International Journal of Engineering Trends and Technology (IJETT) – Volume 67 Issue 11- Nov 2019

Optimal-energy distribution of the line travel time on the interstations train-runtimes using numerical method

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Abstract: *Saving energy is an important criterion, and need to be applied since we build a timetable. In the timetable, there are two important parameters: the line travel time and the interstation runtime, directly related to train energy consumption. This paper proposes a new simple, high-performance algorithm, using numerical method, to distribute the line travel time on the interstation runtimes, when the line travel time is given, so that the train power consumption on the line is minimum. The research results of this paper, applied to the no.2A Catlinh-Hadong in Hanoi Vietnam, show that reallocating the timetable, using the proposed algorithm, will save train energy consumption on the interstation on the line, when compared to the planned timetable of the general contractor company.*

Keywords*:* *optimal energy, timetable, schedule, optimal distribution, numerical method, subway, metro, electric train.*

I. INTRODUCTION

Nowadays, electric subway trains are widely used and popular in many cities in the world, due to the large transport capacity, punctuality and safety. However, the electric subway trains consume a very high power, of which nearly 70% is used to create the electric traction for the train-movement on the inter-stations on the rail-line [1]. Therefore, the development and implementation of energy-saving methods on electric subway trains is very essential. The modern metro subway train system today is equipped with the automatic train operation (ATO) system, in combination with the automatic train protection (ATP) system and the automatic train supervision (ATS), and closely linked together using the communications-based train control (CBTC) system [1,2,3].

ATS ensures that the metro trains run punctually under the planned-timetable on the line. The train-timetable is very important for the subway operation. The planning and creating the train-timetable is carried out before the subway trains are put into operation in reality and optimization of energy saving goals, minimization the train operating costs. The train-timetable contains the following main information: train arrival times at the platforms, train stopping times at platforms, train interval times, the occupancy of the trains, train runtime of each interstation [2,3]. Thus, the train runtime of each interstation is a very important parameter directly related to the power consumption of the electric subway trains. Therefore, it is essential to determine the optimal-energy train runtime of each interstation to minimize the total power consumption of the electric subway trains on the line. The goals of optimization the train-timetable are related to the optimal-energy train runtime on the interstation, the train travel time [4,5], total energy consumption [6]. The other published researches on this issue give general principles [1,2] or basic principles applied to subway trains using mechanical brakes [2,7], not to mention electric brakes. The researches [1-10] have not yet provided the clear algorithm to determine the effective train run-time of each interstation on the urban rail line. Further, in daily subway operation, the timetable is always disrupted by various perturbation (equipment failure, extreme weather, slight accident, mutant passengers, and so on) that may cause the infeasibility of the original plan [11,12]. Therefore, it is necessary to reschedule or adjust the train planned timetable with the real-time train information and perturbation estimations. The research [13] proposed the solution to reschedule the planned-timetable and rearrange trains so that it is not conflictive. The research [15,16] assists the dispatchers to optimize transport capacity, train arriving punctuality and avoid conflicts with other trains. The researches [15,16] deal with selecting the train arrival times, train departure times at each platform, limit speed at each segment. However, these researches proposed the solutions which are implemented in practice are very complex.

This paper presents a simple efficient method to distribute the line travel time on the interstation train runtimes in order to minimize the total power consumption of the subway train on the line using numerical method. The structure of this paper includes five parts. After the first part - introduction, the second part is formation research problem. The third part presents the numerical method and then a new simply high-performance algorithm to solve this problem: optimal-energy distribution of the line travel time on the interstation train runtime. The results of the optimal-energy distribution problem of the line travel time are given in the fourth part.

Finally the conclusions and future studies of the paper are presented in the fifth part.

II. FORMATION OF RESEARCH PROBLEM

In daily operation of subway trains, the timetable is the original legal base for ATS to send the train runtime of each interstation to the ATO equipment in order to optimal-energy control train on each inter-station. From this, a given issue needs to be considered is with the train runtimes of all inter-stations on the line, the power consumption of the train running on the line was minimal or not. Further, the planned-timetable is often interrupted due to various disturbances in the actual train operation, causing some delays in the planned train schedule. Therefore, it is necessary to redistribute the line travel time on the interstations train runtimes (reschedule the timetable) so as to minimize the train power consumption on the line.

It is assumed that there is a relationship value table of the minimum power consumption and train runtime on each interstation of the line, based on the results of the optimal-energy train control problem on interstation [9,10,14,17]. This means that for each individual train runs on the *i*-th interstation of the line, it is obtained a relationship table of the power consumption $A_{Ei}[kWh]$ and train runtime $T_{Xi}[sec]$, calculated with quantity of $(m+1)$ given train-runtimes, increment $\Delta T[sec]$, within $T_{Xi}^{\min} \le T_{Xi} \le T_{Xi}^{\max}$, as described in Tab.1.

Tab.1. Electricity consumption and train runtime on the i-th inter-station

No. j_i	**Runtime on interstation, T_{Xi}**	**Power consumption, $A_{Ei}(T_{Xi})$**	**Power increment $\Delta A_i(j_i)$**
0	$T_{Xi}^{\max}$	$A_{Ei}^{\min}(T_{Xi}^{\max})$	0
1	$T_{Xi}^{\max} - \Delta T$	$A_{Ei}(T_{Xi}^{\max} - \Delta T)$	$\Delta A_i(1)$
2	$T_{Xi}^{\max} - 2\Delta T$	$A_{Ei}(T_{Xi}^{\max} - 2\Delta T)$	$\Delta A_i(2)$
…	…	…	…
j_i	$T_{Xi}^{\max} - j_i\Delta T$	$A_{Ei}(T_{Xi}^{\max} - j_i\Delta T)$	$\Delta A_i(j_i)$
…	…	…	…
m	$T_{Xi}^{\min} = T_{Xi}^{\max} - m\Delta T$	$A_{Ei}^{\max}(T_{Xi}^{\max} - m\Delta T)$	$\Delta A_i(m)$

Formation of the research problem in this paper. Research in this paper is conducted on the subway rail line with $(n+1)$ stations, in two directions (way 1 and way 2). On each way, such as way 1, there is quantity of n interstations; the *i-th* interstation corresponds to rail-track between B_i platform and B_{i+1} platform, with the given train runtime $T_{Xi}[sec]$, as described in Fig.1.

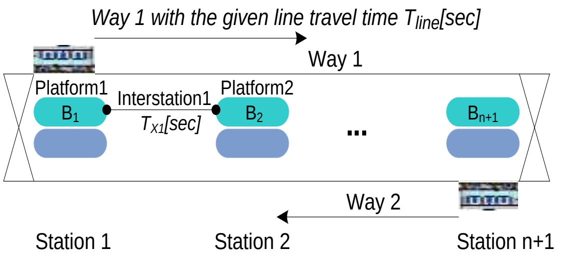

Fig.1 Description of the subway rail line

In actual subway train operation, the line travel time is given $T_{line}[sec]$, based on the planned timetable. The line travel time which train runs from the first platform to the last platform is equal to the sum of all train runtimes on all interstations, as skipping the train stopping-times at the platforms. The line travel time is determined by the following formula (1).

$$T_{line} = \sum_{i=1}^{n} T_{Xi} \tag{1}$$

The total power consumption of the train when the train moves through each interstation on the line with quantity of the n interstation, is determined by the formula (2).

$$A_{line}(T_{line}) = \sum_{i=1}^{n} A_{Ei}(T_{Xi}) \tag{2}$$

Requirement of the research problem in this paper: to determine the train runtime on each interstation in order to reschedule the train timetable so that total train power consumption on the line is minimum.

III. A SIMPLY METHOD FOR OPTIMAL-ENERGY DISTRIBUTION OF THE LINE TRAVEL TIME ON THE INTERSTATION TRAIN RUNTIMES

Optimal energy distribution problem of the line travel time on the interstation train runtime in this paper is solved based on numerical method. The objective function in this study is total energy consumption of the subway train on the line. Hence, optimal energy distribution problem of the line travel time can be expressed as below:

$$\underset{T_{Xi}}{\text{minimize}}\ A_{line}(T_{line}) = \sum_{i=1}^{n} A_{Ei}(T_{Xi}) \tag{3}$$

subject to

$$T_{Xi}^{\min} \le T_{Xi} \le T_{Xi}^{\max}, \tag{4}$$

$$\sum_{i=1}^{n} T_{Xi} = T_{line}, \tag{5}$$

$$T_{Xi} = T_{Xi}^{\min} + j_i\Delta T,\ 0 \le j_i \le m \tag{6}$$

The task of optimal energy distribution problem of the line travel time as: given relationship value table of the minimum power consumption and the train runtime on each interstation of the line (Tab.1), request to find the train runtimes of all interstations of the line, constrained by expression (4), to minimize train power consumption on the line (3), when the line train travel time is given by (5) and the interstation train runtimes (6) is calculated with the increment *ΔT[sec]*, here for subway *ΔT=5* and *m=5*.

From (1), the minimum train travel time on the line is calculated by formula (7).

$$T_{line}^{\min} = \sum_{i=1}^{n} T_{Xi}^{\min} \qquad (7)$$

The maximum train travel time on the line is calculated by formula (8).

$$T_{line}^{\max} = \sum_{i=1}^{n} T_{Xi}^{\max} \qquad (8)$$

Following (6), on each subway interstation of the line, we calculated the minimum electrical power consumptions of the train with the number of 5 runtimes, *m=5*, *ΔT=5*, as presented in Tab.1.

$$T_{Xi} = T_{Xi}^{\max} - j_i \Delta T \quad 0 \le j_i \le m \qquad (9)$$

Therefore, the maximum train runtime on each interstation is calculated by formula (10).

$$T_{Xi}^{\max} = T_{Xi}^{\min} + m\Delta T \qquad (10)$$

And the maximum train travel time on the line can be determined as (11).

$$T_{line}^{\max} = \sum_{i=1}^{n} T_{Xi}^{\max} = \sum_{i=1}^{n} (T_{Xi}^{\min} + m\Delta T) = T_{line}^{\min} + mn\Delta T \qquad (11)$$

The train travel time on the line is calculated by formula (12).

$$T_{line} = \sum_{i=1}^{n} T_{Xi} = \sum_{i=1}^{n} (T_{Xi}^{\max} - j_i \Delta T) = T_{line}^{\max} - \Delta T \sum_{i=1}^{n} j_i \qquad (12)$$

From Tab.1., when the train runtime on the *i-th* interstation decreases by an amount of *ΔT*, the train power consumption increases by an amount of $\Delta A_i(j)$ with $j_i=1,2,..m$; and $\Delta A_i(0)=0$ as formula (13).

$$\Delta A_i(j_i) = A_{Ei}(T_{Xi}^{\max} - j_i \Delta T) - A_{Ei}(T_{Xi}^{\max} - (j_i - 1)\Delta T) \qquad (13)$$

Since $A_{Ei}(T_{Xi})$ is a monotonically decreasing function, $\Delta A_i(j_i)$ monotonically increases with increasing j_i.

Further, it is clear that the minimum train power consumption on the line corresponds to the maximum train runtimes on all interstation as formula (14).

$$A_{line}^{\min}(T_{line}^{\max}) = \sum_{i=1}^{n} A_{Ei}^{\min}(T_{Xi}^{\max}) \qquad (14)$$

And the maximum train power consumption on the line corresponds to the minimum train runtimes on all interstation as (15).

$$A_{line}^{\max}(T_{line}^{\min}) = \sum_{i=1}^{n} A_{Ei}^{\max}(T_{Xi}^{\min}) \qquad (15)$$

Besides, the train power consumption on the *i-th* interstation is calculated by the formula (16).

$$A_{Ei}(T_{Xi} = T_{Xi}^{\max} - j_i \Delta T) = A_{Ei}^{\min} + \sum_{j=1}^{j_i} \Delta A_i(j) \qquad (16)$$

Therefore, the train power consumption on the line is calculated by the formula (17).

$$A_{line}(T_{line} = \sum_{i=1}^{n} (T_{Xi}^{\max} - j_i \Delta T)) = \sum_{i=1}^{n} A_{Ei}^{\min} + \sum_{i=1}^{n} \sum_{j=1}^{j_i} \Delta A_i(j) \qquad (17)$$

where $\Delta A_i(j)$ is determined for each the *i-th* interstation by using formula (13).

Let the train travel time on the line be defined equal to $T_{line}^{\max} - \Delta T$, then minimum train power consumption on the line as

$$\min\{A_{line}(T_{line}^{\max} - \Delta T)\} = A_{line}^{\min} + \min_i\{\Delta A_i(k)\} \qquad (18)$$

where k is an integer value, satisfying $T_{line}^{\min} \le T_{line} \le T_{line}^{\max}$

The $A_{line}^{\min}$ and $\Delta A_i(k)$ values are calculated from Tab.1 on each interstation by formulas (13) and (14).

The objective function (3) achieve the minimum on all interstations, i.e. find on which interstation it is necessary to increase the train runtime by *ΔT* from the minimun interstation train runtime $T_{Xi}^{\min}$. And here, the optimal energy distribution problem of the line train travel time $T_{line}^{\max} - \Delta T$ on the interstation train runtime is solved by decreasing the interstation train runtime by *ΔT*. In this case, the train power comsumption during train movement along the line is minimum, and the number of calculation variants is also equal to the number of interstations, *n*.

Implementing continuously the optimal energy distribution problem of the line train travel time $T_{line}^{\max} - 2\Delta T$ on the interstation train runtime, is reduced to solving the previous problem $T_{line}^{\max} - \Delta T$. Indeed, equation (18) takes the form as

$$\begin{aligned} &\min\{A_{line}(T_{line}^{\max} - 2\Delta T)\} = \\ &= \min\{A_{line}(T_{line}^{\max} - \Delta T)\} + \min_i\{\Delta A_i(d)\} \end{aligned} \qquad (19)$$

where *d=1* for the interstation *i-th* in which the runtimes have not been reduced by *ΔT* in the previous problem. The value *d=2* for interstation in which the runtimes have been reduced by *ΔT* in the previous problem.

It is clear that the first part on the right of (19) was calculated in the previous problem.

Having the second part on the right of (19) for *i=1...n* and compared the results. Let us decrease the interstation train runtime by ΔT on the interstation on which the right part (19) is minimum. The number of calculation variants in this case is also equal to *n*.

Analysing similar to above, the optimal energy distribution problem of the line train travel time $T_{line}^{\max} - 3\Delta T$ and then $T_{line} = T_{line}^{\max} - j_i \Delta T$ on the interstation train runtime, is reduced to solving the previous problem $T_{line}^{\max} - \Delta T$.

Here note that the distribution of $T_{line}^{\max}, T_{line}^{\min}$ is not required. Therefore, the optimal energy distribution problem for the number of (k-2) of the line train travel time on the intersection train runtime when given the line train travel time, within the range $T_{line}^{\max} \div T_{line}^{\min}$ with interval *ΔT*, it is necessary to find in the *i*-th cycle from *1* to *n*, and having all $(k-2)n$ enumeration variants.

As a result, based on the principle of the above numerical method, we obtained a optimal value set of the train runtimes on each interstation, making minimum power consumption of the train on the line, as given line travel time T_{line}

$$T_{Xi}^{*}=\{T_{X1}^{*},T_{X2}^{*},...,T_{Xn}^{*}\} \qquad (20)$$

And optimal total power consumption

$$A_{line}^{*}=A_{line}^{*}(T_{line}^{*})=\sum_{i=1}^{n}A_{Ei}^{*}(T_{Xi}^{*}) \qquad (21)$$

The basic algorithm of the optimal energy distribution problem of the line train travel time on the interstation train runtime is given in Algorithm 1.

Algorithm 1. The optimal energy distribution problem based on numerical method.

```
Data: $A_{Ei}(T_{Xi})$, $T_{line}$, $\Delta T$, m, n
Initialize: i=1, j=0
    Calculate $\Delta A(i,j)=\Delta A_i(j_i)$, $T_{line}^{max}, T_{line}^{min}$
Check: $T_{line}$ within $T_{line}^{max} \div T_{line}^{min}$
Set up: j[1..n]=1
    Assign: $\Delta A_{col}=\Delta A(i,j(i))$
    Seek: $A_{line}^{min}$ in matrix $\Delta A_{col}$.
    Then, getting position j(i) corresponding to $A_{line}^{min}$ and the line travel time $T_{xl}$.
    Increase: j(i)
    Compare: $T_{xl}=T_{line}$?
Finish: when $T_{xl}=T_{line}$ we obtain $A_{line}^{*}$, $T_{Xi}^{*}$
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IV. THE RESULTS OF THE OPTIMAL-ENERGY DISTRIBUTION OF THE LINE TRAVEL TIME ON LINE NO.2A IN HANOI VIETNAM

Researches in this paper is carried out on the subway line no.2A Cat Linh - Ha Dong in Hanoi, Vietnam. These subway trains use three-phase AC traction motor. These trains are formed by 4 cars (2 cars with cabin at the ends and 2 cars with the AC traction motor in the middle) and maximum 8 cars depending on the amount of passengers. These trains use a third rail to receive DC power with a rated voltage of 750VDC [18].

Currently, this subway line no.2A is in the stage of technical test running, not yet put into commercial exploitation for passenger service.

Analysing the interstations of the subway rail line no.2A Cat Linh-Ha Dong [18], we see that this line consists of 12 stations, 22 interstation allocated by two way: way 1 from Cat Linh station to Yen Nghia station and way 2 from Yen Nghia station to Cat Linh station, as described in Fig.3. The total length of the line in both directions of travel and return is 25356 meters, the length of the interstation varies from 930 meters to 1447 meters. The slope of the 22 interstations is in the range from -23‰ to +23‰. Tab.2 presents the characteristics of the interstations of the way 1. Tab.3 presents characteristics of the interstations of the way 2 [18].

The algorithm of the optimal energy distribution problem of the line train travel time on the interstation train runtime on this subway line no.2A is presented in Fig.2.

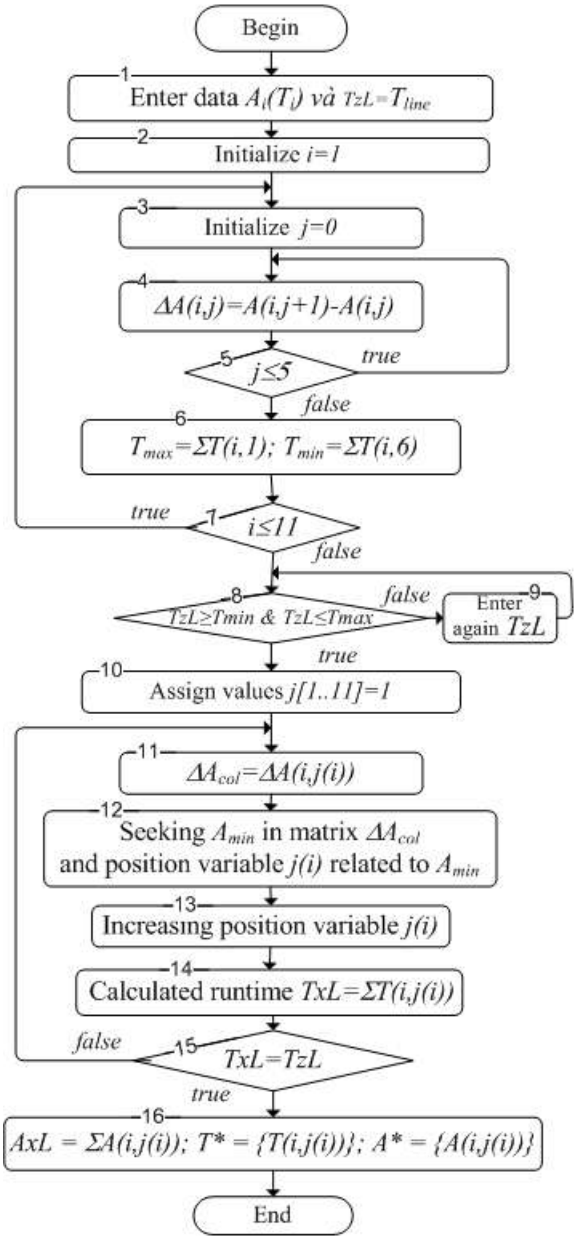
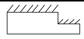
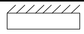
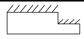
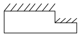
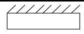
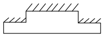
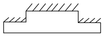
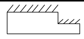
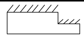
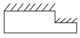
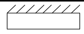


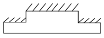
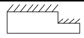
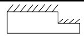
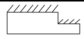
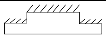
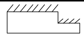
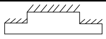
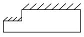
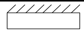
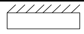
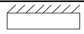
Fig.2. The algorithm of the optimal energy distribution problem on the subway rail line no.2A Cat Linh – Ha Dong in Hanoi, Vietnam

According to the planned timetable by the general contractor [18], the total line travel time on way 1 in the peak time frame $T_{line}=885[s]$, the low-time frame $T_{line}=985[s]$; and on way 2 in the opposite direction of Yen Nghia - Cat Linh in the peak time frame $T_{line}=870[s]$, the low time frame $T_{line}=970[s]$.

Tab.2. The characteristics of the interstations of the way 1 [18]

Interstation name	**Profile form**	**Interstation length (m)**	**Min/max slope (‰)**	**Type of speed limit**
Cat Linh – La Thanh	+	930	-11/+17	
La Thanh – Thai Hà	-	928.5	-8/+12.2	
Thai Ha – Lang	+	1035	-15/+11.35	
Lang – Thuong Dinh	+	1250	-17/+10.27	
Thuong Dinh – Vanh Dai 3	*	1055	-22/+23	
Vanh Dai 3 – Phung Khoang	*	1442	-20/+15	
Phung Khoang – Van Quan	*	1125	-23/+21	
Van Quan – Ha Đông	-	1230	-6.5/+5.95	
Ha Dong – La Khe	+	1235	-15/+15.5	
La Khe – Van Khe	+	1420	-15/+12.1	
Van Khe – Yen Nghia	*	1026.5	-21.5/+22	

Tab.3. The characteristics of the interstations of the way 2 [18]

Interstation name	**Profile form**	**Interstation length (m)**	**Min/max slope (‰)**	**Type of speed limit**
Yen Nghia – Van Khe	*	1015	-22/+21.5	
Van Khe – La Khe	+	1419	-12.1/+15	
La Khe – Ha Dong	+	1239	14.5/14.99	
Ha Dong – Van Quan	-	1231	-3.22/+6.5	
Van Quan – Phung Khoang	*	1133	-21/0	
Phung Khoang –Vanh Đai 3	+	1447	-15/+20	
Vanh Dai 3 – Thuong Dinh	*	1050	-23/+22	
Thuong Dinh – Lang	+	1245	-10.3/+16.9	
Lang – Thai Ha	+	1031	-11.5/+15	
Thai Ha – La Thanh	+	932	-12.3/+8	
La Thanh – Cat Linh	+	937	-16.9/+10.9	

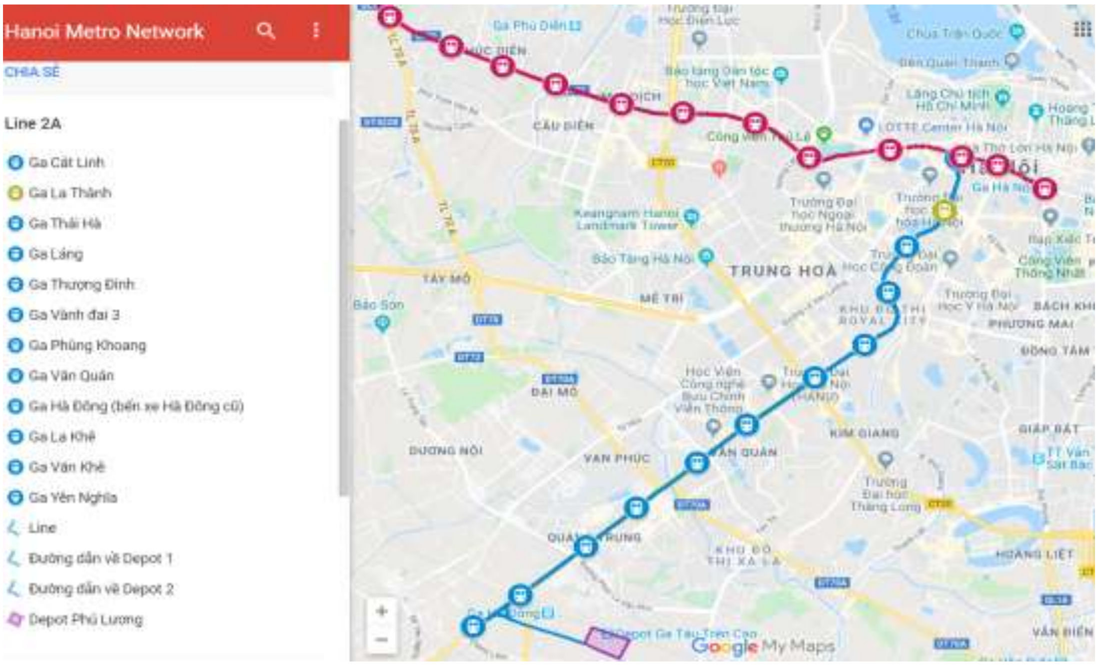


Fig.3. The overview line no.2A Cat Linh – Ha Dong in the Hanoi subway system

The results of the optimal energy distribution problem on the line no.2A is carried out with T_{line} on way 1 and way 2; in the peak-time frame and in the low-time frame; and the values table of the $A_{Ei}(T_{Xi})$ calculated based on the optimal-energy control train on interstation with the regenerative braking energy recovery 25% and non-recovery [9,10,14,17].

The results of the optimal energy distribution problem on the line no.2A are given in Tab.4, Tab.5.

Tab.4. Optimal-energy distribution of the line trave time on way 1 of the line no.2A Hanoi, Vietnam.

No.	Interstation name	Peak-time			Low-time		
		Tx Sched	*Tx η_0*	*Tx η_25%*	*Tx Sched*	*Tx η_0*	*Tx η_25%*
1	Cat Linh-La Thanh	75	75	75	85	85	85
2	La Thanh-Thai Ha	80	80	80	85	95	95
3	Thai Ha-Lang	75	70	70	80	75	75
4	Lang-Thuong Dinh	75	75	75	80	85	85
5	Thuong Dinh-Vanh Dai3	80	80	80	90	85	90
6	Vanh Dai3-Phung Khoang	70	70	70	80	80	80
7	Phung Khoang-Van Quan	80	80	80	90	85	85
8	Van Quan-Ha Dong	75	75	75	90	80	80
9	Ha Dong-La Khe	65	60	60	75	70	70
10	La Khe-Van Khe	95	100	100	105	110	105
11	Van Khe-Yen Nghia	115	120	120	125	135	135
	Line travel time *T_line*	**885**			**985**		

Tab.5. Optimal-energy distribution of the line trave time on way 2 of the line no.2A Hanoi, Vietnam.

No.	Interstation name	Peak-time			Low-time		
		Tx Sched	*Tx η_0*	*Tx η_25%*	*Tx Sched*	*Tx η_0*	*Tx η_25%*
12	Yen Nghia-Van Khe	90	90	90	95	100	100
13	Van Khe-La Khe	55	55	55	60	65	65
14	La Khe-Ha Dong	110	125	125	130	135	135
15	Ha Dong-Van Quan	90	90	90	95	100	100
16	Van Quan-Phung Khoang	70	65	65	75	75	75
17	Phung Khoang-Vanh Dai3	75	75	75	85	85	85
18	Vanh Dai3-Thuong Dinh	75	75	75	85	85	85
19	Thuong Dinh-Lang	75	75	75	85	80	80
20	Lang-Thai Ha	85	80	80	100	90	90
21	Thai Ha-La Thanh	75	75	75	80	80	80
22	La Thanh-Cat Linh	70	65	65	80	75	75
	Line travel time *T_line*	**870**			**970**		

Based on the optimal-energy distribution of the line trave time on the line no.2A Hanoi-Vietnam in both directions, we have the following comments:

- The planned runtimes that the general contractor company given, have not yet optimized the energy consumption of the train on the line.
- Based on the set of the optimal train runtimes on all interstations, it is possible to rearrange the train timetable to save the train power consumption on the subway line no.2A Hanoi.
- The deviation of the optimal train runtimes on all interstations, compared with the planned runtimes, in low-time frame, occurs in more interstations with greater values than in peak-time frame;
- When applying the regenerative braking energy recovery, assuming the energy return rate 25%, the optimal train runtimes on all interstations will change more than as non-recovery.

V. CONCLUSIONS

Timetable, with two important parameters: line travel time and interstation runtimes, is the legal base for dispatching trains on the line. Therefore, these times need to be determined so that the train power consumption is minimal. Based on numerical method, a new simple high-performance algorithm has been developed in this paper, to distribute the line train travel time on the interstation train-runtime so as to minimize the total power consumption of the train on the line.

The paper presented the overview principle to solve the optimal - energy distribution problem of the line train travel time on the interstation train runtimes, and also given a new detail algorithm, applied on the subway line no.2A Catlinh-Hadong in Hanoi-Vietnam. The research results of this paper, calculated using the computer program on Matlab, showed clearly the efficiency of the proposed algorithm: Let's get the optimal runtimes on all interstations so that the train electrical power consumption on the line is minimum; and easily reschedule the energy-saving timetable at any time affected by disturbances on the line.

The future investigation will focus on developing the energy optimal distribution algorithms using the linear programming, neural network, and apply to practical system.

ACKNOWLEDGMENT

This research is funded by University of transport and communications (UTC) under grant number T2019-DT-013.

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