

Error Analysis of Engineering Students' Misconceptions in Algebra

Prof. Cristo Mark D. Ancheta, Ph.D¹, Engr. Gener S. Subia, Ph.D.²

¹ Faculty of Mathematics and Sciences Department, Nueva Ecija University of Science and Technology, Nueva Ecija, Philippines

² Faculty of Mathematics, Wesleyan University Philippines, Cabanatuan City, Philippines

²subiagener@yahoo.com

Abstract - This study dealt with identifying the engineering students' misconceptions in Algebra using the error analysis approach. It also determined the reasons causing students' misconceptions. The study results show that 86% of the students had misconceptions in identifying similar terms and identifying similar radical expressions, 88% for identifying polynomial expressions that are not factorable, and 92% for translating words to mathematics symbols. The underlying reasons for students' misconceptions are due to students' lack of conceptual understanding and retention in their schema. The students easily forget what they previously learned because of rote learning. They tend to memorize the concepts needed for skillful mathematical operation instead of grasping these concepts' meaning. Some are caused by carelessness and incorrect application of laws and principles. In addition, the teaching methodology of the teachers did not focus on retention learning, identifying errors, and improving scores of the students.

Keywords — Algebra, calculus, engineering students, error analysis, mathematics teachers, misconceptions

I. INTRODUCTION

In the mathematics curriculum, subjects are sequenced to prepare students to be more receptive to new concepts, approaches, and applications in math offered in the upcoming years. Calculus is one of the subjects taught in engineering courses that were very relevant and useful to the real world. However, to be able to pass it requires pre-requisite skills in mathematics subjects. The success or failure of any Calculus student is dependent on the extent of competency gained in Pre-Calculus subjects, particularly Algebra. Students' prior knowledge in Pre-Calculus is essential in learning concepts, definitions, theorems, and applications of Calculus [1]. Students who have a deficiency in relevant prior knowledge will have difficulty in acquiring new understandings. Inadequate content knowledge of students in Algebra may cause their pitfall in Calculus. Students build more advanced knowledge from prior understandings [2]. According to the author in [3], "background characteristics of students that have been examined by researchers is the role of prior mathematics

preparation in relation to calculus performance."

Students who passed Pre-Calculus subjects with mediocrity are likely to grapple with Calculus as this subject requires prior familiarity and skill on these subjects. In the study of [1], the students who passed in Calculus were the students with high marks in Pre-Calculus subject, and the students with a low mark in pre-calculus failed in Calculus. A mathematics teacher has to know how mathematical instruction in Pre-Calculus subjects, specifically Algebra, is received, processed, and retained in the students' minds. By doing so, appropriate teaching and learning strategies can be planned and executed to attain more meaningful student learning outcomes.

Viable approaches can be employed to probe students' conceptions of mathematical knowledge. According to [4], "error analysis approach has proven to be an effective tool in analyzing students' errors in mathematics." Error analysis is a type of diagnostic assessment that can help a teacher determine what types of errors a student is making and why. More specifically, it is the process of identifying and reviewing students' errors to determine whether an error pattern exists—whether a student is making the same type of error consistently. Error analysis involves an analysis of error patterns to identify difficulties that students have with facts, concepts, strategies, and procedures. "Identifying the type of error allows teachers to address the learners' needs more efficiently [5]. If a pattern does exist, the teacher can identify the student's misconceptions.

Students who are found weak in basic conceptual knowledge in mathematics commit errors in solving problems ([6],[7]). According to [8] and [9], "the general error categories in mathematics include processing language information, interpreting spatial information, selecting appropriate procedures, making concept associations, and using irrelevant rules or information; calculation errors, procedural errors, and symbolic errors."

From the preceding discussion, it is evident that students commit errors in mathematics due to their misconceptions, and thus important to analyze the nature and sources of these errors. If this is done, measures and interventions can be adopted to overcome them. Hence, a



study is needed to analyze students' misconceptions in Algebra.

II. METHODOLOGY

This study dealt with the identification of the students' misconceptions in Algebra using the error analysis approach. It also determined the reasons causing students' misconceptions. The study used a combination of quantitative and qualitative approaches. Quantitative research, specifically, the descriptive method, was used to describe the misconceptions of the students. Moreover, the study also used a qualitative research design, a case study in particular. According to [10], "a case study design should be considered when : (a) the focus of the study is to answer "how" and "why" questions; (b) you cannot manipulate the behavior of those involved in the study; (c) you want to cover contextual conditions because you believe they are relevant to the phenomenon under study, or (d) the boundaries are not clear between the phenomenon and context." In the case of analyzing the underlying reasons for misconceptions of students, this method is necessary.

The study was conducted at the Nueva Ecija University of Science and Technology, Gen. Tinio Campus, Cabanatuan City. The participants who were purposively chosen as respondents of the study [11] were the 50 second-year engineering students who have experienced difficulties in Algebra; 30 Civil, 10 Electrical, and 10 Mechanical.

Three sets of instruments were used to gather the data needed: The Pre-Calculus Algebra Test (PCAT), Interview Guide for Students, and Interview Guide for the Mathematics Teachers. The contents of the test were drawn from the learning competencies in the syllabi of instruction. The mapping of competencies and skills developed among students served as the first step in developing the test. This mapping was the guide in doing the questions. The items were constructed to measure misconceptions. Table of Specifications was prepared according to the distribution of items as to content and degree of difficulty. For clarity and comprehensiveness of the items and directions, the instruments were "tried out" to engineering students not included in the study. Mathematics teachers and experts also checked the contents of the tests. Using the Kuder-Richardson Formula 20 [12], the computed reliability coefficient of the test was 0.76. The Interview Guide for Students (IGS) helped the researchers direct the conversation towards [13] the underlying reasons for their misconceptions. This was administered after the results of the test were initially analyzed. The answers and solutions provided by the students were also clarified during the interview. The Interview Guide for Mathematics Teachers (IGMT) was utilized to clarify the identified misconceptions. The results of the interview provided consistency of the answers drawn from the other instruments.

To identify the students with misconceptions in Algebra [14], the scale shown in Table 1 was used.

Table 1. Scale Used in Identifying the Number of Students with Misconception

Range of Students Committing the Error	Description/Interpretation
41 to 50	Almost all students with a misconception
31 to 40	Many students with a misconception
21 to 30	Some students with a misconception
11 to 20	Few students with a misconception
1 to 10	Very Few students with a misconception

III. RESULTS AND DISCUSSION

A. Students' Misconceptions in Algebraic Expressions and Functions

In terms of algebraic expressions and functions, 54% have errors in identifying expressions that are not polynomial, and 46% for distinguishing Relation and function.

Table 2. Number and Percentage of Students with Misconceptions in Algebraic Expressions and Functions

Main Topic	Concepts	EE	ME	CE	Total	%	Description of Misconceptions
		n=1 0	n=1 0	n=3 0	n=5 0		
Algebraic Expressions and Functions	Identifying Expressions that are not polynomial	7	6	14	27	54	Some
	Distinguishing Relation and function	7	5	11	23	46	Some
	Identifying Similar Terms	9	10	24	43	86	Almost All

Almost all students had misconceptions in identifying similar terms. For example, in the item, "Which of the following is similar to $5x^2$?"

Which of the following is similar to $5x^2$?

a. $10x$ b. $-3x^2$ c. $5x^3$ d. $5y^2$

Frequency:

a) 22 b) 7 c) 1 d) 20

Only 7 students identified the correct answer: letter b and 43 (86%) committed an error. Twenty-two (22) out of 50 students answered $10x$. They could not recall the concept of similar terms, *that the literal coefficients must be the same for the terms to be called similar*. Instead, they assumed that the terms were similar when the derivative was obtained.

One student said that he obtained the derivative in an interview since he thought that its derivative was similar to the given expression.

Moreover, 20 out of 50 students assumed that the terms were similar when the exponents of the variables involved were the same, as shown in option d, $(5y^2)$. They did not recognize that the variables must be the same and that the corresponding exponent(s) must also be the same for the terms to be similar. One student said that he thought the similarity of variables is not important, but the similarity of the term's exponent and numerical coefficient.

The finding conveys that students have differences in analyzing and understanding the subject. This was clearly emphasized by [15] that there are great differences between and among learners of different countries when it comes to Algebra".

In the interview results with teachers, they said that they were not aware that their students had misconceptions in identifying similar terms. For them, these concepts were introduced to their students very clearly. This shows that teachers need assistance in error identification and determining when the misconceptions of students have occurred [16].

B. Students' Misconceptions in Special Products and Factoring

As to students' misconceptions in special products and factoring, 60% of the engineering students have errors in factoring the sum of two cubes and identifying the perfect square trinomial.

Table 3. Number and Percentage of Students with Misconceptions in Special Products and Factoring

Main Topic	Concepts	EE	ME	CE	Total	%	Description of Misconceptions
		n=10	n=10	n=30	n=50		
Special Products and Factoring	Factoring Sum of Two Cubes	7	9	14	30	60	Many
	Identifying Perfect Square Trinomial	6	5	19	30	60	Many
	Identifying Polynomial Expression which is not Factorable	9	10	25	44	88	Almost All

Almost all students had misconceptions in **identifying polynomial expressions that are not factorable**. To illustrate the misconceptions, consider the item,

Which of the following is not factorable?

a. $x^4 - y^2$ b. $x^2y + xy^2$ c. x^2+y^2 d. $x^3 + y^3$

Frequency:

a) 14 b) 26 c) 6 d) 4

Only 6 out of 50 students got the correct answer to letter c, and 44 (88%) answered it wrong.

Twenty-six (26) out of 50 students answered option b as not factorable. In an interview, a student answered that it was not factorable since the terms had a combination of different variables. They failed to recognize that xy is common to both terms; hence, it could be factored out. They did not realize that $x^2 + y^2$, a sum of two squares, was not factorable. Instead, four students assumed that it could be factored as a product of sum and difference.

14 students answered $x^4 - y^2$ as not factorable since they had different exponents.

The teachers also commented that factoring was one of the weaknesses of students in mathematics. They easily forgot the patterns in factoring as they did not identify polynomial expressions that are not factorable. This finding is similar to the result of the study by [17] where they "seen that the students have major inadequate of the math's fundamentals "the Factorization" and therefore they were not able to conclude although they knew the solution."

C. Students' Misconceptions in Special Products and Factoring

Table 4 reveals the number and percentage of students with misconceptions in exponents and radicals.

Forty-six percent (46%) encountered difficulty simplifying zero exponents, and 48% experienced hardships in identifying imaginary numbers. Further, 78% committed errors in identifying the base of exponential expression, 74% for simplifying a radical expression, and 72% have mistakes on positive and negative exponent with the same base.

Table 4. Number and Percentage of Students with Misconceptions in Exponents and Radicals

Main Topic	Concepts	EE	ME	CE	Total	%	Description of Misconceptions
		n=10	n=10	n=30			
Exponents and Radicals	Simplifying Zero Exponents	6	3	14	23	46	Some
	Identifying Base of Exponential Expression	9	8	22	39	78	Many
	Identifying Relations between Positive and Negative Exponent with the Same Base	7	8	21	36	72	Many
	Identifying Imaginary Numbers	5	2	17	24	48	Some
	Simplifying Radical Expression	8	7	22	37	74	Many
	Identifying Similar Radical Expressions	7	8	26	41	82	Almost All

Almost all students had misconceptions in **identifying similar radical expressions**. To illustrate the misconceptions, consider the item,

Which of the following radicals is similar to $3\sqrt[3]{2}$?

a. $2\sqrt{5}$ b. $3\sqrt{2}$ c. $2\sqrt[3]{3}$ **d. $-\sqrt[3]{2}$**

Frequency:

a) 2 b) 11 c) 27 **d) 10**

Twenty-seven (27) out of 50 students answered $2\sqrt[3]{3}$ as radical expression similar to the given radical. Students assumed that if the roots are equal and with the same sign, then the radicals are similar. In an interview, students said that the expression is similar to option c since they were both positive and with the same root. They did not

realize that for the radicals to be similar, the expressions must have the same root and radicand as in option d.

Moreover, 11 students answered $3\sqrt{2}$. Students assumed that if the expression had the same radicand, then they were similar. In an interview, students said that the expression was similar to option b since they both had 2 as radicand.

In an interview with the teachers about the results that more than 80% of their students have a misconception, they stated that students were not aware of making radical expressions similar. Due to this fact, engineering mathematics teachers should revisit their strategies and realized that the "purpose of mathematics teaching is to develop students' correct thinking, problem-solving skills and to ensure the use of these skills in later life"[18], [19].

D. Students' Misconceptions in Linear Equations, Quadratic and Word Problems

Table 5. Number and Percentage of Students with Misconceptions in Linear Equations, Quadratic and Word Problems

Main Topic	Concepts	EE	ME	CE	Total	%	Description of Misconceptions
		n=10	n=10	n=30			
Linear Equations, Quadratic and Word Problems	Identifying Quadratic Equations	7	7	13	27	54	Some
	Translating Words to Math Symbols: Less Than	9	10	27	46	92	Almost All
	Translating Words to Math Symbols: Sum of the Cubes	5	4	18	27	54	Some
	Identifying Linear Equations	5	5	16	26	52	Some

In the linear equations, quadratic, and word problems, Table 5 shows that the students committed errors in identifying quadratic equations (54%), translating words to math symbols (54%), and identifying linear equations (52%).

Ninety-two percent (92%) of the students had misconceptions in **translating words to mathematics symbols: less than**. To illustrate the misconceptions, consider the item,

In solving equation, what is the mathematical symbol for "14 less than y"?			
a. $y - 14$	b. $14 - y$	c. $14 < y$	d. $y < 14$
Frequency:			
a) 4	b) 10	c) 34	d) 2

Thirty-four (34) out of 50 students answered $14 < y$ as the translation of "14 less than y" in solving the equation. Students translated the phrase into a math symbol. They did not realize that it is in solving an equation; therefore, the inequality symbol will subtract as in $y - 14$. They did not consider the fact that subtraction can be used to mean the statement "14 less than y," which is "y minus 14" or " $y - 14$ ". One cannot solve the problem in the equation with an inequality symbol. In an interview, one (1) student said that he was not aware that less than can be changed to subtraction as an operation.

Moreover, 10 students answered $14 - y$. They used subtraction as an operation, but the order is incorrect. Two (2) students answered $y < 14$. They used the inequality symbol.

Teachers agreed that the students have misconceptions in translating words to mathematics symbols since most of their solving worded problems in class students tend to commit errors. According to the authors in [20], "students perennially demonstrate difficulty incorrectly performing mathematical translations between and among mathematical representations. "Some of these considerations included, defining mathematical errors during the translation process, teacher beliefs and instructional practices, student interpretive and translation activities, and the use of transitional representations."

IV. CONCLUSIONS AND RECOMMENDATIONS

"The teaching and learning of Algebra is a difficult area for study because across different countries and even within countries, what is done in classrooms can be quite different" [21]. Algebra teachers are encouraged to provide immediate feedback to students and their peers on students' misconceptions. Common errors made by [22] them can be addressed early, and difficulty can be lessened. In doing so, the escalation of these problems in higher mathematics subjects can be minimized.

Based on the results, the following conclusions are drawn:

1. Students have misconceptions in the basic concepts and laws in Algebra that are necessary for performing fundamental algebraic operations. They also lack the required skills in performing algebraic operations.

2. The underlying reasons for students' misconceptions

are students' lack of conceptual understanding and retention in their schema. The students easily forget what they previously learned because of rote learning. They tend to memorize the concepts needed for skillful mathematical operation instead of grasping these concepts' meaning. Some are caused by carelessness and incorrect application of laws and principles.

3. Teaching methodology of engineering math teachers [23] did not focus on retention learning, identifying errors, and improving students' scores.

Based on the findings and conclusions, the following recommendations are offered.

1. Additional reinforcement activities in Algebra must be given to students to facilitate learning and the learning tasks and examples provided during classes.

2. Mathematics teachers are encouraged to reflect on and or revisit their teaching practices and attend professional training and rehabilitation [24]. These may provide them sound bases in utilizing other teaching methods that may redound to improved students' mathematics performance.

3. A remediation program can be designed to help students with misconceptions in Pre-calculus subjects.

4. The underlying reasons causing students' misconceptions in Algebra be shared with the other mathematics teachers and students to know what to focus on and emphasize during teaching and learning.

5. The students may also be given training/seminars on time management to balance their studies and other extra-curricular activities.

6. Similar studies are recommended to be conducted on the students' misconceptions and skill deficits in other subject areas of mathematics to design and offer a solution [25] regarding students' common errors.

7. Related studies such as providing a direct and simple [26] mathematical framework [27] that could help reduce errors and enhance the thinking skills of the students [28] could be conducted by engineers and professors.

REFERENCES

- [1] Yeoh, H.B., & Md Yunus, A.S. A holistic approach to Activate and Enhance Prior Knowledge of Tertiary Learners in the Upcoming Lectures of Calculus. *World Applied Sciences Journal*. (Special Issue of Applied Math). 21 (2013)156-161.
- [2] Smith III, J., Sessa, A., and Roschelle, J. Misconceptions Reconceived: a constructivist analysis of knowledge in transition. Published online. (2009) 115-163.
- [3] Mwavita, M. Factors influencing calculus course success among freshmen engineering students. 2005. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.634.2513&rep=rep1&type=pdf>.
- [4] Kingsdorf, S. and Krawek J. Error analyze mathematical word problem solving across students with and without learning disabilities, *Learning Disabilities Research, and Practice*. 29(2) (2014) 66-74.
- [5] Kundu, U., and Sengupta, D. Error analysis in mathematics in Relation to secondary school students. *Indian Journal of Educational Research*. 3 (2014)105-125.
- [6] Kaur, N. A diagnostic study of errors in mathematics among ninth-grade students at different levels of intelligence. *Researcher's Tandem*. 2(8) (2011) 22-31.

- [7] Luneta, K. Understanding students' misconceptions: an analysis of final grade 12 examination questions in geometry. *Pythagoras*. 36(1) (2015), 11 pages.
- [8] Elbrink, M. Analyzing and addressing common mathematical errors in secondary education. *B.S. Undergraduate Mathematics Exchange*. 50(1) (2008) 2-4.
- [9] Marshall, S.P. Sex differences in mathematical errors: An analysis of distracter choices. *Journal for Research in Mathematics Education*. 14 (1983) 325-336.
- [10] Yin, R. K. Case study research: Design and methods (3rd ed.). Thousand Oaks, CA: Sage, 2003.
- [11] Subia, G. Comprehensible Technique in Solving Consecutive Number Problems in Algebra. *Journal of Applied Mathematics and Physics*. 6 (2018) 447-457.
- [12] Allen, M. Reliability, Kuder-Richardson Formula. *The SAGE Encyclopedia of Communication Research Methods*, 2017, <https://dx.doi.org/10.4135/9781483381411.n493>.
- [13] Kennedy, M. A Guide to Interview Guides. 2006, <https://msu.edu/user/mkennedy/digitaladvisor/Research/interviewing.htm>.
- [14] Mulungye, M., Connor, M. & Ndethiu, S. Sources of Student Errors and Misconceptions in Algebra and Effectiveness of Classroom Practice Remediation in Machakos County- Kenya. *Journal of Education and Practice*. 7(10) (2016) 31-33.
- [15] Grønmo L.S. The Role of Algebra in School Mathematics. In: Kaiser G., Forgasz H., Graven M., Kuzniak A., Simmt E., Xu B. (eds) *Invited Lectures from the 13th International Congress on Mathematical Education*. ICME-13 Monographs. Springer, Cham. (2018) 175-193.
- [16] Osborne, R. J., & Wittrock, M. C. Learning science: A generative process. *Science Education*. 67(4) (1983) 498-508.
- [17] Burhanzade, H. & Aygor, N. Difficulties That Students Face During Factorization Questions. *Procedia - Social and Behavioral Sciences* 191 (2015) 859 – 864.
- [18] Yılmaz, K. Öğrencilerin Epistemolojik ve Matematik Problemi Çözümlerine Yönelik İnançlarının Problem Çözme Sürecine Etkisinin Araştırılması. Yüksek Lisans Tezi. Marmara Üniversitesi. Eğitim Bilimleri Enstitüsü. İstanbul. (2007).
- [19] Salangsang, L., & Subia, G. Mathematical thinking on problem-solving and self-regulation strategies of Filipino primary grade pupils. *International Journal of Scientific & Technology Research*. 9(2) (2020) 4000-4004.
- [20] Bosse, M., Adu-Gyamfi, K., & Cheetham, M. Assessing the Difficulty of Mathematical Translations: Synthesizing the Literature and Novel Findings. *International Electronic Journal of Mathematics Education*. 6(3) (2011) 114-133.
- [21] Chazan, D., et al., Research and Development in the Teaching and Learning of Algebra. Conference: Proceedings of the 10th International Congress on Mathematical Education At: Copenhagen, Denmark. 1 (2004).
- [22] Morgan, A. A study of the difficulties experienced with mathematics by engineering students in higher education, *International Journal of Mathematical Education in Science and Technology*. 21(6) (1990) 975-988.
- [23] Baig, F. Application of Teaching Methods in Mathematics at Secondary Level in Pakistan. *Pakistan Journal of Social Sciences (PJSS)*. 35 (2) (2015) 935-946.
- [24] Ibrahim Moh'D Arman, Ashraf Moh'D Abukhayran Efficiency of Using "Blended Learning" Strategy on the Achievement of the 6th Grade Students on Mathematics *International Journal of Engineering Trends and Technology*. 67 (9) (2019) 44-50.
- [25] Russell N. Alfonso, Marlon C. Leyesa, Donald M. Lapiguera, Noel Florencondia, Gener S. Subia Proposed Design for Framework Management of Cryptocurrency: Study of the World's First Digital Currency *International Journal of Engineering Trends and Technology*. 68 (1) (2020) 57-63.
- [26] Subia, G. Comprehensible Technique in Solving Consecutive Number Problems in Algebra. *Journal of Applied Mathematics and Physics*. 6 (2018) 447-457.
- [27] Sravya, K, V.Santhosh Kumar Tap Wavelet Filters Using Algebraic Integers, *International Journal of Engineering Trends and Technology (IJETT)*. 25(2) (2015) 82-88.
- [28] Subia, G. S. Treasure Chess: Worthy Contributions of the Game in the Lives of Student Champions. *The Normal Lights*. 14(1) (2020) 100-121.