International Journal of Engineering Trends and Technology
ISSN: 2231 – 5381 /doi:10.14445/22315381/IJETT-V69I11P214

Volume 69 Issue 11, 113-121, November, 2021


# Cosparse Analysis Model-Based Compressive Sensing With Optimized Projection Matrix

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***Abstract*** — *The Compressive Sensing (CS) technique provides a signal acquisition dimensional reduction by multiplying a projection matrix with the signal. Until now, the projection matrix optimization is commonly performed using the Sparse Synthesis Model-Based (SSMB), where it takes a linear combination of a few atoms in a synthesis dictionary to form a signal. The Cosparse Analysis Model-Based (CAMB) provides an alternative model where the multiplication of the signal with an analysis dictionary (operator) produces a cosparse coefficient. The CAMB-CS method is proposed in this paper by taking into account the amplified Cosparse Representation Error (CSRE) parameter and the relative amplified CSRE optimize the projection matrix, in addition to the mutual coherence parameter. The optimized projection matrix in CAMB-CS is obtained using an alternating minimization algorithm and nonlinear conjugation gradient method. In the optimization algorithm, the Gaussian random matrix is used as the initial projection matrix. The simulation results showed an increase in the Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM) of the reconstructed image in the CAMB-CS system up to 10.23% and 8.46%, respectively, compared to the Gaussian random matrix. Compared to the SSMB-CS optimized projection matrix, the developed method increases the PSNR and SSIM of the recovered image up to 21.21% and 17.11%, respectively*.

**Keywords —** *Compressive Sensing, Cosparse Analysis Model, Cosparse Representation Error, Projection Matrix Optimization.*

## I. INTRODUCTION

The Shannon-Nyquist sampling theorem underlies the conventional way of sampling a signal in which the minimum sampling frequency is twice the maximum frequency of the signal [1]. The theorem underlies almost all signal acquisition protocols used in electronic audio-visual equipment, medical imaging equipment, radio receivers, and so on. Compressive Sensing (CS) or Compressed Sensing, a term introduced by Donoho, is a sampling technique in which the signal can be reconstructed in a much smaller number of samples than the Shannon-Nyquist method [2]. Since the important results published by Donoho [2] and Candès et al. [3]–[5], CS has attracted a lot of new attention in the last two decades and has been widely applied in various fields such as compressive imaging, biomedical applications, communication systems, pattern recognition, and so on [6]–[10].

A signal that can be generated from a few coefficients using an appropriate dictionary is called a sparse signal. The Sparse Synthesis Model-Based (SSMB) states that signals can be formed from a few linear combinations of signal atoms derived from synthesis-dictionary [8], [11]. The Cosparse Analysis Model-Based (CAMB) as an alternative to SSMB states that multiplication of the signal by an analysis-dictionary or operator yields a sparse analysis coefficient [12]–[15]. Currently, SSMB is the basis for CS research in general, but CAMB is starting to become an alternative for CS research because it provides better results than SSMB, as shown in [16]–[19].

The three main issues in CS research are: 1) how to establish an appropriate dictionary; 2) designing the optimal projection matrix; and 3) designing a CS signal reconstruction algorithm. In SSMB, algorithms such as maximum likelihood, method of optimal directions, K-SVD, and its development, are used to construct synthesis-dictionary [20]–[23]. Various algorithms such as analysis operator learning, K-SVD analysis, and sparsifying-transform learning have been proposed to construct an analysis dictionary for CAMB [24], [25], [34]–[37], [26]–[33]. The sparse recovery algorithms such as convex and relaxation, greedy, Bayesian, and their combination can be used to recover the signal in SSMB-CS [38].

The equivalent algorithm for CAMB-CS uses several methods such as convex and relaxation analysis, greedy, Bayesian analysis, and their combinations. can be seen in [13], [14], [39]–[41].

Compressive Sensing (CS), either on SSMB or CAMB, is done by multiplying the signal by a projection matrix, thereby reducing signal dimensions. In SSMB-CS, the projection matrix is designed to be incoherent with the synthesis dictionary so that the signal can be reconstructed accurately [42]. Random matrices were taken from independent and identically Gaussian or Bernoulli distributions, and partial Fourier matrices are often used as projection matrix in SSMB-CS because with a high probability, the matrix is incoherent with most synthesis-dictionaries [5], [42]. A deterministic and structured projection matrix was also developed because it has advantages for practical applications but requires a higher number of measurements than random matrices [6], [10], [51], [52], [43]–[50]. While the optimization of the projection matrix on SSMB-CS has been proposed as in [53], [54], [63]–[72], [55], [73]–[78], [56]–[62], but how to optimize the projection matrix on CAMB-CS, has not been widely studied. The projection matrix optimization method for CAMB-CS is proposed in this paper. The simulation results show that the proposed method produces better CS performance in terms of signal reconstruction quality compared to the previous methods.

The rest of the paper is arranged as follows: Section II explains the CS theory and projection matrix optimization in SSMB-CS. The proposed method is presented in section III. Section IV discusses the comparison results of the proposed method and the previous ones. Finally, some conclusions are given in section V.

## II. COMPRESSIVE SENSING THEORY

The compressive measurement of signal $\mathbf{x} \in \mathbb{R}^{N\times1}$ in SSMB-CS can be done by projecting it into a projection matrix $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ and producing a compressed signal $\mathbf{y} \in \mathbb{R}^{M\times1}$ $M < N$ as in Equation (1).

$$\mathbf{y} = \mathbf{\Phi x} \tag{1}$$

The synthesis dictionary $\mathbf{\Psi} \in \mathbb{R}^{N\times K}$ and the sparse coefficients $\boldsymbol{\theta} \in \mathbb{R}^{K\times1}$ $K \geq N$ are used in the SSMB to model the signal $\mathbf{x}$ as in Equation (2) with $\mathbf{e_s} \in \mathbb{R}^{N\times1}$ is the sparse representation error (SRE) of the signal $\mathbf{x}$.

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\theta} + \mathbf{e_s} \tag{2}$$

The signal $\mathbf{x}$ in Equation (1) can be substituted by using Equation (2) so as to produce Equation (3) with $\mathbf{D} = \mathbf{\Phi\Psi} \in \mathbb{R}^{M\times L}$ is an equivalent dictionary and $\mathbf{\Phi e_s} = \boldsymbol{\sigma}_\mathbf{s} \in \mathbb{R}^{M\times1}$ is amplified SRE.

$$\mathbf{y} = \mathbf{\Phi\Psi}\boldsymbol{\theta} + \mathbf{\Phi e_s} = \mathbf{D}\boldsymbol{\theta} + \mathbf{\Phi e_s} \tag{3}$$

The $\ell_0-$minimization problem in Equation (4) needs to be solved to get the sparse coefficients $\hat{\boldsymbol{\theta}}$ from the compressed signal $\mathbf{y}$ in Equation (3) so that the reconstructed signal is $\hat{\mathbf{x}}$ obtained as in Equation (5).

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_0 \ s.t. \ \|\mathbf{y} - \mathbf{D}\boldsymbol{\theta}\|_2 \leq \|\boldsymbol{\sigma}_\mathbf{s}\|_2 \tag{4}$$

$$\hat{\mathbf{x}} = \mathbf{\Psi}\hat{\boldsymbol{\theta}} \tag{5}$$

However, the solution of Equation (4) is NP-hard [79], so an approximate solution of Equation (4) is needed. Orthogonal matching pursuit (OMP) as one of the greedy pursuit algorithms is often used as an approach method to solve Equation (4) [80].

Equation (6) models the signal $\mathbf{x} \in \mathbb{R}^{N\times1}$ in CAMB with $\mathbf{z} \in \Re^{N\times1}$ is the cosparse signal and $\mathbf{e_{cs}} \in \mathbb{R}^{N\times1}$ is the cosparse representation error (CSRE) of the signal $\mathbf{x}$.

$$\mathbf{x} = \mathbf{z} + \mathbf{e_{cs}} \tag{6}$$

The signal $\mathbf{x}$ in Equation (1) can be substituted by using Equation (6) so as to produce Equation (7) with $\mathbf{\Phi e_{cs}} = \boldsymbol{\sigma}_\mathbf{cs} \in \mathbb{R}^{M\times1}$ is amplified CSRE.

$$\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi z} + \mathbf{\Phi e_{cs}} \tag{7}$$

Multiplication between the analysis dictionary or operator $\mathbf{\Omega} \in \mathbb{R}^{K\times N}$ with $K \geq N$ and the cosparse signal $\mathbf{z} \in \mathbb{R}^{N\times1}$ produces the cosparse coefficients $\boldsymbol{\alpha} \in \mathbb{R}^{K\times1}$. The $\ell_0-$minimization problem in Equation (8) needs to be solved to get the reconstructed signal $\hat{\mathbf{x}}$.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{\Omega x}\|_0 \ s.t. \ \|\mathbf{y} - \mathbf{\Phi x}\|_2 \leq \|\boldsymbol{\sigma}_\mathbf{cs}\|_2 \tag{8}$$

However, the same as in Equation (4), the solution of Equation (8) is NP-hard [14], so an approximate solution of Equation (8) is needed. Greedy analysis pursuit (GAP) is often used as an approach method to solve Equation (8) [13], [14].

The extent to which the projection matrix used in either the SSMB-CS or CAMB-CS satisfies several properties such as *null space property* (NSP), *restricted isometry property* (RIP) [2], [3], [81] and the *spark of a matrix* [82], [83] determines the quality of the reconstructed signal $\hat{\mathbf{x}}$. However, it is not easy to determine whether a projection matrix satisfies those properties. Another property that is widely used in projection matrix optimization is mutual coherence because it can be calculated and fulfilled more easily [53], [82], [84], [85]. In the SSMB-CS, the equivalent dictionary can be written as $\mathbf{D} = [\mathbf{d_1} \ \mathbf{d_2} \ \ldots \ \mathbf{d_K}] \in \mathbb{R}^{M\times K}$ the $(i, j)$ element of the Gram matrix of $\mathbf{D}$ is $g_{s-ij} \triangleq \mathbf{d_i}^T\mathbf{d_j}$ and $\mathbf{D_s} \triangleq diag\left(g_{s-11}^{-1/2} \ \ldots \ g_{s-kk}^{-1/2} \ldots \ g_{s-KK}^{-1/2}\right)$ is a diagonal matrix.

The normalized equivalent dictionary is $\bar{\mathbf{D}} = \mathbf{DD_s}$ , and the normalized Gram matrix of $\mathbf{D}$ is $\bar{\mathbf{G}}_\mathbf{s} = \bar{\mathbf{D}}^T\bar{\mathbf{D}}$ with $\bar{g}_{s-kk} = 1, \ \forall k$ The Equation (9) defines the mutual coherence of $\mathbf{D}$.

$$\mu(\mathbf{D}) = \max_{i\neq j} |\bar{g}_{s-ij}| \tag{9}$$

If the signal $\mathbf{x}$ is exactly $S-sparse$ that is $\mathbf{e_s} = 0$ , then the reconstructed signal $\hat{\mathbf{x}}$ can be exactly recovered if Equation (10) is fulfilled [82].

$$S < \frac{1}{2}\left(1 + \frac{1}{(\mu(\mathbf{D}))}\right) \tag{10}$$

The value range of the mutual coherence $\mu(\mathbf{D})$ is $\mu_B \leq \mu(\mathbf{D}) \leq 1$ where $\mu_B$ defined in equation (11) is the Welch bound.

$$\mu_B \triangleq \sqrt{\frac{K - M}{M\left(K-1\right)}} \qquad (11)$$

The Welch bound can be achieved by the optimal *Grassmannian* frame, which is an equiangular tight frame (ETF) [58].

Equation (12) defines $t-$ *averaged mutual coherence* $\mu_t\left(\mathbf{D}\right)$, which is used as a parameter in projection matrix optimization [53].

$$\mu_t\left(\mathbf{D}\right) = \frac{\sum_{i\neq j,1\leq i,j\leq K}\left(\left|\bar{g}_{s-ij}\right| \geq t\right).\left|\bar{g}_{s-ij}\right|}{\sum_{i\neq j,1\leq i,j\leq K}\left(\left|\bar{g}_{s-ij}\right| \geq t\right)} \qquad (12)$$

The threshold value $t$ is usually chosen equal to $\mu_B$ where the indicator function $\left(\left|\bar{g}_{s-ij}\right| \geq t\right) = 1$ if the condition is true and otherwise is zero.

Equation (13) is an optimization problem with an objective function $\Im\left(\mathbf{\Phi},\mathbf{G_t}\right)$ that is required to be solved to get the optimized projection matrix in SSMB-CS.

$$\min_{\mathbf{G_t}\in\mathbf{S_t},\mathbf{\Phi}}\left[\Im\left(\mathbf{\Phi},\mathbf{G_t}\right)\right]$$

$$\Im\left(\mathbf{\Phi},\mathbf{G_t}\right) = \zeta_1\left\|\mathbf{G_t} - \mathbf{\Psi^T\Phi^T\Phi\Psi}\right\|_F^2 + \zeta_2\left\|\mathbf{\Phi}\left(\mathbf{X} - \mathbf{\Psi\Theta}\right)\right\|_F^2 \qquad (13)$$

$\mathbf{S_t}$ is a set of Gram matrix ETF, $\zeta_1,\zeta_2$ are weighting factors, training signals $\mathbf{X}\in\mathbb{R}^{N\times P}$ are used to build the synthesis dictionary $\mathbf{\Psi}\in\mathbb{R}^{N\times K}$, $\mathbf{\Theta}\in\mathbb{R}^{K\times P}$ is sparse coefficients, and $\left\|\mathbf{M}\right\|_F$ denotes the *Frobenius* norm of matrix $\mathbf{M}$.

The iterative shrinkage algorithm as in Equation (14) where $0<\gamma<1$ was used in [53] to solve Equation (13) with $\zeta_1 = 1$ and $\zeta_2 = 0$ by minimizing $t-$ *averaged mutual coherence*.

$$\bar{g}_{s-ij} = \begin{cases} \gamma\bar{g}_{s-ij}, & \left|\bar{g}_{s-ij}\right| \geq t \\ \gamma t\ sign\left(\bar{g}_{s-ij}\right), & \gamma t \leq \left|\bar{g}_{s-ij}\right| \leq t \\ \bar{g}_{s-ij}, & \left|\bar{g}_{s-ij}\right| \leq \gamma t \end{cases} \qquad (14)$$

The alternating minimization method was used in [63] to solve Equation (13) with $\zeta_1 = 1$ and $\zeta_2 = 0$ by using $\mathbf{G_t}$ its identity matrix $\mathbf{I_K}$. The alternating minimization method was also used in [74] by rewriting Equation (13) into Equation (15).

$$\min_{\mathbf{\Phi},\mathbf{D_t}}\left[\Im\left(\mathbf{\Phi},\mathbf{D_t}\right) = \left\|\mathbf{\Phi A} - \mathbf{B}\right\|_F^2\right] \qquad (15)$$

Where $\mathbf{A}\triangleq\left[\sqrt{\zeta_1}\mathbf{\Psi} \quad \sqrt{\zeta_2}\mathbf{E_s}\right]$, $\mathbf{B}\triangleq\left[\sqrt{\zeta_1}\mathbf{D_t} \quad \mathbf{0}\right]$, the SRE matrix $\mathbf{E_s}\triangleq\mathbf{X}-\mathbf{\Psi\Theta}$, and $\mathbf{D_t}$ is an ETF. Equation (15) can be solved efficiently for the large set of training signals by using an identity matrix $\mathbf{I_N}$ to replace $\mathbf{E_s}$ [78].

## III. PROPOSED METHOD

As far as is known from the literature, the projection matrix optimization method in CAMB-CS has only been proposed in [86], where the equivalent operator $\mathbf{O}\in\mathbb{R}^{M\times K}$ which is defined as $\mathbf{O} = \mathbf{\Phi\Omega}^T$ was introduced. The novel method of projection matrix optimization for CAMB-CS is proposed by taking into account the relative amplified CSRE and the amplified CSRE energy as in Equation (16).

$$\min_{\mathbf{G_t}\in\mathbf{S_t},\mathbf{\Phi}}\left[\Im\left(\mathbf{\Phi},\mathbf{G_t}\right)\right]$$

$$\Im\left(\mathbf{\Phi},\mathbf{G_t}\right) = \zeta_1\left\|\mathbf{G_t} - \mathbf{\Omega\Phi^T\Phi\Omega^T}\right\|_F^2 + \zeta_2\left\|\mathbf{\Phi E_{cs}}\right\|_F^2 + \zeta_3\frac{\left\|\mathbf{\Phi E_{cs}}\right\|_F^2}{\left\|\mathbf{\Phi Z}\right\|_F^2} \qquad (16)$$

$\mathbf{S_t}$ It is a set of Gram matrix ETF, $\zeta_1,\zeta_2,\zeta_3$ are weighting factors, training signals $\mathbf{X}\in\mathbb{R}^{N\times P}$ are used to build operator $\mathbf{\Omega}\in\mathbb{R}^{K\times N}$, cosparse signals $\mathbf{Z}\in\mathbb{R}^{N\times P}$ can be obtained from $\mathbf{X}$ by using a cosparse coding [87], and $\mathbf{E_{cs}} = \mathbf{X}-\mathbf{Z}$ is CSRE matrix.

The projection matrix optimization for CAMB-CS in [86] adopted method in [74] to solve Equation (16) with $\zeta_1 = 1$, $\zeta_2 = \zeta_3 = 0$. The proposed method extends the work in [86] by using the alternating minimization algorithm to solve Equation (16) with $\zeta_1,\ \zeta_2,\ \zeta_3 \neq 0$ where $\zeta_1+\zeta_2+\zeta_3 = 1$. The method in [74] was adopted by the proposed method to update the target Gram matrix $\mathbf{G_t}$ while optimized projection matrix $\mathbf{\Phi}$ was obtained by using the nonlinear conjugate gradient (NCG) method with extension to two dimensional (matrix) case to solve Equation (16) [88]. Fig. 1 shows the flowchart of the alternating minimization algorithm that is used in the proposed method.

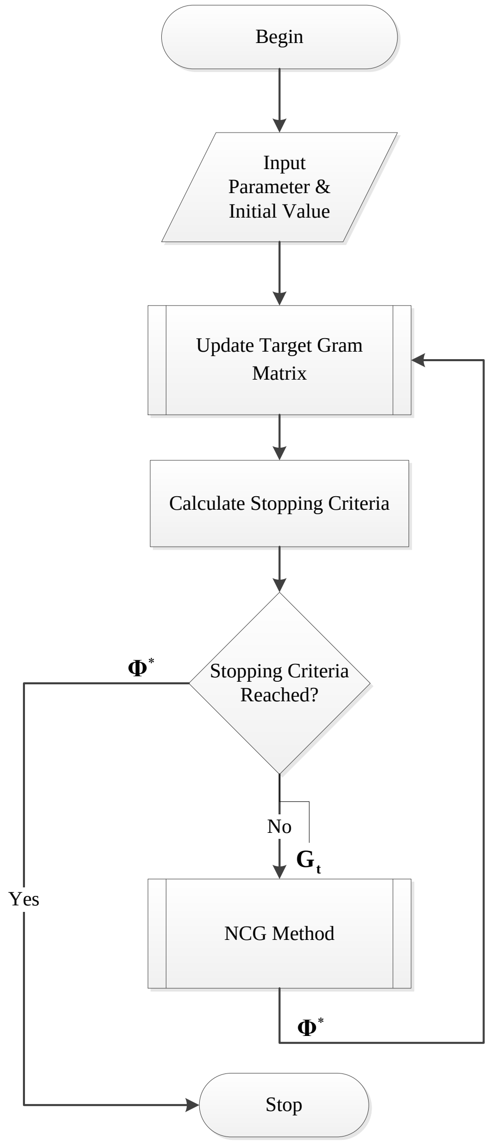


**Fig 1: Flowchart of the alternating minimization algorithm**

At first, input parameters are needed, which include analysis-dictionary $\mathbf{\Omega}$, cosparse signals $\mathbf{Z}_{tr}$ and CSRE signals $\mathbf{E}_{\mathbf{cs}-tr}$ from training signals, weight factors $\zeta_1, \zeta_2, \zeta_3$, positive constants $\varepsilon_G$, and $\varepsilon_{NCG}$ to determine whether the alternating minimization algorithm stopping criteria and the NCG method stopping criteria are reached, and the parameters needed for NCG. While the initial values of the variables needed are the initial projection matrix $\mathbf{\Phi}_{(0)}$ and the initial target Gram matrix $\mathbf{G}_{\mathbf{t}(0)}$. The next stage is updating the target Gram matrix $\mathbf{G}_{\mathbf{t}(r)}$ so that it is obtained at the $r$-th iteration. Next is to calculate the stopping criteria $E_{\mathbf{G}_{\mathbf{t}(r)}}$ in Equation (17) at the $r$-th iteration.

$$E_{\mathbf{G}_{\mathbf{t}(r)}} = \left\| \mathbf{G}_{\mathbf{t}(r)} - \mathbf{G}_{\mathbf{t}(r-1)} \right\|_F^2 \quad (17)$$

If $E_{\mathbf{G}_{\mathbf{t}(r)}} \le \varepsilon_G$ then the algorithm will stop, which means the target Gram matrix $\mathbf{G}_{\mathbf{t}(r)}$ has met the set convergence limit. Meanwhile, if $E_{\mathbf{G}_{\mathbf{t}(r)}} > \varepsilon_G$ then the stopping criteria have not been met so that the target Gram matrix $\mathbf{G}_{\mathbf{t}(r)}$ value obtained will be used by the NCG method to solve Equation (16). In the $k$-th iteration, the optimum projection matrix $\mathbf{\Phi}^*$ is obtained. In the next iteration $r = r+1$, the optimum projection matrix obtained $\mathbf{\Phi}^*$ will be used to update the target Gram matrix $\mathbf{G}_{\mathbf{t}(r)}$ again. The same process is carried out again, calculating the stopping criteria $E_{\mathbf{G}_{\mathbf{t}(r)}}$. If the stopping criteria are not met, then the NCG method will be carried out again using the new target Gram matrix $\mathbf{G}_{\mathbf{t}(r)}$ to get the optimum projection matrix $\mathbf{\Phi}^*$ which will be used to update the target Gram matrix again in the next iteration.

## IV. RESULTS AND DISCUSSION

### A. *Experimental Setup*

The 40000 training images in LabelMe training data set [89], [90] were used to obtain the set of non-overlapping $8 \times 8$ patches by extracting the patches randomly from each training image. The training signals $\mathbf{X} \in \mathbb{R}^{64\times320000}$ are formed by reshaping each patch $8 \times 8$ as a vector of $64 \times 1$. KSVD algorithm [22] and the algorithm in [36] were used to build synthesis dictionary $\mathbf{\Psi}$ and operator $\mathbf{\Omega}$, respectively with $P = 320000$, $N = 64$, $K = 96$, $S = 4$, and $C = 64 - 4 = 60$ by using the training signals $\mathbf{X}$. The backward greedy algorithm [87] was performed to obtain the cosparse signals $\mathbf{Z}$ from $\mathbf{X}$ by using the built operator $\mathbf{\Omega}$. The CSRE matrix $\mathbf{E_{cs}}$ can be calculated by using $\mathbf{E_{cs}} = \mathbf{X} - \mathbf{Z}$. The computational complexity of the proposed algorithm can be reduced by replacing $\mathbf{Z}$ and $\mathbf{E_{cs}}$ with $\mathbf{Z_r}$ and $\mathbf{E_{cs-r}}$, respectively. The mean value of $n-$th row of $\mathbf{Z}$ and $\mathbf{E_{cs}}$ are $z_n$ and $e_{cs-n}$, respectively. The $z_n$ and $e_{cs-n}$ are used as diagonal entries of $\mathbf{Z_r}$ and $\mathbf{E_{cs-r}}$, respectively.

The SSMB-CS uses random Gaussian optimization algorithms in [53], [63], and [74], [78] as projection matrix denoted by SSMB-CS-RG, SSMB-CS-Elad, SSMB-CS-LZYB, and SSMB-CS-BLH-HZ, respectively. The CAMB-CS uses random Gaussian and proposed algorithm as projection matrix denoted by CAMB-CS-RG and CAMB-CS-Proposed, respectively. The stopping criteria $\varepsilon_G = 10^{-3}$ and $\varepsilon_{NCG} = 10^{-3}$ were used in the CAMB-CS-Proposed.

### B. Test Images and Evaluation Parameters

In the CAMB-CS and SSMB-CS systems, the test $\mathbf{I}_{test}$ images used came from two groups of images. Group 1 test images consisted of 10 thousand test images taken from the LabelMe test data set [89], [90]. Blocking is performed on each test image to produce non-overlapping $8 \times 8$ patches. For each test image that has been blocked, $p$ patches are taken randomly, and then each patch is formed into a vector $64 \times 1$ that produces a test signal matrix $\mathbf{X}_{1-test} \in \mathbb{R}^{64 \times 10000p}$. Group 2 test images come from standard test images such as *lena, peppers, barbara, cameraman*, and so on. For each test image used in group 2, the same process is carried out as in the test image for group 1. The difference is that all patches are taken from each test image in group 2, resulting from a test signal matrix $\mathbf{X}_{2-test} \in \mathbb{R}^{64 \times q}$ where $q$ is the number of patches in each test image group 2. The test signal matrices from the two groups are projected into the projection matrix of each method on CAMB-CS and SSMB-CS to generate a compressed signal matrix $\mathbf{Y}_{tes}$. The reconstructed signal $\tilde{\mathbf{X}}_{test}$ is recovered from the compressed signal matrix $\mathbf{Y}_{tes}$ by using OMP and GAP for SSMB-CS and CAMB-CS, *respectively*. The deblocking process is carried out on the reconstructed signal $\tilde{\mathbf{X}}_{test}$ to obtain a reconstructed image $\tilde{\mathbf{I}}_{test}$. Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM) is used to measure the quality of the reconstructed image.

SSIM was defined in [91] while PSNR in decibels (dB) is defined as in Equation (18), where $W$ and $H$ is the number of pixels in the row and column, so that $H \times W$ is the total pixels in the image. The higher value of PSNR and SSIM means the quality of the reconstructed image is getting better and closer to the test image. However, SSIM provides a better measurement in assessing the similarity of two images because it also takes into account the quality of visual perception.

$$\mathrm{PSNR} = 10\log\left(\frac{\max\left(\mathbf{I}_{test}\right)^2}{\frac{\sum_{y=1}^{H}\sum_{x=1}^{W}\left(\mathbf{I}_{test}(x,y)-\tilde{\mathbf{I}}_{test}(x,y)\right)^2}{(H \times W)}}\right) \quad (18)$$

### C. Weighting Factors of the Proposed Algorithm

The combinations of weighting factors $\zeta_1$ $\zeta_2$ and $\zeta_3$ are shown in Table 1, where a step change of 0.50 is assigned empirically to each combination.

**TABLE 1**
**THE COMBINATIONS OF WEIGHTING FACTORS**

| Weighting Factors Combinations | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ |
|---|---|---|---|
| WF-1 | 1 | 0 | 0 |
| WF-2 | 0.50 | 0.50 | 0 |
| WF-3 | 0.50 | 0 | 0.50 |
| WF-4 | 0.50 | 0.05 | 0.45 |
| WF-5 | 0.50 | 0.10 | 0.40 |
| WF-6 | 0.50 | 0.15 | 0.35 |
| WF-7 | 0.50 | 0.20 | 0.30 |
| **WF-8** | **0.50** | **0.25** | **0.25** |
| WF-9 | 0.50 | 0.30 | 0.20 |
| WF-10 | 0.50 | 0.35 | 0.15 |
| WF-11 | 0.50 | 0.40 | 0.10 |
| WF-12 | 0.50 | 0.45 | 0.05 |

The random Gaussian matrix is used as the initial projection matrix $\mathbf{\Phi}_{(0)}$, and the optimized projection matrix $\mathbf{\Phi}^*$ is obtained by using CAMB-CS-Proposed with the number of measurements $M = 20$. Using $\mathbf{\Phi}_{(0)}$ as the projection matrix on the CAMB-CS system provide PSNR = 28.41 dB and SSIM = 0.9553 for Group 1 the test images with $p = 8$ while PSNR = 32.54 dB and SSIM = 0.9351 for *cameraman* image that is used as Group 2 test images. The PSNR and SSIM values of the optimized projection matrix $\mathbf{\Phi}^*$ on the CAMB-CS system for the test images of group 1 and group 2 are provided in Table 2 and Table 3. Both tables show that for all combinations of weight factors, the PSNR value is > 28.41 dB, and the SSIM is > 0.9553. This shows that the optimized projection matrix provides a better reconstruction signal quality than the random Gaussian matrix that is used as the initial projection matrix.

WF-8 with weighting factors $\zeta_1 = 0.5$, $\zeta_2 = 0.25$, and $\zeta_3 = 0.25$ as shown in Table 2 and Table 3, provide the largest PSNR and SSIM values, meaning that they provide the best reconstructed signal quality.

For the next simulation, the CAMB-CS projection matrix optimization method uses WF-8 as a weighting factors combination.

**TABLE 2**
**RECONSTRUCTION IMAGES FOR GROUP 1 TEST IMAGES**

| Weighting Factors Combinations | PSNR (dB) | SSIM |
|---|---|---|
| FB-1 | 31.00 | 0.9727 |
| FB-2 | 31.15 | 0.9737 |
| FB-3 | 31.04 | 0.9731 |
| FB-4 | 30.95 | 0.9725 |
| FB-5 | 31.18 | 0.9738 |

| | | |
|---|---|---|
| FB-6 | 31.20 | 0.9739 |
| FB-7 | 31.07 | 0.9734 |
| **FB-8** | **31.21** | **0.9740** |
| FB-9 | 31.17 | 0.9738 |
| FB-10 | 31.15 | 0.9737 |
| FB-11 | 31.19 | 0.9727 |
| FB-12 | 31.19 | 0.9737 |

**TABLE 3**
**RECONSTRUCTION IMAGES FOR GROUP 2 TEST IMAGES**

| Weighting Factors Combinations | PSNR (dB) | SSIM |
|---|---|---|
| WF-1 | 35.30 | 0.9598 |
| WF-2 | 35.62 | 0.9617 |
| WF-3 | 35.47 | 0.9609 |
| WF-4 | 35.29 | 0.9598 |
| WF-5 | 35.69 | 0.9619 |
| WF-6 | 35.77 | 0.9639 |
| WF-7 | 35.75 | 0.9635 |
| **WF-8** | **36.02** | **0.9648** |
| WF-9 | 35.81 | 0.9629 |
| WF-10 | 35.75 | 0.9628 |
| WF-11 | 35.71 | 0.9627 |
| WF-12 | 35.82 | 0.9633 |

### *D. Performance Comparison of SSMB-CS and CAMB-CS Systems*

CAMB-CS and SSMB-CS use the same random Gaussian matrix as the initial projection matrix $\Phi_{(0)}$ in the projection matrix optimization.

The PSNR and SSIM comparison of the SSMB-CS and CAMB-CS systems for the test images of group 1 $p = 8$ for ten trials provide in Table 4 and Table 5. CAMB-CS-Proposed increased average PSNR by 2.79 dB (9.83%) and increased average SSIM by 0.0187 (1.96%) from the CAMB-CS-RG. Meanwhile, the increase from SSMB-CS-BLH-HZ, which is the best projection matrix optimization method of SSMB-CS, is 5.45 dB (21.21%) for PSNR and 0.0542 (5.89%) for SSIM.

Table 6 shows the 53 test images of group 2, while the average PSNR and SSIM comparison of the SSMB-CS and CAMB-CS systems for those test images is provided by Table 7 and Table 8.

**TABLE 4**
**PSNR COMPARISON OF SSMB-CS AND CAMB-CS FOR GROUP 1 TEST IMAGES**

| Trial | PSNR (dB) | | | | | |
|---|---|---|---|---|---|---|
| | SSMB-CS- | | | | CAMB-CS- | |
| | RG | Elad | LZYB | BLH-HZ | RG | Proposed |
| 1 | 23.92 | 24.31 | 24.57 | 25.73 | 28.41 | **31.21** |
| 2 | 23.88 | 24.25 | 24.56 | 25.72 | 28.34 | **31.12** |
| 3 | 23.89 | 24.27 | 24.54 | 25.73 | 28.36 | **31.18** |
| 4 | 23.87 | 24.30 | 24.52 | 25.68 | 28.33 | **31.12** |
| 5 | 23.93 | 24.36 | 24.59 | 25.79 | 28.42 | **31.21** |
| 6 | 23.83 | 24.24 | 24.49 | 25.65 | 28.33 | **31.12** |
| 7 | 23.83 | 24.25 | 24,49 | 25.64 | 28.34 | **31.11** |
| 8 | 23,93 | 24.27 | 24.56 | 25.68 | 28.36 | **31.15** |
| 9 | 23.88 | 24.28 | 24.57 | 25.74 | 28.39 | **31.19** |
| 10 | 23.86 | 24.30 | 24.50 | 25.70 | 28.41 | **31.19** |
| **Mean** | 23.88 | 24.28 | 24.54 | 25.71 | 28.37 | **31.16** |

**TABLE 5**
**SSIM COMPARISON OF SSMB-CS AND CAMB-CS FOR GROUP 1 TEST IMAGES**

| Trial | PSNR (dB) | | | | | |
|---|---|---|---|---|---|---|
| | SSMB-CS- | | | | CAMB-CS- | |
| | RG | Elad | LZYB | BLH-HZ | RG | Proposed |
| 1 | 0.8781 | 0.8949 | 0.8990 | 0.9196 | 0.9553 | **0.9740** |
| 2 | 0.8783 | 0.8947 | 0.8995 | 0.9201 | 0.9552 | **0.9740** |
| 3 | 0.8788 | 0.8951 | 0.8993 | 0.9205 | 0.9555 | **0.9744** |
| 4 | 0.8775 | 0.8947 | 0.8984 | 0.9194 | 0.9550 | **0.9738** |
| 5 | 0.8796 | 0.8966 | 0.9003 | 0.9209 | 0.9558 | **0.9744** |
| 6 | 0.8771 | 0.8943 | 0.8985 | 0.9191 | 0.9552 | **0.9740** |
| 7 | 0.8776 | 0.8949 | 0.8986 | 0.9192 | 0.9553 | **0.9740** |
| 8 | 0.8789 | 0.8954 | 0.8995 | 0.9200 | 0.9553 | **0.9740** |
| 9 | 0.8782 | 0.8951 | 0.8997 | 0.9203 | 0.9555 | **0.9743** |
| 10 | 0.8780 | 0.8954 | 0.8989 | 0.9200 | 0.9557 | **0.9744** |
| **Mean** | **0.8782** | **0.8951** | **0.8992** | **0.9199** | **0.9554** | **0.9741** |

**TABLE 6**
**TEST IMAGES OF GROUP 2**

| Test Image | | No | Test Image |
|---|---|---|---|
| 1 | *Cable car* | 28 | *Brick* |
| 2 | *Cornfield* | 29 | *Knob* |
| 3 | *Flower* | 30 | *Hats* |
| 4 | *Fruits* | 31 | *Red riding hood* |
| 5 | *Pens* | 32 | *Motor cross* |
| 6 | *Boat* | 33 | *Boat zentime* |
| 7 | *Cameraman* | 34 | *Flower & sill* |
| 8 | *Crowd* | 35 | *Seifenfabrikation alfred* |
| 9 | *Elaine* | 36 | *Sailboats* |
| 10 | *House* | 37 | *Sailboat* |
| 11 | *Jet plane* | 38 | *Zentime at the peer* |
| 12 | *Lake* | 39 | *Beach bums* |
| 13 | *Lena* | 40 | *Mountain stream* |
| 14 | *Lighthouse* | 41 | *White water rafting* |
| 15 | *Livingroom* | 42 | *Covello girl* |
| 16 | *Mandrill* | 43 | *Land ahoy* |
| 17 | *Peppers* | 44 | *Roman statue* |
| 18 | *Pirate* | 45 | *Country style* |
| 19 | *Splash* | 46 | *Light home* |
| 20 | *Tank* | 47 | *Six more shot* |
| 21 | *Tiffany* | 48 | *Lighthouse* |
| 22 | *Truck* | 49 | *Rustic dream* |
| 23 | *Walk bridge* | 50 | *Cockatoo* |
| 24 | *Woman dark hair* | 51 | *Little red riding home* |
| 25 | *Zelda* | 52 | *Monarch* |
| 26 | *Barbara* | 53 | *Moon* |
| 27 | *Gold hill* | | |

**TABLE 7**
**AVERAGE PSNR COMPARISON OF SSMB-CS AND CAMB-CS FOR GROUP 2 TEST IMAGES**

| Average PSNR (dB) | | | | | |
|---|---|---|---|---|---|
| SSMB-CS- | | | | CAMB-CS- | |
| RG | Elad | LZYB | BLH-HZ | RG | Proposed |
| 24.72 | 24.88 | 25.20 | 26.41 | 28.68 | **31.61** |

**TABLE 8**
**AVERAGE SSIM COMPARISON OF SSMB-CS AND CAMB-CS FOR GROUP 2 TEST IMAGES**

| Average PSNR (dB) | | | | | |
|---|---|---|---|---|---|
| SSMB-CS- | | | | CAMB-CS- | |
| RG | Elad | LZYB | BLH-HZ | RG | Proposed |
| 0.6819 | 0.6922 | 0.7070 | 0.7622 | 0.8231 | **0.8927** |

CAMB-CS-Proposed increased average PSNR by 2.93 dB (increase 10.23%) and increased average SSIM by 0.0696 (increase 8.46%) from the CAMB-CS-RG. Meanwhile, the increase from SSMB-CS-BLH-HZ, which is the best projection matrix optimization method of SSMB-CS, is 5.20 dB (19.69%) for PSNR and 0.1305 (17.11%) for SSIM. Fig. 2 shows the reconstructed image comparison of the SSMB-CS and CAMB-CS systems for *cameraman* image.


**a. PSNR = 25.25 dB SSIM = 0.8039** **b. PSNR = 26.44 dB SSIM = 0.8351**


**c. PSNR = 26.48 dB SSIM = 0.8394** **d. PSNR = 27.28 dB SSIM = 0.8597**


**e. PSNR = 32.54 dB SSIM = 0.9351** **f. PSNR = 36.02 dB SSIM = 0.9648**

**Fig 2: Reconstructed *cameraman* image comparison SSMB-CS- a. RG b. Elad c. LZYB d. BLH-HZ CAMB-CS- e. RG f. Proposed**

## V. CONCLUSIONS

A new method of projection matrix optimization for CAMB-CS based on alternating minimization algorithm and nonlinear conjugate gradient method has been explained in this paper. The simulation results show that the proposed method can improve the quality of the reconstructed image of the CAMB-CS system compared to the non-optimized one of up to 10.23% and 8.46% in terms of PSNR and SSIM, respectively. There was an increase of up to 21.21% and 17.11% compared to the best projection matrix optimization of SSMB-CS in terms of PSNR and SSIM, respectively. The future work on the projection matrix optimization method of CAMB-CS can be further developed by combining analysis-dictionary learning problems into the objective function of the CAMB-CS projection matrix optimization so that it becomes a joint optimization of analysis-dictionary and projection matrix and will improve the quality of the reconstructed image.

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