Inflation Based replacement model for cutting tools using Markov Stochastic process

Dr S Gajanana[#1],Yayavaram Revanth Sai[#2], K Rahul[#3], S Rohith Yadav[#4]

[#]1 Professor & Dean, Mechanical Department, MVSR Engineering College
Nadergul, Hyderabad, India

Abstract- *Deterioration of the equipment in any industry is inevitable and is proportional to time. In order to maintain a smooth flow in operations of the machines, it is mandatory to have a continuous monitoring. If there arises a situation where the machine or the equipment requires any kind of repair then it may delay the production. The increase in repairing and the maintenance cost demands the replacement of items. The present paper focuses on three different states of repairs of a single point cutting tool. Markov model, which is a stochastic model used to model randomly changing systems in an assumption that future states depend only on the current state is applied in generating the probabilities of items falling in different states. Based on the average cost the replacement decision is taken considering macroeconomic variable "Inflation."*

Keywords

n = time period
ϕ = inflation
R = real interest rate
i = nominal interest rate
ν = present worth factor = 1/(1+i)
C_1 = Individual replacement cost per item
C_2 = Minor Repair Cost
C_3 = Medium Repair Cost
C_4 = Major Repair Cost
C_5 = Group replacement Cost
X_i^I = Probability of item in functional state at i^{th} period
X_i^{II} = Probability of item in minor repairable state at i^{th} period
X_i^{III} = Probability of item in medium repairable state at i^{th} period
X_i^{IV} = Probability of item in major repairable state at i^{th} period
X_i^{V} = Probability of item in irreparable state at i^{th} period
M = Transition probability matrix

I. INTRODUCTION

Replacement theory deals with analysis of materials and machines that deteriorate with time and optimal time of their replacement such that the total cost is minimum. A set up cost for replacement that is independent of number replaced may be advantageous to replace all items at fixed intervals. This kind of policy is called as a group replacement and is attractive when the value of individual item is very small. Inflation is a quantitative measure of rate at which average cost increases over a period of time. In other words, it is a gradual rise in the level of prices. Nominal interest rate (i_n), according to Irving Fisher, is given by the difference between nominal interest rate(i_n) and inflation (ϕ). It is given by the following formula

$$(1+i) = (1+R)\,(1+\phi)$$

Which is equivalent to $i = R + \phi\,(1+R)$

In the current work, it is assumed that the inflation, a macro-economic variable follows the non-linear pattern as given in equation (1) and forecasting of inflation is carried out for future periods.

$$\phi(n) = \alpha_0 + \alpha_1 n + \alpha_2/n \qquad (1)$$

here $\phi(n)$ is the is the inflation, 'n' is a variable and $\alpha_0, \alpha_1, \alpha_2$ are coefficients.

The following set of equations are considered in getting the values of coefficients

$\alpha_0, \alpha_1, \alpha_2$

$$\Sigma\phi = n\alpha_0 + \alpha_1\,\Sigma n + \alpha_2\,\Sigma(1/n) \qquad (2)$$

$$\Sigma\phi n = \alpha_0\Sigma n + \alpha_1\,\Sigma n^2 + \alpha_2 \qquad (3)$$

$$\Sigma\phi n^2 = \alpha_0\Sigma n^2 + \alpha_1\,\Sigma n^3 + \alpha_2\,\Sigma n \qquad (4)$$

By solving these equations, we get the following values for the coefficients

$\alpha_0 = 1.429$, $\alpha_1 = 0.449$, $\alpha_2 = 0.003$

the final regression equation for inflation is

$\phi(n) = 1.429 + 0.449n + 0.003/n$

Using above regression equation, inflation is forecasted for the next period as given below.

TABLE 1: *PREDICTED INFLATION DATA*

Period	1	2	3	4	5
Inflation	1.881	2.329	2.777	3.226	3.675
Period	6	7	8	9	10
Inflation	4.124	4.572	5.021	5.470	5.919

II. MARKOV STOCHASTIC PROCESS

A Stochastic system is called a Markov process if the occurrence of a future state depends on the immediately preceding state only on it.

Thus if '$t_0 < t_1 <\ldots\ldots\ldots < t_n$ represents the points in time scale, then family of random variables [$X(t_n)$] is said to be a Markov process provided it holds the Markovian property.

$$P\,[\,X(t_n)] = x_n\ \{X\,(t_{n-1}) = X_{n-1}\}$$

For all $X(t_0)$. $X(t_1)$, ……$X(t_n)$.

$X_{0,}\ X_{1,}\ X_2$………...$X_{n,}\ X_{n+1}$ are called the states of the process. if the random process at time t_n is in the state $X_{n,}$ the future state of the random process X_{n+1} at time depends only on the present state X_n and not on the past states X_{n-1}, X_{n-2}……X_0. Higher order Markov process assumes that the probability of next outcome can be calculated by obtaining and taking account of the outcomes of the past 'k' outcomes.

Transition Probability:

The probability of moving from one state to another or remaining in the same state during a single time period is called the transition probability,

Mathematically the probability

$P\,(X_{n-1,}\ Xn) = P\,\{X(t_n)_= X_{n/}X(t_{n-1}) = X_{n-1}\}$ is called the transition probability. The transition probabilities can be arranged in matrix form and such a matrix is called one step transition probability matrix denoted by

$$\begin{bmatrix} P11 & P12 & P13 & P14 & P15 \\ P21 & P22 & P23 & P24 & P25 \\ P31 & P32 & P33 & P34 & P35 \\ P41 & P42 & P43 & P44 & P45 \\ P51 & P52 & P53 & P54 & P55 \end{bmatrix}$$

The matrix P is a square matrix in which each element is non-negative and sum of each row is unity.

III. MODEL DEVELOPMENT

$$M = \begin{bmatrix} P11 & P12 & P13 & P14 & P15 \\ P21 & P22 & P23 & P24 & P25 \\ P31 & P32 & P33 & P34 & P35 \\ P41 & P42 & P43 & P44 & P45 \\ P51 & P52 & P53 & P54 & P55 \end{bmatrix}$$

Each row is dedicated to five different states respectively.

Probability of items in different states are computed as follows:

$[\,X_i^{I}\ \ X_i^{II}X_i^{III}\ X_i^{IV}\ X_i^{V}] = [\,X_0^{I}\ \ X_0^{II}\ \ X_0^{III}\ \ X_0^{IV}\ \ X_0^{V}\,]\ M^n$

Where $0 \leq i \leq n$

<u>Number of individual replacements</u>

1st period, $f_1 = N_1\,X_1^{V}$

2nd period, $f_2 = N_1\,X_2^{V} + f_1\,X_1^{V}$

3rd period, $f_3 = N_1\,X_3^{V} + f_1\,X_2^{V} + f_2\,X_1^{V}$

… …. …

In the same manner values of "g, h, k" are calculated.

By using MATLAB code, the values of future states have been calculated.

IV. CUTTING TOOL

Cutting tool is a wedge shaped and sharp-edged device that is used to remove excess layer of material from the workpiece by shearing during machining in order to obtain desired shape, size and accuracy. Some of the common materials available are

- High Speed Steel (HSS)
- Diamond
- Tungsten Carbide
- Ceramics
- Carbon Boron Nitride (CBN)

Cutting tool is used for shaping up different materials and perform various operations such as facing, taper turning, cutting, drilling, knurling. Special cutting tools are used to perform desired operations. Every tool material must possess certain properties such as high hardness, high hot hardness, high strength, higher melting point and chemically inert even at high cutting temperature. During these operations, part of cutting tool remains contact with work piece and thus experiences severe cutting temperatures and insistent rubbing. Due to excessive usage of tool, it undergoes deformation such as crater wear and flank wear. Crater wear is more or less circular and occurs on the rake face whereas flank wear or wear land is on clearance surface of the tool. The former one is considered to be minor repair as it increases the cutting forces and modifies the tool geometry whereas flank wear modifies tool geometry and changes cutting parameters, such as depth of cut and hence it is considered to be a medium repair. If both of them occur together then it is considered to be major repair. Fig 1, Fig 2 shows the wear of single point cutting tool.

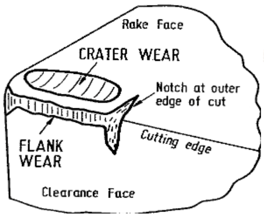


Fig1

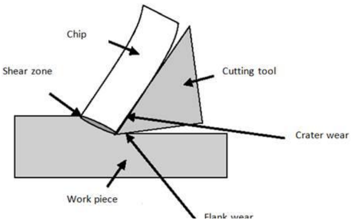
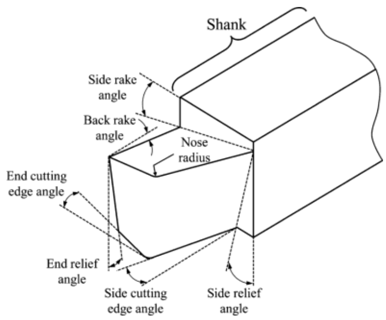


Fig 2

Tool wear is affected by feed rate even. If the surface hardness of tool is greater than surface hardness of workpiece then there is no substantial increase in tool wear with speed increase. If the surface hardness of tool is lesser than surface hardness of workpiece then there is substantial increase in tool wear with speed increase.



General Profile of single point cutting tool is shown in Fig 3

The following model can be applied for maintenance of cutting tools at a big manufacturing unit. The different stages are:

I = Functional State

II = Minor repairable State

III = Medium repairable State

IV = Major repairable State

V = Irreparable State

The initial state probabilities and transition matrix are assumed as given below

$$M = \begin{bmatrix} 0.31 & 0.07 & 0.02 & 0.27 & 0.33 \\ 0.63 & 0.01 & 0.05 & 0.2 & 0.11 \\ 0.14 & 0.3 & 0.4 & 0.06 & 0.1 \\ 0.04 & 0.5 & 0 & 0.32 & 0.14 \\ 0.23 & 0.02 & 0.25 & 0.1 & 0.4 \end{bmatrix}$$

Number of tools falling in different states in future period is calculated as given below

To find out the value of

$$[X_1^{I}\ \ X_1^{II}\ \ X_1^{III}\ X_1^{IV}\ X_1^{V}] = [X_0^{I}\ \ X_0^{II}\ \ X_0^{III}\ X_0^{IV}\ X_0^{V}] \begin{bmatrix} 0.31 & 0.07 & 0.02 & 0.27 & 0.33 \\ 0.63 & 0.01 & 0.05 & 0.2 & 0.11 \\ 0.14 & 0.3 & 0.4 & 0.06 & 0.1 \\ 0.04 & 0.5 & 0 & 0.32 & 0.14 \\ 0.23 & 0.02 & 0.25 & 0.1 & 0.4 \end{bmatrix}$$

$$=[0.43 \quad 0.21 \quad 0.13 \quad 0.11 \quad 0.12]$$

$$\begin{bmatrix} 0.31 & 0.07 & 0.02 & 0.27 & 0.33 \\ 0.63 & 0.01 & 0.05 & 0.2 & 0.11 \\ 0.14 & 0.3 & 0.4 & 0.06 & 0.1 \\ 0.04 & 0.5 & 0 & 0.32 & 0.14 \\ 0.23 & 0.02 & 0.25 & 0.1 & 0.4 \end{bmatrix}$$

C_1 = 5000, C_2 = 750, C_3 = 1500, C_4 = 2500, C_5 = 3500, N = 100, i = 15%

MATLAB CODE

```
clc
clear all

x = [ ]; /* Enter the values for x */
cnt = size(x,2);
N = [ ]; /* enter the number of values required */
for it1 = 1:cnt
   f(it1) = N * x(it1);
   for it2 = 1:it1-1
       f(it1) = f(it1) + f(it2) * x(it1-it2);
   end
end
f
```

Outputs

The outputs obtained are under the column 7,8,9,10 in the TABLE 2 below:

Table-2: Calculation of replacement period with inflation.

1	2	3	4	5	6	7	8	9	10
Period **(n)**	Inflation **(ϕ)** %	Real Interest Rate **(R)**	Present Worth factor **(v)**	Present Worth factor **(v)**	Dividing Discount Factor **(Σ(v^(n-1)))**	f	g	h	k
1	1.881	0.129	0.886	1.000	1.000	24.140	21.310	10.110	12.860
2	2.329	0.124	0.890	0.890	1.890	31.310	25.476	12.372	18.164
3	**2.777**	**0.119**	**0.894**	**0.799**	**2.689**	**38.273**	**30.065**	**14.650**	**20.808**
4	3.226	0.114	0.898	0.723	3.412	47.620	36.156	16.516	24.061
5	3.675	0.109	0.902	0.661	4.072	59.307	43.530	18.524	27.832
6	4.124	0.104	0.905	0.608	4.681	73.913	52.419	20.764	32.316

11	12	13	14	15= (11+12+13+14)	16
Individual replacement **(f*C1*(v^(n-1)))**	Minor Repair **(g*C2*(v^(n-1)))**	Medium Repair **(h*C3*(v^(n-1)))**	Major Repair **(k*C4*(v^(n-1)))**	maintenance cost **(M)**	cummilative maintenance cost **(ΣM)**
120700	15982.5	15165	32150	183997.5	75345
139300.232	17001.66	16513.15	40405.97	213221.01	288566.01
152847.628	**18010.21**	**17552.3**	**41549.57**	**229959.7**	**518525.71**
172198.758	19611.57	17917.17	43503.51	253231	771756.71
195872.773	21564.93	18353.92	45960.27	281751.89	1053508.6
224877.199	23922.39	18952.44	49160.04	316912.07	1370420.67

17	18= 16+17	19 = 18/6
Group Replacement **(N*C5*(v^(n-1)))**	Total Cost **(TC)**	Average Cost **(AC)**
350000	425345	425345
311434.57	600000.57	317492.03
279553.05	**798078.76**	**296845.1**
253127.11	1024883.82	300397.72
231188.46	1284697.06	315472.51
212972.06	1583392.73	338274.94

The above table indicates the variation in cost with respect to the change in Inflation.

The following is a graph of the cost and replacement period when inflation is considered.

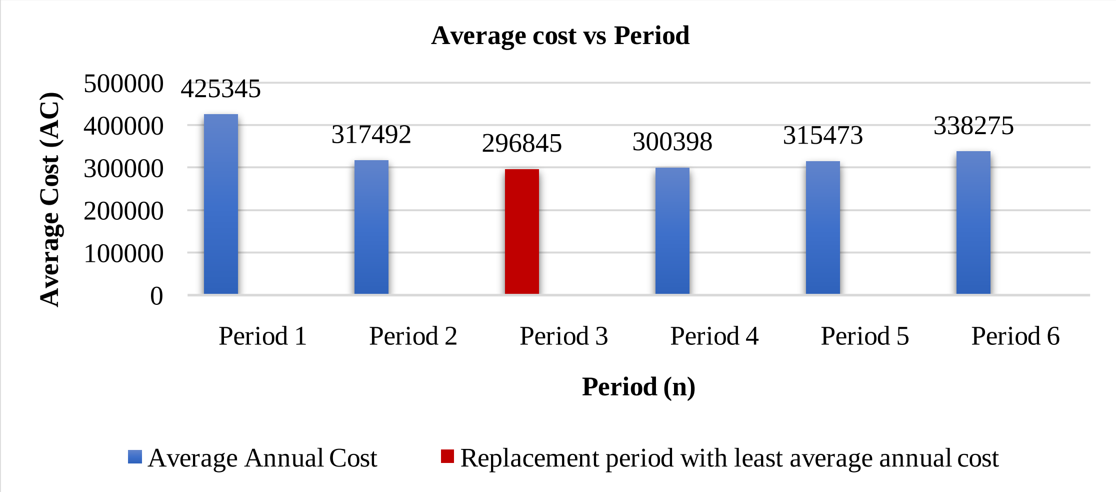


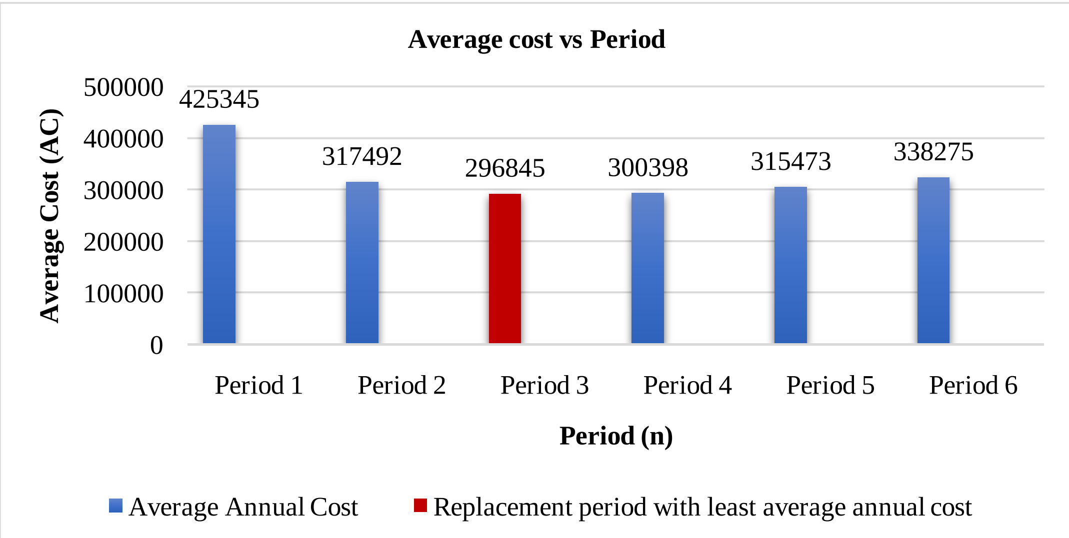
Table-3: Calculation of replacement period without inflation.

1	2	3	4	5	6	7	8	9	10
Period **(n)**	Inflation **(ϕ)** %	Real Interest Rate **(R)**	Present Worth factor **(v)**	Discount Factor **(v^(n-1))**	Dividing Discount Factor **(Σ(v^(n-1)))**	f	g	h	k
1	0.000	0.150	0.870	1.000	1.000	24.140	21.310	10.110	12.860
2	0.000	0.150	0.870	0.870	1.870	31.310	25.476	12.372	18.164
3	0.000	**0.150**	**0.870**	**0.756**	**2.626**	**38.273**	**30.065**	**14.650**	**20.808**
4	0.000	0.150	0.870	0.658	3.283	47.620	36.156	16.516	24.061
5	0.000	0.150	0.870	0.572	3.855	59.307	43.530	18.524	27.832
6	0.000	0.150	0.870	0.497	4.352	73.913	52.419	20.764	32.316

11	12	13	14	15= (11+12+13+14)	16
Individual replacement (f*C1*(v^(n-1)))	Minor Repair (g*C2*(v^(n-1)))	Medium Repair (h*C3*(v^(n-1)))	Major Repair (k*C4*(v^(n-1)))	maintenance cost **(M)**	cummilative maintenance cost (ΣM)
120700.000	15982.50	15165.00	32150.00	183997.50	75345.00
136130.435	16614.78	16137.39	39486.52	208369.13	283714.13
144699.433	**17050.09**	**16616.60**	**39334.59**	**217700.72**	**501414.85**
156554.615	17829.87	16289.41	39551.25	230225.13	731639.98
169544.849	18666.31	15886.91	39782.59	243880.66	975520.64
183739.120	19546.13	15485.36	40166.91	258937.52	1234458.17

17	18= 16+17	19 = 18/6
Group Replacement **(N*C5*(v^(n-1)))**	Total Cost **(TC)**	Average Cost **(AC)**
350000.00	425345.00	425345.00
304347.83	588061.96	314544.77
264650.28	**766065.13**	**291755.55**
230130.68	961770.66	292934.73
200113.64	1175634.28	304965.21
174011.86	1408470.02	323625.88

The following is a graph of the cost and replacement period when inflation is not considered.



V. CONCLUSION

From the tables 2 and 3 it is evident that the average cost is gradually decreasing until certain period and then increases unexpectedly. Although the pattern and the replacement period are similar during both the cases, it is clear that the average cost varied and the least average cost obtained was 291755.55, which was when the inflation was not considered. Since the model is developed using Markov stochastic process, better probabilities of various states are obtained.

References

[1] Xitong Zheng & Nasser Fard (1991): "A Maintenance Policy for Repairable Systems based on Opportunistic Failure-rate Tolerance", IEEE Transactions on Reliability, Vol-40, No.2, June, 1991.

[2] Isha Bagai and Kanchan Jain (1994): "Improvement, Deterioration and Optimal Replacement Under Age-Replacement With Minimal Repair", IEEE Transactions on Reliability, Vol.43, No.1, P-156 to 162, March, 1994.

[3] Thomas W. Archibald and Rommert Dekker (1996): "Modified Block-Replacement for Multiple-Component Systems", IEEE Transaction on Reliability, Vol-45, No.1, P-75 to 83, March, 1996.

[4] Nadamuni Reddy. C and C. Uma Sankar (1996): "Markov-Chains in Group replacement decisions", paper presented at 38th National Convention of Indian Institution of Industrial Engineering, Bombay, December, 1996.

[5] Nadhamuni Reddy.C.(1997): "Block, Combined influence of inflation and cost of money in replacement decisions", National Conference on Statistical Quality Control, Guntur. India.

[6] Gerry Trodd (1998): "Practical Implementation of Predictive Maintenance", IEEE Transactions on Semiconductor Manufacturing, P-29 to 37, 1998.

[7] ToyonariShimakage, Kai Wu, Takeyoshi Kato, Tatsuki Okamoto and YasuoSuzuoki (2002): "Economic Evaluation of Cable Replacement Considering Annual Failure Probability", Conference Record of the 2002, IEEE International Symposium on Electrical Insulation, Boston , MA USA, P-1472 to 475, April, 2002.

[8] Jong Min Lee, Seung-Jong Kim, Chang-Seop Song &Yoha Hwang (2003): "Diagnosis of mechanical fault signals using continuous hidden Markov model" Journal of Sound and Vibration 276 (2004) 1065–1080, Elsevier, August 2003.

[9] S.Gajanana, Dr. G. Krishnaiah& Dr. C.Nadhamuni Reddy (2005): "Prediction of Inflation using Fore casting Techniques for Developing Societies" International conference on Operations Research Applications in Infrastructure Development (ICORAID-2005), IISc, Bangalore, India.

[10] C. Richard Cassady and ErhanKutanoglu (2005): Integrating Preventive Maintenance Planning and Production Scheduling for a Single Machine", IEEE Transactions on reliability, Vol-54, No.2, June, 2005.

[11] RueyHaueiYeh, Gaung-Chen and Ming-Yuh Chen (2005): "Optimal Age-Replacement Policy for Non repairable Products Under Renewing Free-Replacement Warranty", IEEE Transactions on Reliability, Vol.54, No.1, P-92 to 97, March 2005.

[12] Antonio G. Vallejo Jr., Juan A. Nolazco-Flores, Rub´en Morales-Men´endez, L. Enrique Sucar&Ciro A. Rodr´ıguez (2005): "Tool-Wear Monitoring Based on Continuous Hidden Markov Models" Springer-Verlag Berlin Heidelberg 2005.

[13] S.Gajanana, Dr. G. Krishnaiah& Dr. C.Nadhamuni Reddy (2006): "Development of Mathematical Model For Inflation Prediction Using Forecasting Technique" Industrial Engineering Journal, Vol. XXXV NO.5, May-2006, P-28 to 31.

[14] A. A. Kassim, Zhu Mian&M. A. Mannan : "Tool condition classification using Hidden Markov Model based on fractal analysis of machined surface textures" Machine Vision and Applications (2006) 17:327–336, Springer, July 2006.

[15] S.Gajanana, Dr. G. Krishnaiah& Dr. C.Nadhamuni Reddy (2006): "Development Of Replacement Model Using Predicted Inflation" Indian Journal of Mathematics and Mathematical Sciences.

[16] Tony Boutros, & Ming Liang (2011): "Detection and diagnosis of bearing and cutting tool faults using hidden Markov models" Mechanical Systems and Signal Processing 25 (2011) 2102–2124, Elsevier, February 2011.