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Some Aspects of the Shear Stress Distribution in Non-Uniform Gradually Varied Open Flows

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Abstract *- The effect of non-uniformity on the shear stress distribution in open channels was investigated using a polynomial approximation of the shear stress profile. The features of the shear stress distribution were revealed in non-uniform gradually varied turbulent and laminar flows in open channels. The parameter of non-uniformity determining the shape of the shear stress profile in accelerated and decelerated flows was obtained. It was revealed that in the accelerated flow, the shear stress reaches its maximum at the bottom, while in a decelerated flow, it can be maximal both at the bottom and at some distance from the bottom.*

Keywords - *hydraulics, shear stress, non-uniform flow, turbulent flow, laminar flow.*

I. INTRODUCTION

Non-uniform water movement is present in almost all areas of natural channels [1]. Therefore, the first engineering problems associated with this type of movement were solved in relation to natural watercourses. In hydraulics [2], it is customary to divide non-uniform flows into gradually and sharply varying ones. The boundary between these two flow types is very conditional and is usually determined by the specifics of the tasks being solved. According to the classical works of J.B. Belanger [3], gradually varying flows must satisfy the conditions of small curvature of current lines and angles between adjacent current lines. For flows satisfying these conditions, the dependence, commonly known as the basic differential equation of non-uniform motion, was obtained from the Bernoulli equation [2]:

$$J = \frac{d}{dx}\left(\frac{V^2}{2g}\right) + \frac{V^2}{C^2 R}, \tag{1}$$

Where J = i - dh/dx is the slope of the free surface of the water; x, y are the longitudinal and transverse coordinates; h is the flow depth; i is the slope of the bottom, g is the gravitational acceleration, V is the mean velocity, R is the hydraulic radius; C is the Chezy coefficient.

Using the traditional one-dimensional formulation of engineering problems of gradually varied flows has led to aberration from the real distributions of kinematic and dynamic quantities and the neglecting of some specific features of the flows under consideration. First of all, this refers to the last term in equation (1), which determines the energy loss. When determining the Chezy coefficient, the assumption is usually made about the equality of the work of the resistance forces of inhomogeneous and equivalent homogeneous flows having the same average cross-sectional velocities per unit of length, as well as hydraulic radii. This leads to the conclusion about the equality of the Chezy coefficients of non-uniform and equivalent uniform flows. This assumption has not been confirmed by theory or experiment. It was assumed that its validity, as a basis for design, was verified by many years of solving practical problems. Nevertheless, more recent studies of this issue have revealed a significant influence of the non-uniformity of the flow both on the waterway capacity [4], [5] and on the magnitude of shear stresses [6], [7].

II. SHEAR STRESS DISTRIBUTION IN A UNIFORM FLOW

Let first consider a uniform steady flow in open channels. Here the shear stresses are distributed according to the linear law [1]:

$$\frac{\tau}{\tau_{0p}} = 1 - \eta, \tag{2}$$

Where τ is the shear stress; τ0pis the shear stress at the bottom of a uniform flow; η=y/h *is the* dimensionless distance from the bottom.

Experiments show (Fig. 1) that in non-uniform gradually varied open flows, there is a noticeable deviation from the law (2) [8], [9].

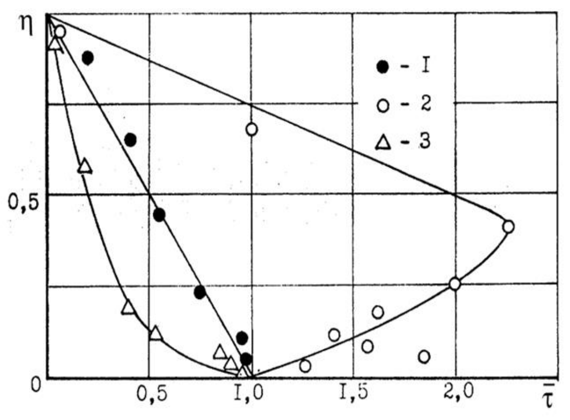

Fig. 1 Shear stress diagrams for open flows in a hydraulically smooth channel according to data of E.V. Zalutsky: 1 – uniform flow; 2 – decelerated flow; 3 – accelerated flow

Experiments on studying turbulent flows were carried out in a tray that had glass walls and a bottom made of painted cement coating. A uniform flow was reached at the initial section of the 9.5 m long tray. Flow acceleration or deceleration was achieved by giving positive (i = 0.02) or negative (i = -0.02) slopes to the bottom of the tray in a section of 1.5 m. The kinematic characteristics of the flow in the studied area were determined by photographing markers introduced into the flow.

III. SHEAR STRESS DISTRIBUTION IN A NON-UNIFORM GRADUALLY VARIED FLOW

Consider a non-uniform, gradually varied flat open flow. The flow projection along the x-axis directed along with the flow motion (while the y-axis is directed pointing upwards) is described by the differential equation [10]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=gJ+\frac{1}{\rho}\frac{\partial \tau}{\partial y}, \qquad (3)$$

Where u, v are projections of the velocity vectors on the x-axis and y-axis, respectively; ρ is the liquid density.

To approximate the shear stress profile along with the flow depth, following [11], we use the polynomial;

$$\overline{\tau}=\tau/\tau_0=A_0+A_1\eta+A_2\eta^2+\ldots+A_n\eta_n^n, \qquad (4)$$

Where τ_0 is the shear stress at the bottom of a non-uniform flow.

We restrict ourselves to three terms in the decomposition (2.22) [138]:

$$\overline{\tau}=A_0+A_1\eta+A_2\eta^2. \qquad (5)$$

Next, it is necessary to determine the coefficients A_0, A_1, and A_2 from the boundary conditions:

<u>At the bottom of the channel:</u>

a) y= 0;
b) $\tau=\tau_0$-by definition;
c) $\partial\tau/\partial y=-\rho gJ$ – according to equation (3).

<u>On the free surface of the flow:</u>

d) y=h; τ=0 (under the usual assumption of neglecting friction on a free surface).

After substituting (a) and (b) into equation (4), we get $A_0 = 1$. From condition (c), it follows that

$$A_1=-\frac{\rho ghJ}{\tau_0}=A\,. \qquad (6)$$

From the condition on the flow surface (d), we find

$$A_2=-(1+A). \qquad (7)$$

Substituting the found values of the coefficients into the polynomial (5), we obtain

$$\overline{\tau}=1+A\eta-(1+A)\eta^2. \qquad (8)$$

Thus, the shape of the shear stress profile is determined by the value of the form parameter A (the parameter of non-uniformity).

The polynomial (8) is defined at $-2 \le A<\infty$, which follows from the condition $\overline{\tau}\ge 0$. In the particular case of uniform motion, from (6), we have A = -1. After substituting value A in (8), we obtain the well-known linear law of the shear stress distribution:

$$\overline{\tau}=1-\eta \qquad (9)$$

Rewrite the formula (6) in the form

$$A=-\frac{\rho ghi-\rho gh(dh/dx)}{\tau_0}=\frac{(\rho ghdx)i-\rho gd(h^2/2)}{\tau_0 dx}=\frac{dQ_x+dP_x}{dF_{fx}}. \qquad (10)$$

In x-axis projections, expression (10) represents the ratio of the sum of gravity forces ($dQ_x=\rho ghidx$) and pressure forces ($dP_x=-\rho gd(h^2/2)$)to the friction forces ($dF_{\text{тр}\,x}=-\tau_0 dx$), acting on an elementary liquid volume of a non-uniform flow of length dx.

Expression (10) can also be obtained from the general equation of dynamic equilibrium of forces acting on the concerned liquid volume, projected on the x-axis [12]:

$$dQ_x+dP_x+dF_{fx}+dI_x=0, \qquad (11)$$

Where dI_x is the inertial force.

From (11) follows

$$\frac{dQ_x+dP_x}{dF_{fx}}=1-\frac{dI_x}{dF_{fx}}. \qquad (12)$$

From the comparison of (10) and (12), it can be seen that parameter A characterizes the correlation between inertia forces and friction forces:

In accelerated flows $dI_x< 0$; $dF_{fx}< 0$ and, consequently, A < -1.

In decelerated flows $dI_x>O$; $dF_{fx}< 0$ and A > -1.

IV. SHEAR STRESSES AT THE BOTTOM OF A NON-UNIFORM GRADUALLY VARIED FLOW

Let us find the shear stress value at the bottom of the flow. For this, we use a one-dimensional momentum equation [10] for stationary flow (the Boussinesq coefficient β assumed to be a constant in the considered section of the flow):

$$J=\frac{\tau_0}{\rho gh}+\frac{\beta}{2g}\frac{dV^2}{dx}=\frac{\tau_0}{\rho gh}+\frac{\beta V}{g}\frac{dV}{dx}, \qquad (13)$$

Where V is the average velocity in the cross-section.

Differentiating the continuity equation Vh = const, we get

$$\frac{dV}{dx}=-\frac{V}{h}\frac{dh}{dx}. \qquad (14)$$

Substituting (14) into (13), we find

$$J = \frac{\tau_0}{\rho g h} - \frac{\beta V^2}{g h}\frac{dh}{dx}. \qquad (15)$$

And after transformation (15)

$$\tau_0 = \rho g h J + \beta \rho V^2 \frac{dh}{dx}. \qquad (16)$$

A similar equation was obtained in [13] when considering the dynamic equilibrium of a fluid volume of a non-uniform flow. Equation (15) can be represented as

$$\tau_0 = \tau_{0p} - \rho g h \frac{dh}{dx}(1 - \beta Fr), \qquad (17)$$

Where $\tau_{0p} = \rho g h i$ is the shear stress at the bottom of an equivalent uniform flow; $Fr = V^2/gh$is the Froude number.

It follows from (17) that for $Fr < 1/\beta$the shear stress at the bottom of the decelerated flow ($dh/dx > 0$) is less, and for the accelerated flow ($dh/dx < 0$) is greater than in the equivalent uniform flow. The reverse pattern is true when $Fr > 1/\beta$: $\tau_0>\tau_{0p}$ for decelerated flows and $\tau_0<\tau_{0p}$ – for accelerated flows.

V. THE SHAPE OF SHEAR STRESS PROFILES

Consider the shear stress profiles. The second derivative (8) has the form

$$\frac{\partial^2 \bar{\tau}}{\partial \eta^2} = -2(1 + A). \qquad (18)$$

The sign of the derivative determines the concavity or convexity of the function (8). The analysis of the expression (18) shows that:

For decelerated flows $A > -1$, and according to the consequences of (12),$\partial^2\bar{\tau}/\partial\eta^2 < 0$ thus the function (8) is convex;

For accelerated flows, $A<-1$, and$\partial^2\bar{\tau}/\partial\eta^2 > 0$

Hence the function (2.26) is concave.

Consider the changes in the value of τ near the channel bottom. At the channel bottom the $u = v = 0$, and from equation (3) we get

$$\frac{\partial \tau}{\partial y} = -\rho g J = -\rho g\left(i - \frac{dh}{dx}\right). \qquad (19)$$

To determine dh/dx, we use, as a first approximation, the equation of non-uniform motion in a wide rectangular channel [1]

$$\frac{dh}{dx} = i\frac{h^3 - h_0^3}{h^3 - h_c^3}, \qquad (20)$$

Where $h_0$and $h_c$are the normal and critical depths of the non-uniform flow.

After substituting (20) into (19), we obtain

$$\frac{\partial \tau}{\partial y} = \rho g i \frac{h_c^3 - h_0^3}{h^3 - h_c^3}. \qquad (21)$$

Channels with different bottom slopes were considered:

I. **Channels with i> 0.** At that, three cases were considered:

1) $h_0 > h_c$, $i_0 < i_c$. In this case, the depth h can be located in two zones (relative to h_c):

a) $h>h_c$. This case is satisfied by the backwater curve a_1 and the depletion curve b_1. At that, the numerator in (21) is less than zero, and the denominator is greater than zero. Hence, $\partial\tau/\partial y< 0$;

b) $h<h_c$. In this case, we have a backwater curve c_1. From (21) we get $\partial\tau/\partial y>0$.

2) $h<h_c$ and $i_0 > i_c$. Depth h can be located in two zones:

a) $h>h_c$. A backwater curve a_2 is possible. At that $\partial\tau/\partial y>0$;

b) $h<h_c$. A depletion curve b_2 and a backwater curve c_2 are possible. At that, $\partial\tau/\partial y>0$;

3) $h_0 = h_c$ and $i_0 = i_c$. Backwater curves a_3 and c_3 are possible. At that, $\partial\tau/\partial y = 0$.

II. **Channels with i = 0.** For channels with a horizontal bottom, we find from (19)

$$\frac{\partial \tau}{\partial y} = \rho g \frac{dh}{dx}. \qquad (22)$$

Therefore, for decelerated flows ($dh/dx>0$) we have $\partial\tau/\partial y>0$, and for accelerated flows ($dh/dx <0$) $\partial\tau/\partial y<0$.

III. **Channels with i< 0.** For decelerated flows at a negative bottom slope, from equation (19), we obtain $\partial\tau/\partial y>0$. For accelerated flows, $J > 0$ and hence $\partial\tau/\partial y<0$.

Based on the conducted analysis, we conclude that while for accelerated flows $\partial\tau/\partial y<0$, i.e., the shear stress reaches its maximum value at the bottom, then for decelerated flows, two cases are possible $\partial\tau/\partial y<0$ and $\partial\tau/\partial y\geq 0$, and τ can be maximal both at the bottom, and some distance from the bottom.

Let's find the value η_m at which maximum shear stresses can occur in decelerated flows. To do this, we differentiate (8):

$$\frac{\partial \bar{\tau}}{\partial \eta} = A - 2(1 + A). \qquad (23)$$

At the maximum point $\partial\bar{\tau}/\partial\eta = 0$and, therefore,

$$\eta_m = \frac{A}{2(1 + A)}. \qquad (24)$$

Thus, a well-defined nature of the shear stress distribution over the depth of non-uniform flows is revealed. Since no special reservations were made above concerning the friction mechanism in non-uniform flows, the conclusions obtained about the shear stress distribution are equally valid for both laminar and turbulent flows.

The experimental shear stress profiles in non-uniform open flows, shown in Fig. 1, are qualitatively consistent with the curves corresponding to formula (8), as well as with the profiles obtained in [12].

VI. PRACTICAL APPLICATION

The obtained data can be used to calculate the distribution of local flow velocities.

Laminar flow. In the case of laminar flow, we use Newton's law of viscous friction

$$\tau = \rho \nu \left(\frac{\partial u}{\partial y}\right), \qquad (25)$$

Where ν is the kinematic viscosity.

Substituting (25) into (8), we find

$$\frac{\nu}{hU_*^2}\frac{\partial u}{\partial \eta} = 1 + A\eta - (1+A)\eta^2, \qquad (26)$$

or

$$\frac{\partial u}{\partial \eta} = \frac{U_*^2 h}{\nu}[1 + A\eta - (1+A)\eta^2], \qquad (27)$$

were $U_* = \sqrt{\tau_0/\rho}$ Is the dynamic velocity.

After integration, equation (27) takes the form

$$u = \frac{U_*^2 h}{\nu}\left(1 + \frac{A}{2}\eta - \frac{1+A}{3}\eta^2\right)\eta + C. \qquad (28)$$

From the conditions at the flow bottom η = 0 and u = 0, we can find C = 0. The final dependence for the velocity distribution has the form

$$\frac{u}{U_*} = \frac{U_* h}{\nu}\left(1 + \frac{A}{2}\eta - \frac{1+A}{3}\eta^2\right)\eta. \qquad (29)$$

After substituting the value A=-1 for a uniform flow in (29), we obtain a known parabolic velocity profile [2]:

$$\frac{u}{U_*} = \frac{U_* h}{\nu}\left(\eta - \frac{1}{2}\eta^2\right). \qquad (30)$$

Turbulent flow. In the case of turbulent flow, shear stresses are determined by the formula

$$\tau = \rho \nu_T \frac{\partial u}{\partial y}, \qquad (31)$$

were ν_T is the coefficient of turbulent kinematic viscosity.

The distribution of the turbulent viscosity coefficient over the depth of a non-uniform flow approximately corresponds to the parabolic law. Following the data [10],

$$\nu_T = \kappa U_* h\eta(1-\eta), \qquad (32)$$

Where k is the Karman constant, whose value depends on the degree of non-uniformity of the flow.

Experimental data of I. Nikuradze (processed by M. Hirano and A. Kaneko [14]), G.A. Gurjienko (processed by K.V. Grishanin [10]), and F. Donch [15] confirm this conclusion.

Substituting (31) and (32) into (7), we find

$$\frac{\rho\kappa U_*\eta(1-\eta)}{\tau_0}\frac{\partial u}{\partial \eta} = (1-\eta)(1+\eta+A\eta). \qquad (33)$$

Equation (33) can be transformed as follows

$$\frac{1}{U_*}\frac{\partial u}{\partial \eta} = \frac{1}{\kappa}\left[\frac{1}{\eta} + (A+1)\right]. \qquad (34)$$

After integrating (34) with the variable η, we find the distribution law of the averaged velocities:

$$\frac{u}{U_*} = \frac{1}{\kappa}[\ln \eta + (1+A)\eta] + C. \qquad (35)$$

The constant C is defined from the conditions on the free surface of the flow η =1 and u = $Umax$. In this case, equation (35) takes the form:

$$C = \frac{U_{max}}{U_*} - \frac{1}{\kappa}(1+A). \qquad (36)$$

Substituting (36) into (35), we find

$$\frac{U_{max} - u}{U_*} = -\frac{1}{\kappa}[\ln \eta - (1+A)(1-\eta)]. \qquad (37)$$

The resulting expression corresponds to the formula of J.C. Rotta [16] for flows with a moderate pressure gradient

$$\frac{U_{max} - u}{U_*} = -\frac{1}{\kappa}[\ln \eta - T(1-\eta)], \qquad (38)$$

where T is the correction parameter

VII. CONCLUSIONS

1. The features of the shear stress distribution in non-uniform gradually varied open flows under turbulent and laminar flow regimes are revealed.
2. The shear stress profile of a flat open flow is completely determined by the dh/dx parameter.
3. In length-accelerated flows, shear stresses always have a maximum at the bottom; in decelerated flows, shear stresses can reach a maximum both at the bottom and at some distance from it.

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