Application with Implications of Log-Periodic Trends to Financial Bubbles: A Case for Econoplysics

[1]Debasis Patnaik, [2] U Siddharth
[1]*Ph.D. Associate Professor,*[2]*M.Sc. Economics*
Dept. of Economics, BITS Pilani, Goa Campus, India

Abstract: *This paper attempts to understand the phenomenon of market crashes in BSE India. This involves the identification of its causes, mechanism, properties that the market exhibits as well as the identification of the market indicators and its success in the predetermination of market crashes. An analysis on the time series data of the Bombay Stock Exchange is also tested for log-periodic trends to assess market stability.*

Keywords: *Log periodic, bubbles, anti bubbles, econophysics, BSE-India, Time series.*

I. INTRODUCTION

Both physics and economics deal with systems of many interacting components that obey specific rules. Finance deals with empirical observation of regularities in market data, the dynamics of price formation and understanding of bubbles and panics. . The power laws and associated complex exponents and log-periodic patterns were found to perform quite reliably for prediction purposes. Feigenbaum, Freund and Sornette independently suggested that the same concept be applied to study large financial crashes. The existence of stock market bubbles is at odds with the assumptions of efficient market theory which assumes rational investor behavior. Behavioral finance theories attribute stock market bubbles to cognitive biases that lead to 'groupthink' and 'herd' behavior. Bubbles occur not only in real-world markets, with their inherent uncertainty and noise, but also in highly predictable experimental markets. The two most famous bubbles of the twentieth century, the bubble in American stocks in the 1920s, he Nifty Fifty stocks in the early 1970s, Taiwanese stocks in 1987, Japanese stocks in the late 1980s and the Dot-com bubble of the late 1990s were based on speculative activity surrounding the development of new technologies.

Stock market bubbles frequently produce hot markets in Initial Public Offerings, since investment bankers and their clients see opportunities to float new stock issues at inflated prices. These hot IPO markets misallocate investment funds to areas dictated by speculative trends, rather than to enterprises generating longstanding economic value. . The bubble in closed-end country funds in the late 1980s were the bubbles that occurred in experimental asset markets. For closed-end country funds, observers compare the stock prices to the net asset value per share (the net value of the fund's total holdings divided by the number of shares outstanding). For experimental asset markets, observers compare the stock prices to the expected returns from holding the stock (which the experimenter determines and communicates to the traders). In both instances, closed-end country funds and experimental markets, stock prices diverge from fundamental values.

Given the puzzling and violent nature of stock market crashes, it is worth investigating whether it is entirely possible to build a dynamic model of the stock market exhibiting well defined critical points that lie within the strict confines of rational expectations, a landmark of economic theory, and is also intuitively appealing.

A crash happens when large groups of agents place sell orders simultaneously. This group of agents must create enough of an imbalance in the order book for market makers to be unable to balance the sell orders without lowering prices substantially. One curious fact is that the agents in this group typically do not know each other.

Sornette claimed that any model of a crash would combine the following features: A system of noise traders who are influenced by their neighbors. Local imitation propagating spontaneously into global cooperation.Global cooperation among noise traders causing a crash. Prices related to the properties of this system. System parameters evolving slowly through time.

Such a system would display certain characteristics, namely prices following a power law in the neighborhood of some critical date, with either a real or complex critical exponent. What all the models in this class would have in common is that the crash is most likely to occur when the locally imitative system goes through a critical point.

In the spirit of "mean field" theory of collective systems, the simplest way to describe an imitation process is to assume that the hazard rate h(t) evolves according to the following equation $dh / dt = C h^{\delta}$ with $\delta>1$ and $2< \delta < <+ \infty$ -(1) where h(t) is called as the hazard rate defined to be the probability per unit time that the crash would happen in the next instant, provided it has not already happened, δ is the number of market players he exchanges information with, apart from himself. C is a positive constant. The mean field theory amounts to embody the diversity of trader actions by a single effective representative behaviour determined from an average interaction among the traders. The network of friends/media reports could be such an indicator. In this sense, h (t) is the collective result of the interactions among traders. The term h^{δ} in the r.h.s. of (1) accounts for the fact that the hazard rate will increase or decrease due to the presence of *interactions* between the traders. The condition $\delta>1$ is crucial to model interactions and is essential to obtain a singularity (critical point) in finite time. Integrating (1) we get $h(t) = B / (tc – t)^{\alpha}$ with $\alpha \equiv 1 / (\delta – 1)$ - (2)

The critical time t_c is determined by the initial conditions at some origin of time. The exponent α must lie between zero and one for an economic reason: otherwise the price would go to infinity when approaching t_c (if the bubble has not crashed in the mean time). This condition translates into $2 < \delta < + \infty$: a typical trader must be connected to more than one other trader.

The critical time t_c signals the death of the speculative bubble but t_c is not the time of the crash because the crash could happen at any time before t_c, even though this is not very likely.

So t_c is the most probable time of the crash. Thus there exists a finite probability

$1 - \int_{t_0}^{t_c} h(t)\, dt > 0$ - (3) of "landing" smoothly, i.e. of attaining the end of the bubble without crash. This residual probability is crucial for the coherence of the model, because otherwise agents would anticipate the crash and not remain in the market.

As a first-order approximation of the market organization, we assume that traders do their best and price the asset so that a fair game condition holds. Mathematically, this stylized rational expectation model is equivalent to the familiar martingale hypothesis which states that "in a stylised framework of a purely speculative asset that pays no dividends, and ignoring the interest rate, risk aversion, information symmetry and the market-clearing condition, rational expectations are given by: $\forall t` > t$ $E_t[p(t`)] = p(t)$ - (4) where Et[·] denotes the expectation conditional on information revealed up to time t. Furthermore, assuming for simplicity: during a crash, the price drops by a fixed percentage $\kappa \varepsilon (0, 1)$, say between 20 and 30% of the price increase above a reference value p_1. μ is the returns and p(t) refers to price of the asset at time t.Then, the dynamics of the asset price before the crash are given by $dp = \mu(t)\, p(t)\, dt - \kappa[p(t) - p_1]dj$ - (5) where j denotes a jump process whose value is zero before the crash and one afterwards, μ is the returns and p(t) refers to price of the asset at time t. In this simplified model, we neglect interest rate, risk aversion, information asymmetry, and the market clearing condition.

If we do not allow the asset price to fluctuate under the impact of noise, the solution to Equation (5) is a constant: $p(t) = p(t_0)$, where t_0 denotes some initial time. p(t) can be interpreted as the price in excess of the fundamental value of the asset. Taking $E_t [dp] = 0$ and $E_t [dj] = h(t)dt$ on combining the equation with the martingale hypothesis we get

$\mu(t)p(t) = \kappa[p(t) - p_1]\, h(t)$ - (6)

In words, if the crash hazard rate h(t) increases, the return μ increases to compensate the traders for the increasing risk. Plugging (6) into (5), an ordinary differential equation is obtained. For $p(t) - p(t_0) < p(t_0) - p_1$, its solution is $p(t) \approx p(t_0) + \kappa[p(t_0) – p_1] \int_{t_0}^{t} h(t`)\, dt`$ before the crash -(7) The higher the probability of a crash, the faster the price must increase (conditional on having no crash) in order to satisfy the martingale (no free lunch) condition. Intuitively, investors must be compensated by the chance of a higher return in order to be induced to hold an asset that might crash. This effect may go against the naive preconception that price is adversely affected by the probability of the crash. Using (2) into (7) gives the following price law: $p(t) \approx p_c – \kappa B/\beta \times (t_c – t)^{\beta}$ before the crash -(8) where $\beta = 1 – \alpha\ \varepsilon\ (0, 1)$ and p_c is the price at the critical time (conditioned on no crash having been triggered). The price before the crash follows a power law with a finite upper bound p_c. The trend of the price becomes unbounded as we approach the critical date. This is to compensate for an unbounded crash rate in the next instant. Market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory "fingerprints" observable in the stock market prices i.e. market prices contain information on impending crashes. The results obtained by Sornette suggest a weaker form of the "weak efficient market hypothesis", according to which the market prices contain, in addition to the information generally available to all, subtle information formed by the global market that most or all individual traders have not yet learned to decipher and use. Instead of the usual interpretation of the efficient market hypothesis in which traders extract and incorporate consciously (by their actions) all information contained in the market prices, it may be that the market as a whole can exhibit an "emergent" behavior not shared by any of its constituents. In other words, we have in mind the process of the emergence of intelligent behaviors at a macroscopic scale of which the individuals at the microscopic scale have no idea. Blanchard (1979) and Blanchard and Watson (1982) introduced the concept of rational expectation (RE) bubbles, which allow for arbitrary deviations from fundamental prices while keeping the fundamental anchor point of economic modeling. A market crash is a "critical event" or a "phase transition" considering the market to be a physical system. At a critical point one expects a scale invariance to set in. A market can also be considered to be to a hierarchical model with investors which range from the individual small investor to the largest of mutual funds. The stocks in the market then arrange themselves into sectors, subsectors, industries etc - a fibre bundle-like model. As a rule, discrete scaling is connected with such hierarchical models. A mathematical model that encompasses all of these properties is that of log-periodicity.

The suggestion that log-periodicity may be associated with bubbles would thus provide a tool for their characterization and detection. In other words, the suggestion that the conundrum of bubble definition and detection could be resolved by using the log-periodic power law structures as one of the qualifying signatures. Imitation between investors and their herding behavior not only lead to speculative bubbles with accelerating overvaluations of financial markets possibly followed by crashes, but also to "anti-bubbles" with decelerating market devaluations following market peaks. There is thus a certain degree of symmetry between the speculative behavior of the "bull" and "bear" market regimes.

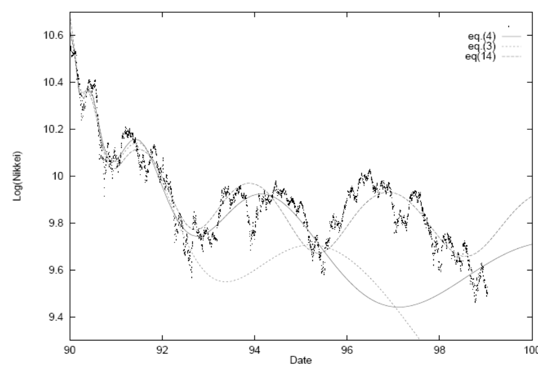
This degree of symmetry, after the critical time t_c, corresponds to the existence of "anti-bubbles," characterized by a power law decrease of the price (or of the logarithm of the price) as a function of time $t>t_c$, down from a maximum at t_c (which is the beginning of the anti-bubble) and by decelerating/expanding log-periodic oscillations.

The classic example of such an anti-bubble is the long-term depression of the Japanese index, the Nikkei that has decreased along a downward path marked by a succession of ups and downs since its all-time high of 31Dec1989. This is however a rather rare occurrence, probably because accelerating markets with log-periodicity often end-up in a crash, a market rupture that thus breaks down the symmetry ($t_c - t$ for $t<t_c$ into $t - t_c$ for $t>t_c$) Herding behavior can occur and progressively weaken from a maximum in "bearish" (decreasing) market phases, even if the preceding "bullish" phase ending at t_c was not characterized by an imitation run-away. The symmetry is thus statistical or global in general and holds in the ensemble rather than for each single case individually. The decrease of the Nikkei index has been analyzed, starting from 1Jan. 1990, using the first-order expression based on discrete scale invariance of stock indices read as $\text{Ln } p(t) \approx A1 + B_1\tau^{\alpha} + C\tau^{\alpha} \text{Cos}(\acute{\omega} \ln(\tau) - \Phi_1)$ - (2) where $\tau = t - t_c$, t_c is the time of the beginning of the anti-bubble. The inclusion of a non-linear quadratic term in the Landau expansion leads to the second-order log-periodic formula

$$\ln p(t) \approx A_2 + \frac{\tau^{\alpha}}{\sqrt{1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}}}\left\{B_2 + C_2 \cos\left[\omega \ln(\tau) + \frac{\Delta_{\omega}}{2\alpha}\ln\left(1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}\right) + \phi_2\right]\right\}$$

- (3) where $\tau = t_c - t$. The log-periodic frequency ω is related to the preferred scaling ratio by $\text{Ln } \lambda = 2\pi/\omega$

These two equations (2, 3) describe the evolution of the price prior to a time t_c, where a large crash may occur, i.e., $t < t_c$. Equation 2 has been found to describe the price evolution up to 3 years prior to large crashes and eq. 3 up to 8 years.



In the figure, we see the logarithm of the Nikkei from 31 Dec. 1989 until 31 Dec. 1998. The fits are equations (2), (3) respectively with all nonlinear variables free for the two equations and where the interval used for the first equation is until mid-1992 and for the second equation until mid-1995. Not only do the equations (2) and (3) agree remarkably well with respect to the parameter values produced by the fits, but they are also in good agreement with previous results obtained from stock market and foreign exchange bubbles with respect to the values of exponent α. Furthermore, the value obtained for $\omega \approx 4.9$ correspond to a scaling ratio $\lambda \approx 3.6$

Application and Implication of Log-Periodic theory

The Bombay Stock Exchange is one of the oldest in Asia and is also the biggest stock exchange in the world in terms of listed companies with 4,800 listed companies as of August 2007. It is therefore a prime indicator of India's growth and economic prosperity. Recently, India has experienced phenomenal economic growth coupled with a high influx of foreign institutional investment (FII) as well as foreign direct investment (FDI). These investors have invested large sums of money in the Indian market and have a considerable influence on the market. If these players were to behave irrationally or respond to other exogenous factors (like the US subprime crisis in early 2008) in ways that would be detrimental to the Indian market, the dangers of the Indian market crashing are indeed high. Therefore it is of importance to check the stability of the Indian market at the present moment, and log-periodicity seems to be a rational testing methodology, both for practical as well as academic purposes

The following is a comprehensive analysis of the Bombay Stock Exchange for any log –periodic trends that might be present in it. It is based partly on my understanding of the various papers that I have had the fortune to study and some assumptions made by me.

Specifications

Econophysics is a field wherein the researchers are mostly physicists. Due to their insufficient knowledge of economics, they are prone to err while interpreting the results that they obtain. The assumptions that they make while hypothesizing models may also not be in line with economic theory. Furthermore physicists are always in search of universal laws which may not always be the case in the social sciences. A law that applies to one system may not be relevant in another. Keeping these points in mind I have tried my best to maintain the spirit of both these disciplines.

I have chosen to fit the acquired data with the given Landau function; however my assumptions of the constants are not without economic justification. In some other cases however, the assumed values are nothing more than accidents that produced results that are hard to ignore. I have presented these results because they open up scope for further research and analysis.

The following are the various cases (Some cases that did not show any semblance to any kind of fit that might seem a logical fit to the data used have not been presented)

For Monthly Data

CASE1

a) On the Y axis we have the monthly average sensex value
b) On the X axis t runs from 1 to 208(t_c). This is not as required by the equation, however this type of fit has also been followed in all other publications on this topic, and thus has been tested for here as well
c) A = 1st value of the monthly average sensex value, B = slope of trend line plotted for the monthly average sensex value

General model

f(x) = 990.72+(x^alpha)*(41.75+C*cos(omega*log(x)-phi))

where f(x) is the monthly average sensex value and x = t (time)

CASE2

a) On the Y axis we have the natural logarithm of the monthly average sensex value
b) On the X axis t runs from 1 to 208(t_c). This is not as required by the equation; however this type of fit has also been followed in all other publications on this topic, and thus has been tested for here as well.
c) A = 1st value of the natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the monthly average sensex value

General model

f(x) = 6.898431952 + (x^alpha) * (0.007 + C * cos (omega * log (x) -phi))

where f(x) is the monthly average sensex value and x = t (time)

CASE3

a) On the y axis we have the monthly average sensex value
b) On the x axis t runs from 208(t_c) to 1. This is as required by the equation
c) A = 1st value of the monthly average sensex value, B = slope of trend line plotted for the monthly average sensex value

General model

f(x) = 990.72+(x^alpha)*(41.75+C*cos(omega*log(x) -phi))

where f(x) is the monthly average sensex value and x = t (time)

CASE4

a) On the y axis we have the monthly average sensex value
b) On the x axis t runs from 208(t_c) to 1. This is as required by the equation
c) A = 1st value of the revered monthly average sensex value, B = slope of trend line plotted for the reverse monthly average sensex value. This

assumption has no economics significance, but was chosen simply because it gives a better fit to the curve

General model

f(x) = 17676.54 +(x^alpha)*(-45.23+C*cos(omega*log(x)-phi))

where f(x) is the monthly average sensex value and x = t (time)

CASE 5

On the y axis we have the natural logarithm of the monthly average sensex value. On the x axis t runs from 208(t_c) to 1. This is as required by the equation: A = 1st value of the natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the monthly average sensex value. **General model** f(x) = 6.898431952 +(x^alpha)*(0.007+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time).

CASE 6

On the y axis we have the natural logarithm of the monthly average sensex value. On the x axis t runs from 208(t_c) to 1. This is as required by the equation A = 1st value of the revered natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the reverse monthly average sensex value. This assumption has no economics significance, but was chosen simply because it gives a better fit to the curve. **General model** f(x) = 9.6945 +(x^alpha)*(-0.007+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)

<u>For Daily Data</u>

CASE1

a) On the y axis we have the daily average sensex value
b) On the x axis t runs from 1 to 1338(t_c). This is not as required by the equation, however this type of fit has also been followed in all other publications on this topic
c) A = 1st value of the daily average sensex value, B = slope of trend line plotted for the daily average sensex value

General model

f(x) = 3388.94 +(x^alpha)*(11.12+C*cos(omega*log(x)-phi))

where f(x) is the daily average sensex value and x = t (time)

CASE2

a) On the y axis we have the natural logarithm of the daily average sensex value
b) On the x axis t runs from 1 to 1338(t_c). This is not as required by the equation, however this type of fit has also been followed in all other publications on this topic
c) A = 1st value of the natural logarithm of the daily average sensex value, B = slope of trend line plotted for the natural logarithm of the daily average sensex value

General model

f(x) = 8.128270992 +(x^alpha)*(0.001+C*cos(omega*log(x)-phi))

where f(x) is the daily average sensex value and x = t (time)

CASE 3

a) On the y axis we have the daily average sensex value
b) On the x axis t runs from 1338(t_c) to 1. This is as required by the equation
c) A = 1st value of the daily average sensex value, B = slope of trend line plotted for the daily average sensex value.

General model

f(x) = 3388.94 +(x^alpha)*(11.12+C*cos(omega*log(x)-phi))

where f(x) is the daily average sensex value and x = t (time)

CASE 4

a) On the y axis we have the daily average sensex value
b) On the x axis t runs from 1338(t_c) to 1. This is as required by the equation
c) A = 1st value of reversed daily average sensex value, B = slope of trend line plotted for the reversed daily average sensex value.

General model

f(x) = 17936+(x^alpha)*(-11.16 +C*cos(omega*log(x)-phi))

where f(x) is the daily average sensex value and x = t (time)

CASE 5

a) On the y axis we have the natural logarithm of the daily average sensex value
b) On the x axis t runs from 1338(t_c) to 1. This is as required by the equation
c) A = 1st value of the natural logarithm of the daily average sensex value, B = slope of trend line plotted for the natural logarithm of the daily average sensex value.

General model

f(x) = 8.128270992 +(x^alpha)*(0.001+C*cos(omega*log(x)-phi)) where f(x) is the daily average sensex value and x = t (time)

CASE 6

On the y axis we have the natural logarithm of the daily average sensex value. On the x axis t runs from 1338(t_c) to 1. This is as per required by the equation. A = 1st value of the natural logarithm reversed daily average sensex value, B = slope of the trend line plotted for the natural logarithm of the reversed daily data.

General model

F(x) = 9.7945+(x^alpha)*(-0.001+C*cos(omega*log(x)-phi)) where f(x) is the daily average sensex value and x = t (time)

II. ANALYSIS OF LOG-PERIODIC TRENDS IN THE BOMBAY STOCK EXCHANGE

Methodology

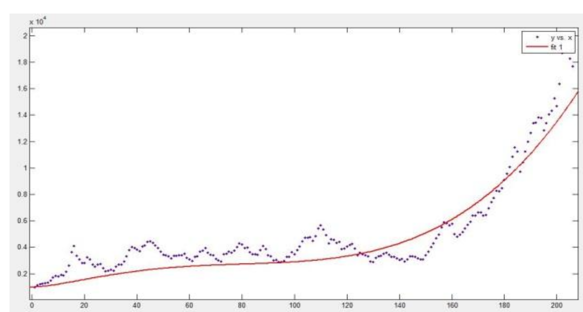
Step 1: Average of monthly and daily, market high and low were taken as a measure of market volatility (Using Microsoft Excel)-**Step 2:** Natural logarithm of theses average values were taken (Using Microsoft Excel) - **Step 3:** The crash date was estimated for both the daily as well as monthly data sets. **Step 4:** Curve fitting of the data to the 1st order Landau expansion was carried out using MATLAB by setting the values of A and B (not in all cases) from which the values of α, ω, Φ and C were estimated. **Step 5:** The hypothesis of log-periodicity was accepted or rejected based on the estimated value of α. If α was estimated to lie between 0 and 1, the hypothesis of log-periodicity was accepted, else it was rejected.

Assumptions Made: The monthly data set was taken from January 1991 because log periodic behavior is expected to have a better fit when data is taken for longer time spans. The crash date for the monthly data set was assumed to be, March 2008 (t_c)because the average Sensex value dropped from 17676.54 to 16228.28, a drop of 8.19%, the maximum in the given time period. Although this does not satisfy the afore mentioned definition of a market crash wherein it was defined a fall in the Sensex for over 10%, this working assumption can be made because we are testing for the presence of log-periodic trends in the market rather than estimate the time of the crash. The daily data was taken from January 2nd 2003 because it is from the year 2003 that the market shows a long run bullish trend. This does not contradict our starting choice of January 1991 as the starting date for the monthly data because of the time scale invariance that the market is said to exhibit according to theory. The crash date for the daily data set was assumed to be, January 21st 2008 (t_c) because the average Sensex value dropped from 17935.54 to 16200.50, a drop of 9.67%, the maximum in the given time period. This gives 208 data points. Although this does not satisfy the afore mentioned definition of a market crash wherein it was defined a fall in the Sensex for over 10%, this working assumption can be made because we are testing for the presence of log-periodic trends in the market rather than estimate the time of the crash. The curve fitting toolbox in the software MATLAB R2007A is robust enough to perform the necessary curve fitting operations. The formulae used for curve fitting were: $I(t) = A + B\tau^{\alpha} + C\tau^{\alpha}\cos(\acute{\omega}\ln(\tau) - \Phi_1)$ and $\ln I(t) = A + B\tau^{\alpha} + C\tau^{\alpha}\cos(\acute{\omega}\ln(\tau) - \Phi_1)$. The values of A and B were estimated where A was assumed to be the first value of the average Sensex value and B the slope of the trendline that was fitted for the data (using Microsoft Excel). The estimate of C is assumed to represent the extent of log-periodic behavior present in the fit. The value of Φ is assumed to represent the whether the market is undergoing an initial upturn or downturn in view of the time scale considered.

The following analysis has been done for monthly data for various cases as mentioned.

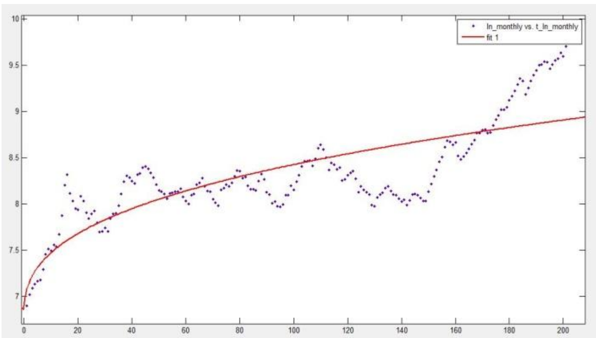
CASE1 **Specification:** On the Y axis we have the monthly average sensex value. On the X axis t runs from 1 to 208(t_c). This is not as required by the equation, however this type of fit has also been followed in all other publications on this topic, and thus has been tested for here as well. A = 1st value of the monthly average sensex value, B = slope of trend line plotted for the monthly average sensex value

General model f(x) = 990.72+(x^alpha)*(41.75+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



Results obtained: Coefficients (with 95% confidence bounds): C = 41.71 ; alpha = 2.266; omega = 0.09055; phi = -2.701 **Interpretation:** alpha>1 thus the data does not exhibit log – periodicity

CASE2 **Specification:** On the Y axis we have the natural logarithm of the monthly average sensex value .On the X axis t runs from 1 to 208(t_c). This is not as required by the equation; however this type of fit has also been followed in all other publications on this topic, and thus has been tested for here as well. A = 1^{st} value of the natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the monthly average sensex value. **General model** f(x) = 6.898431952 +(x^alpha)*(0.007+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



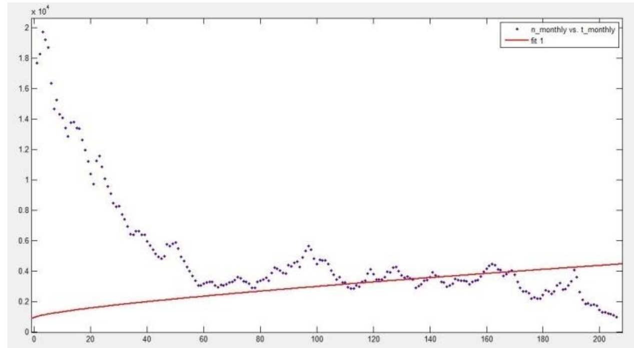
Results obtained: Coefficients (with 95% confidence bounds): C = 14.19; alpha = 0.2711 ; omega = 0.004001 ; phi = 1.559

Interpretation: Alpha is between 0 and 1, thus there is log periodicity. The Value of C is high, at 14.19 which show the importance of the log-periodic character in the data . According to this fit, the market is highly unstable and is on the verge of a crash/has crashed according to the assumption of the crash date

Comments: The graph that has been plotted here is in doubt because the model predicts the presence of log-periodicity; however the graph shows no such trends. The estimated value of omega is very close to zero, thereby causing the cosine part of the Landau function to lose its periodicity. The cosine part of the function behaves in a manner very similar to the '$B\tau^{\alpha}$ 'part of the equation.

CASE3 **Specification:** On the y axis we have the monthly average sensex value. On the x axis t runs from 208(t_c) to 1. This is as required by the equation A = 1^{st} value of the monthly average sensex value, B = slope of trend line plotted for the monthly average sensex value

General model f(x) = 990.72+(x^alpha)*(41.75+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



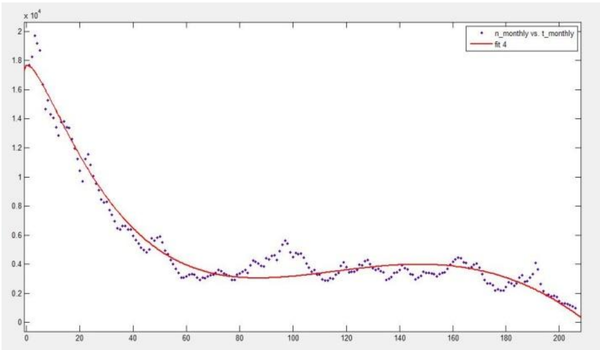
Results obtained: Coefficients (with 95% confidence bounds): C = 18.05 (-6.236e+004, 6.239e+004)
alpha = 0.8925 (-356.5, 358.3); omega = -0.2942 (-503.1, 502.5); phi = 7.009 (-3816, 3830)

Interpretation: 1. Alpha is between 0 and 1. Thus the data does show log-periodic nature. 2. The value of C is high at 18.05, which shows the importance of the log-periodic character in the data. 3. According to this fit, the market is highly unstable and is on the verge of a crash/has crashed according to the assumption of the crash date.

Comments: Even though the results show the presence of log-periodicity, the graph does not seem to exhibit a log-periodic trend. This is because of the choice of A and B, due to which the fit is trying to fall into an overall trend before going into the perturbations. This is evident from the fact that the estimated value of omega is close to zero. This proves that the best estimate for the its value is 0, making the value of cos(0) as 1. This would imply an almost linear fit similar to the $B\tau^{\alpha}$ part of the equation. Thus this choice of A and B is absurd and irrelevant

CASE4 **Specification:** On the y axis we have the monthly average sensex value. On the x axis t runs from 208(t_c) to 1. This is as required by the equation A = 1^{st} value of the revered monthly average sensex value, B = slope of trend line plotted for the reverse monthly average sensex value. This assumption has no economics significance, but was chosen simply because it gives a better fit to the curve

General model f(x) = 17676.54 +(x^alpha)*(-45.23+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



Results obtained: Coefficients (with 95% confidence bounds): C = 40.3; alpha = 1.529;omega = 0.878
phi = 4.672

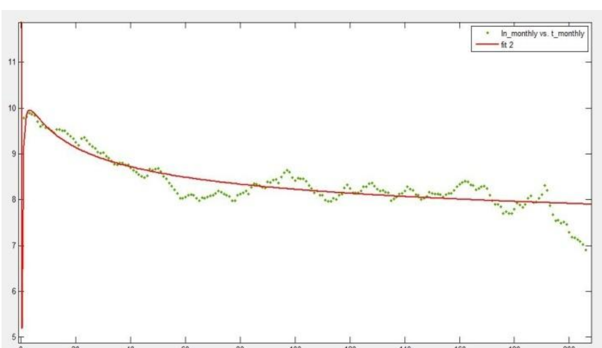
Interpretation: 1. The value of alpha is >1. Thus the data does not exhibit log-periodic behavior

Comments: Compared to the previous fit, this fit is more accurate. Here the values of A and B have been chosen to reflect the choice of direction that the time dimension has taken i.e. backwards (t_c to 1) as dictated by theory.

***CASE 5*__Specification:** On the y axis we have the natural logarithm of the monthly average sensex value

a) On the x axis t runs from 208(t_c) to 1. This is as required by the equation. A = 1st value of the natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the monthly average sensex value.

General model f(x) = 6.898431952 +(x^alpha)*(0.007+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



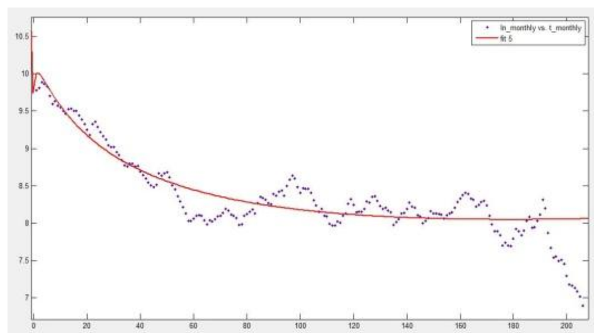
Results obtained: Coefficients (with 95% confidence bounds): C = 23.88; alpha = -0.525;omega = 0.1259 ;phi = 1.479

Interpretation: 1. The value is alpha<0. Thus the data does not exhibit log-periodic behaviour.

Comments: Even though the results show the presence of log-periodicity, the graph does not seem to exhibit a characteristic log-periodic trend. This is because of the choice of A and B due to which the fit is trying to fall into an overall trend before going into the perturbations. This is evident from the fact that the estimate value of omega is close to zero. This proves that the best estimate for the its value is 0, making the value of cos(0) as 1. This would imply an almost linear fit similar to the $B\tau^{\alpha}$ part of the equation. Thus this choice of A and B is absurd and irrelevant.

***CASE 6-*__Specification:** On the y axis we have the natural logarithm of the monthly average sensex value.On the x axis t runs from 208(t_c) to 1. This is as required by the equation. A = 1st value of the revered natural logarithm of the monthly average sensex value, B = slope of trend line plotted for the natural logarithm of the reverse monthly average sensex value. This assumption has no economics significance, but was chosen simply because it gives a better fit to the curve

General model f(x) = 9.6945 +(x^alpha)*(-0.007+C*cos(omega*log(x)-phi)) where f(x) is the monthly average sensex value and x = t (time)



Results obtained: Coefficients (with 95% confidence bounds): C = 0.3171; alpha = 0.3362;omega = 0.6574; phi = -0.2144

Interpretation: 1. Alpha is between 0 and 1. Thus the data does show log-periodic nature. 2. The value of C is low at 0.3171, which shows the growing importance of the log-periodic character in the data. 3. According to this fit, the market is slightly unstable and is on the verge of a crash/has crashed according to the assumption of the crash date**. Comments:** Compared to the previous fit, this fit is more accurate. Here the values of A and B have been chosen to reflect the choice of direction that the time dimension has taken i.e. backwards (t_c to 1) as dictated by theory.

REFERENCES

[1] A. Johansen and D. Sornette, "Significance of log-periodic precursors to financial crashes", Institute of Geophysics and Planetary Physics and Department of Earth and Space Science, , June 26 2001, http://arxiv.org/abs/cond-mat/0106520

[2] Johansen and D. Sornette, "Critical Crashes", Institute of Geophysics and Planetary Physics University of California, Los Angeles, California

[3] Wei-Xing Zhou, Didier Sornette, "Testing the Stability of the 2000 US Stock Market "Antibubble", 16 May 2006, http://www.sciencedirect.com/science?_ob=ArticleURL&_udi=B6TVG-4DNW4RF-4&_user=1556284&_rdoc=1&_fmt=&_orig=search&_sort=d&view=c&_acct=C000053686&_version=1&_urlVersion=0&_userid=1556284&md5=ed187af73505426f1e74c99a56564732

[4] Didier Sornette and Wei-Xing Zhou," The US 2000-2002 Market Descent: How Much Longer and Deeper?", 3 September 2002, http://arxiv.org/abs/cond-mat/0209065

[5] James A. Feigenbaum and Peter G.O. Freund, "Discrete Scale Invariance and the "Second Black Monday"", October 1997, http://adsabs.harvard.edu/abs/1997cond.mat.10324F

[6] S. Drozdz, F. Grummer, F. Ruf, J. Speth "Log-Periodic self similarity: and emerging financial law", 12 January 2008, http://arxiv.org/abs/cond-mat/0209591

[7] Thomas Lux, "Applications Of Statistical Physics In Finance And Economics", Department of Economics University of Kiel, Germany, June 22 2006, http://www2.warwick.ac.uk/fac/soc/wbs/research/wfri/wpaperseries/wf06-256.pdf

[8] Zinn-Justin, Jean ; "Quantum Field Theory and Critical Phenomena", Oxford University Press (2002), accessed using Google Books

[9] Didier Sornette, "Prediction : The Future of the USA Stock Market", 24 November 2003, http://www.safehaven.com/article-934.htm

[10] A. Johansen and D. Sornette, "Financial "Anti-Bubbles": Log-Periodicity in Gold and Nikkei collapses", 22 May 2006, arxiv.org/abs/cond-mat/9901268

[11] J. Voit, "The Statistical Mechanics of Financial Markets", Third Edition, Section 9.4, Springer

[12] James A. Feigenbaum and Peter G.O. Freund, "Discrete Scaling in Stock Markets Before Crashes", 6th September 1995

[13] Didier Sornette, Anders Johansen and Jean-Philippe Bouchaud, "Stock market crashes, Precursors and Replicas", 6th October 1995

[14] Didier Sornette, Anders Johansen, "The NASDAQ crash of April 2000: Yet another example of log-periodicity in a speculative bubble ending in a crash", May 8th 2000

[15] For obtaining data regarding the Bombay Stock Exchange this site was used http://www.bseindia.com/histdata/hindices.asp

[16] S. Drozdz, F. Gr¨ummer, F. Ruf, J. Speth, "Log-periodic self-similarity: an emerging financial law?", 26 September 2002