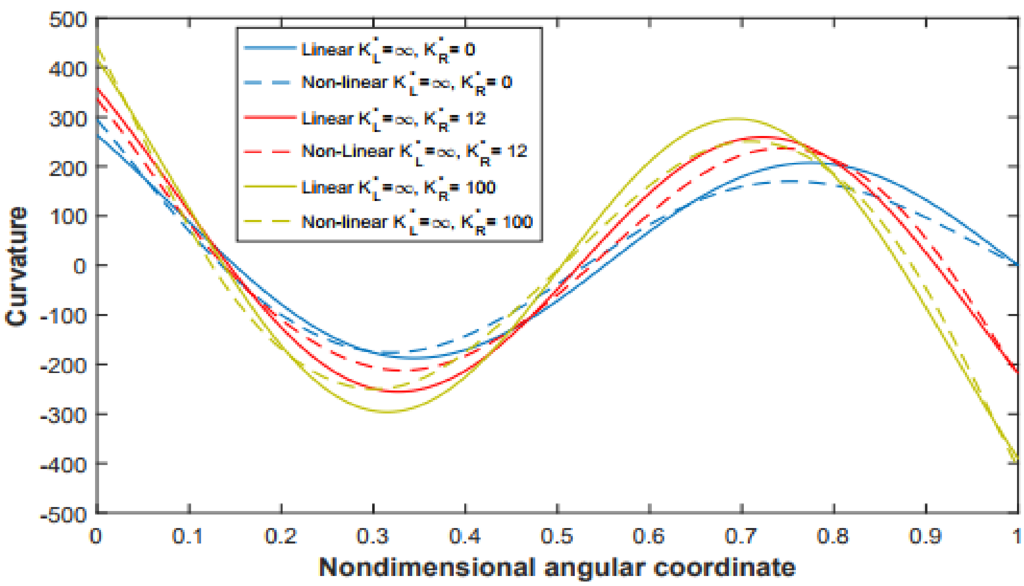

Fig.10: **The curvature associated to the first non-linear modes shape for** $K_L^* = \infty$ **and various values of** K_R^*

Table 8: effect of torsion spring stiffness K^* on non-linear curvature and frequency ration of circular arch

	Curvature at $\varphi = 0$			ω_l^*	ω_{nl}^*	$\omega_l^* / \omega_{nl}^*$
	Linear	Non-linear	difference			
$K^*=12$	207.50	211.39	3.85	2.875	3.726	1.295
$K^*=100$	380.47	399.13	18.66	4.018	4.770	1.187
$K^*=\infty$	430.80	458.78	27.93	4.390	5.168	1.177

IV. CONCLUSION

The geometrical non-linearity in free In-plane vibration of inextensible circular arch elastically restrained at the ends against rotation has been investigated. The generalized transcendent equation of frequency has been solved iteratively using the Newton-Raphson algorithm. The analysis of the geometrical non-linearity was performed based on the Euler Bernoulli theory and the Von Karman geometrical non-linearity assumptions. The linear frequencies have been obtained for various values of the dimensionless rotational stiffness of springs at the two ends. The linear modes shapes have been plotted and used as a basic function on non-linear analysis. The kinetic and total strain energy were discretized into a set of non-linear algebraic equations and derived using a Hamilton's principle energy and spectral analysis. The problem is reduced into a set of non-linear algebraic equations solved numerically by an approximate explicit method the so-called second formulation. Based on multi-mode approach, the effect of the dimensionless rotational stiffness of springs at the two ends on non-linear behavior of arch has been illustrated on the backbone curves of the frequency-amplitude dependence for various cases. Also the non-linear amplitude and curvature associated to the first non-linear deflection has been presented in case of CC, SS and CS circular arch.

REFERENCES

[1] Chidamparam, P. & Leissa, A. W. Vibrations of Planar Curved Beams, Rings, and Arches. Appl. Mech. Rev 46, 467–483 (1993).

[2] Auciello, N. M. & De Rosa, M. A. Free Vibrations Of Circular Arches: A Review. Journal of Sound and Vibration 176, 433–458 (1994).

[3] Laura, P. A. A., Ercoli, L., Baron, J., Sanchez Sarmiento, G. & Utjes, J. C. A note on the determination of the fundamental frequency of vibration of a rectangular plate with a free, semicircular cut-out along the edge. Journal of Sound and Vibration 104, 1–8 (1986).

[4] Henrych, J. The Dynamics of Arches and Frames. (Elsevier Science Ltd, 1981).

[5] Archer, R. R. Small vibrations of thin incomplete circular rings. International Journal of Mechanical Sciences 1, 45–56 (1960).

[6] Wang, T. M. & Lee, J. M. Forced vibrations of continuous circular arch frames. Journal of Sound and Vibration 32, 159–173 (1974).

[7] Free in-plane vibration of curved beam structures: A tutorial and the state of the art - F Yang, R. Sedaghati, E. Esmailzadeh, 2018.

[8] Chengyi, C., Genshu, T. & Lei, Z. In-plane nonlinear buckling analysis of circular arches considering shear deformation. Journal of Constructional Steel Research 164, 105762 (2020).

[9] Three-dimensional free vibrations of a circular arch using the theory of a Cosserat point. Journal of Sound and Vibration 286, 799–816 (2005).

[10] Mau, S. T. & Williams, A. N. Frequency response functions of circular arches for support motions. Engineering Structures 10, 265–271 (1988).

[11] Yau, J. D. & Yang, Y. B. Geometrically nonlinear analysis of planar circular arches based on rigid element concept — A structural approach. Engineering Structures 30, 955–964 (2008).

[12] Dimopoulos, C. A. & Gantes, C. J. Nonlinear in-plane behavior of circular steel arches with hollow circular cross-section. Journal of Constructional Steel Research 64, 1436–1445 (2008).

[13] Benamar, A. B., Rhali. Semi-Analytical Solution of In-Plane Vibrations of Circular Arches Carrying Added Point Masses

[14] El kadiri, M., Benamar, R. & White, R. G. IMPROVEMENT OF THE SEMI-ANALYTICAL METHOD, FOR DETERMINING THE GEOMETRICALLY NON-LINEAR RESPONSE OF THIN STRAIGHT STRUCTURES. PART I: APPLICATION TO CLAMPED–CLAMPED AND SIMPLY SUPPORTED–CLAMPED BEAMS. Journal of Sound and Vibration 249, 263–305 (2002).

[15] Fakhreddine, H., Adri, A., Rifai, S. et al. A Multimode Approach to Geometrically Non-linear Forced Vibrations of Euler–Bernoulli Multispan Beams. J. Vib. Eng. Technol. 8, 319–326 (2020).

[16] El kadiri, M., Benamar, R. & White, R. G. IMPROVEMENT OF THE SEMI-ANALYTICAL METHOD, FOR DETERMINING THE GEOMETRICALLY NON-LINEAR RESPONSE OF THIN STRAIGHT STRUCTURES. PART I: APPLICATION TO CLAMPED–CLAMPED AND SIMPLY SUPPORTED–CLAMPED BEAMS. Journal of Sound and Vibration 249, 263–305 (2002).

[17] Zhao, Y. & Kang, H. In-plane free vibration analysis of cable–arch structure. Journal of Sound and Vibration 312, 363–379 (2008).

[18] Benamar, A. B., Rhali. The Efficiency of the Rayleigh-Ritz Method Applied to In-Plane Vibrations of Circular Arches Elastically Restrained against Rotation at the Two Ends. International Journal of Engineering Trends and Technology - IJETT.

[19] Karami, G. & Malekzadeh, P. In-plane free vibration analysis of circular arches with varying cross-sections using differential quadrature method. Journal of Sound and Vibration 274, 777–799 (2004).

[20] Benedettini, F., Alaggio, R. & Zulli, D. Nonlinear coupling and instability in the forced dynamics of a non-shallow arch: theory and experiments. Nonlinear Dyn 68, 505–517 (2012).