Investigation of the Nature of Universe and Kasner Solution on Bianchi Type V Cosmology

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Abstract — *In this paper we have searched the existence of Bianchi Type V cosmological model with power law expansion. We have also discussed the nature of universe and physical properties of the Universe. Einstein's field equations are solved by Bianchi Type-v metric for equation of state. It is also examined the cosmological dynamics of the anisotropic Universe with Kasner solution.*

Keywords — *Bianchi Type - V metric, Power law expansion, Kasner solution.*

I. INTRODUCTION

The Bianchi-V cosmology is a generalization of FRW cosmology. Several authorsYadav, Kumar, Suresh 2011 in [1] and Bali, Raj, Pratibha Singh, and J. P. Singh 2012 in [2] have studied Bianchi-V cosmological model using different physical parameter. It is known from astronomical observations of SNe Ia(Riess et al.1998)[3] that the Universe is undergoing an accelerated expansion and it is not cleared why the Universe is expanding. Using modified theory of Gravity and existence of Dark energy. Many authors (Caldwell et al.2006) [4] have studied several cosmological model. In the study of Dark energy the cosmological constant is assumed to be the simplest candidate. For this assumption two problems occurs on theoretical grounds, which are tuning and cosmic coincidence. The Bianchi-Vcosmology describes a homogeneous and anisotropic Universe that has different scale factors along each spatial direction. That's why it is a particular interest on study with Bianchi-V cosmological model. For some suitable case Bianchi-V model also convert to Bianchi type I model. Singh, Chaubey (2007) in [5] have studied the evolution of a homogeneous, anisotropic universe given by a Bianchi Type-V cosmological model filled with viscous fluid, in the presence of cosmological constant Λ. Singh, J.P. & Baghel (2009) in [6] investigated Bianchi type V cosmological models with bulk viscous fluid source. Exact solutions of the Einstein field equations are presented via a suitable power law assumption for the Hubble parameter. Lorenz, Dieter (1981) [7] presented a cosmological solution of the source-free Einstein-Maxwell equations with "stiff" matter and an electromagnetic null field, which is a locally rotationally symmetric tilted Bianchi type-V universe.Beesham (1986) in [8] derived tilted Bianchi type V cosmological model in the scale covariant theory. The solution for radiation is discussed separately from that for the non radioactive case.Singh, C. P., Mohd Zeyauddin, and Shri Ram (2008) in [9] studied the variation law for generalized mean Hubble's parameter in a spatially homogeneous and anisotropic Bianchi type V space-time with perfect fluid along with heat-conduction.

Einstein explains gravitation in his general theory of Relativity with the help of geometry of space time.Inspired by Einstein Weyl [10] proposed a new modification of Einstein's theory of Relativity including gravitation and electromagnetism. But it is not taken seriously for nonintegrability of length transfer. Under vector parallel transport Lyra[11] proposed a new modification of Riemannian Geometry including gauge function later. The Bianchi-V cosmological model with Lyra geometry under different parameters have studied by Larenz[12]. Some of authors have found contracting universes from infinite volume to zero volume.

In relativistic cosmology [13-17] kasner solution is a first known exact solution. It is one of the most important exact solutions in General relativity also. The kasner solution is a good approximation to a cosmological singularity for almost all matter sources except for a stiff fluid though it is a vacuum solution. It is expect that kasner solution should change the behavior near a cosmological singularity. For investigating this kasner solution in modified gravity is important area. for an anisotropically expanding Universe with scale factors Kasner solution is a solution changing as powers of time. In kasner solution there are two different types of situations in case of power exponents. The power law solution for scale factor is an asymptotic solution if the equation of motion is second order. GR kasner solutionare different from two conditions of power law.

II. FIELD EQUATIONS

We consider the Bianchi Type V metric of the form

$$ds^2 = dt^2 - \sum_{i=1}^{3} e^{2a_i x}\, b_i^2 dx_i^2 \quad \dots\dots\dots\dots\dots\dots (1)$$

where a_i s are constants, $b_i = b_i(t)$ are scale factors in x, y and z directions, $i = 1, 2, 3$. We choose

$a_1 = 0, a_2 = a_2 = a$

Einstein's field equations are represented by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G\, T_{ij} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

We take the perfect fluid form for the energy momentum tensor as

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

together with commoving coordinates $u_i u^i = 1$.

The equation of state for the fluid is considered as

$$p = \gamma \rho \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

where γ is a constant and $0 \leq \gamma \leq 1$.

The field equations are

$$\frac{\ddot{b_1}}{b_1} + \frac{\ddot{b_2}}{b_2} + \frac{\dot{b_1}\dot{b_2}}{b_1 b_2} - \frac{{a_i}^2}{{b_1}^2} = -\chi p \ldots \ldots \ldots (5)$$

$$\frac{\ddot{b_1}}{b_1} + \frac{\ddot{b_3}}{b_3} + \frac{\dot{b_1}\dot{b_3}}{b_1 b_3} - \frac{{a_i}^2}{{b_1}^2} = -\chi p \ldots \ldots \ldots \ldots (6)$$

$$\frac{\ddot{b_2}}{b_2} + \frac{\ddot{b_3}}{b_3} + \frac{\dot{b_2}\dot{b_3}}{b_2 b_3} - \frac{{a_i}^2}{{b_1}^2} = -\chi p \ldots \ldots \ldots \ldots (7)$$

$$\frac{\ddot{b_1}}{b_1} + \frac{\ddot{b_2}}{b_2} + \frac{\dot{b_1}\dot{b_2}}{b_1 b_2} - \frac{{a_i}^2}{{b_1}^2} = -\chi p \ldots \ldots \ldots \ldots (8)$$

$$\frac{2\dot{b_1}}{b_1} - \frac{\dot{b_2}}{b_2} - \frac{\dot{b_3}}{b_3} = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9)$$

The Expansion scalar (θ) and shear scalar (σ^2) have the form

$$\theta = 3H = \frac{\dot{b_1}}{b_1} + \frac{\dot{b_2}}{b_2} + \frac{\dot{b_3}}{b_3} \ldots \ldots \ldots \ldots \ldots \ldots (10)$$

$$\sigma^2 = \left[\frac{{\dot{b_1}}^2}{{b_1}^2} + \frac{{\dot{b_2}}^2}{{b_2}^2} + \frac{{\dot{b_3}}^2}{{b_3}^2} \right] - \frac{\theta^2}{3} \ldots \ldots \ldots (11)$$

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III. SOLUTIONS OF FIELD EQUATIONS

Using equation state (4) and defining

$$V^3 = b_1 b_2 b_3 e^{2ax} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (12)$$

Integrating (9)

$$ {b_1}^2 = b_2 b_3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (13)$$

In the literature, the law of variation of Hubble parameter which gives the constant value of the deceleration parameter. This law deals with two types of cosmology: Power law cosmology and Exponential law cosmology. It is known that the exponential law represent the dynamic of future universe and such type of model does not have consistency with present day observations.

We define average scale factor (R) and mean Hubble parameter(H) of Bianchi Type V cosmological model as

$$R = (b_1 b_2 b_3)^{\frac{1}{3}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)$$

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \ldots \ldots \ldots \ldots (15)$$

Where $H_1 = \frac{\dot{b_1}}{b_1}, H_2 = \frac{\dot{b_2}}{b_2}, H_3 = \frac{\dot{b_3}}{b_3}$ are the directional Hubble parameters in the direction of x, y and z respectively.

From (5) and (6) ,

$$\frac{b_2}{b_3} = A \exp\left(x_1 \int \frac{dt}{b_1 b_2 b_3} \right) \ldots \ldots \ldots \ldots \ldots \ldots (16)$$

where A and x_1 are constants of integration.

Integrating (9) we get

$$ {b_1}^2 = b_2 b_3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (17)$$

$$b_1 = (nDt)^{\frac{1}{n}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (18)$$

where D is constant.

$$\dot{b_1} = \frac{1}{n}(nDt)^{\frac{1}{n}-1} \cdot nD$$

$$\ddot{b_1} = D \cdot \left(\frac{1}{n} - 1 \right) (nDt)^{\frac{1}{n}-2} \cdot nD$$

$$= nD^2 \cdot \frac{1-n}{n} \cdot (nDt)^{\frac{1-2n}{n}}$$

$$= D^2 (1-n)(nDt)^{\frac{1-2n}{n}}$$

$$b_2 = \sqrt{A}(nDt)^{\frac{1}{n}} \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right] \ldots \ldots \ldots \ldots (19)$$

$$\dot{b_2} = \sqrt{A} \cdot \frac{1}{n} \cdot (nDt)^{\frac{1}{n}-1} \cdot nD \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right] + \sqrt{A} \cdot (nDt)^{\frac{1}{n}} \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right] \cdot \frac{x_1}{2D(n-3)} \cdot (nD)^{\frac{n-3}{3}} \cdot \left(\frac{n-3}{3} \right) \cdot t^{\frac{n-3}{3}-1}$$

$$= \sqrt{A} \cdot D \cdot (nDt)^{\frac{1}{n}-1} \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right] + \sqrt{A} \cdot (nDt)^{\frac{1}{n}} \cdot \frac{x_1}{2D(n-3)} \cdot (nD)^{\frac{n-3}{3}} \cdot \left(\frac{n-3}{3} \right) \cdot t^{\frac{n-6}{3}}$$

$$= \sqrt{A} \cdot D \cdot (nDt)^{\frac{1}{n}-1} \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right] + \frac{\sqrt{A}(n-3)x_1}{6D(n-3)} \cdot (nD)^{\frac{1}{n}+\frac{n-3}{3}} \cdot t^{\frac{1}{n}} \cdot t^{\frac{n-6}{3}} \cdot exp\left[\frac{x_1}{2D(n-3)} \cdot (nDt)^{\frac{n-3}{3}} \right]$$

$$= \sqrt{A} \cdot D \cdot (nD)^{\frac{1-n}{n}} \cdot L \cdot t^{\frac{1-n}{n}} + \frac{\sqrt{b_1} x_1}{6D} \cdot (nD)^{\frac{3+n^2-3n}{3n}} \cdot t^{\frac{3+n^2-6n}{3n}} \cdot L$$

$$= \left[\sqrt{A} \cdot D^{1+\frac{1-n}{n}} \cdot n^{\frac{1-n}{n}} \cdot t^{\frac{1-n}{n}} + \frac{\sqrt{A} x_1}{6} \cdot n^{\frac{3+n^2-3n}{3n}} \cdot D^{\frac{3+n^2-3n}{3n}-1} \cdot t^{\frac{3+n^2-6n}{3n}} \right] L$$

$$b_3 = \frac{1}{\sqrt{A}}(nDt)^{\frac{1}{n}} \cdot exp\left[-\frac{x_1}{2D(x-3)} (nDt)^{\frac{n-3}{3}} \right] \ldots \ldots \ldots \ldots \ldots (20)$$

From field equation (5) and using (18),(19),(20) ,we get the value of pressure

$$p = -\frac{1}{\gimel}\left[\frac{1-n}{n^2}t^{-2} + \frac{1}{n^2}t^{-1} + \frac{x_1 D^{n-6}/_{3n^{3(1-n)}}}{6L}t^{\frac{-(6-n)}{3}} + MM_1(nD)^{\frac{-1}{n}}t^{\frac{-(24-4n)}{3}} + \frac{M_2Mx_1}{6}t^{-(24-4n)} + \frac{1}{n^2}t^{-2} + \frac{x_1 D^{n-6}/_{3n^{\frac{n-6}{3}}}}{6}t^{-6} - \frac{a^2}{(nD)^{\frac{2}{n}}}t^{\frac{2}{n}}\right]$$

Where

$$L = exp\left[\frac{x_1}{2D(n-3)}(nDt)^{\frac{n-3}{3}}\right] \dots\dots\dots..(21)$$

$$M = \frac{\sqrt{A}x_1}{6}.n^{\frac{3+n^2-4n}{3n}}.D^{\frac{3+n^2-6n}{3n}} \dots\dots\dots..(22)$$

$$M_1 = \frac{n^2-6n+3}{3n\sqrt{b_1}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(23)$$

And,

$$\rho = -\frac{1}{\gimel\gamma}\left[\frac{1-n}{n^2}t^{-2} + \frac{1}{n^2}t^{-1} + \frac{x_1 D^{n-6}/_{3n^{3(1-n)}}}{6L}t^{\frac{-(6-n)}{3}} + MM_1(nD)^{\frac{-1}{n}}t^{\frac{-(24-4n)}{3}} + \frac{M_2Mx_1}{6}t^{-(24-4n)} + \frac{1}{n^2}t^{-2} + \frac{x_1 D^{n-6}/_{3n^{\frac{n-6}{3}}}}{6}t^{-6} - \frac{a^2}{(nD)^{\frac{2}{n}}}t^{\frac{2}{n}}\right]$$

Case 1: When $n \neq 3$ is a positive constant and $n > 3$

$R = (b_1b_2b_3)^{\frac{1}{3}} = (nDt)^{\frac{1}{n}}$ can be easily obtained by law of variation of Hubble parameter

Let $n = 5$, we get

$$b_1 = (5Dt)^{\frac{1}{5}} \dots\dots\dots\dots\dots(24)$$

$$b_2 = \sqrt{A}(5Dt)^{\frac{1}{5}} \cdot exp\left[\frac{x_1}{4D}\cdot(5Dt)^{\frac{2}{3}}\right] \dots\dots\dots.(25)$$

$$b_3 = \frac{1}{\sqrt{A}}(5Dt)^{\frac{1}{5}} \cdot exp\left[-\frac{x_1}{4D}(5Dt)^{\frac{2}{3}}\right] \dots\dots\dots(26)$$

$$p = -\frac{1}{\gimel}\left[\frac{-4}{25}t^{-2} + \frac{1}{25}t^{-1} + \frac{x_1 D^{-1/3}}{600L}t^{\frac{-1}{3}} + MM_1(5D)^{\frac{-1}{5}}t^{\frac{-4}{3}} + \frac{M_2Mx_1}{6}t^{-4} + \frac{1}{25}t^{-2} + \frac{x_1 D^{-1}/_{3.5^{\frac{-1}{3}}}}{6}t^{-6} - \frac{a^2}{(5D)^{\frac{2}{5}}}t^{\frac{2}{5}}\right]$$

$$\rho = -\frac{1}{\gamma\gimel}\left[\frac{-4}{25}t^{-2} + \frac{1}{25}t^{-1} + \frac{x_1 D^{-1/3}}{600L}t^{\frac{-1}{3}} + MM_1(5D)^{\frac{-1}{5}}t^{\frac{-4}{3}} + \frac{M_2Mx_1}{6}t^{-4} + \frac{1}{25}t^{-2} + \frac{x_1 D^{-1}/_{3.5^{\frac{-1}{3}}}}{6}t^{-6} - \frac{a^2}{(5D)^{\frac{2}{5}}}t^{\frac{2}{5}}\right]$$

From this observation it is notice that the scale factor increase with time but pressure and density decrease with time. This gives the present physical situation of the Universe the universe is accelerating as the second and first derivative of scale factor is positive. If we get increasing pressure then its mean that volume and amount of matter reduce. So density also increase.

Case2: When $n \neq 3$ is a positive constant and $n < 3$

Let $n = 2$

$$b_1 = (2Dt)^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(27)$$

$$b_2 = \sqrt{A}(2Dt)^{\frac{1}{2}} \cdot exp\left[\frac{-x_1}{2D}\cdot(2Dt)^{\frac{-1}{3}}\right] \dots\dots(28)$$

$$b_3 = \frac{1}{\sqrt{A}}(2Dt)^{\frac{1}{2}} \cdot exp\left[\frac{x_1}{2D}\cdot(2Dt)^{\frac{-1}{3}}\right] \dots\dots(29)$$

$$p = -\frac{1}{\gimel}\left[\frac{-1}{4}t^{-2} + \frac{1}{4}t^{-1} + \frac{x_1 D^{-2}}{24L}t^{\frac{-4}{3}} + MM_1(2D)^{\frac{-1}{2}}t^{\frac{-16}{3}} + \frac{M_2Mx_1}{6}t^{-16} + \frac{1}{4}t^{-2} + \frac{x_1 D^{-4}/_{3}2^{-\frac{4}{3}}}{6}t^{-6} - \frac{a^2}{2D}t\right] \dots(30)$$

$$\rho = -\frac{1}{\gamma\lambda}\left[\frac{-1}{4}t^{-2} + \frac{1}{4}t^{-1} + \frac{x_1 D^{-2}}{24L}t^{\frac{-4}{3}} + MM_1(2D)^{\frac{-1}{2}}t^{\frac{-16}{3}} + \frac{M_2 M x_1}{6}t^{-16} + \frac{1}{4}t^{-2} + \frac{x_1 D^{-4/3} 2^{-\frac{4}{3}}}{6}t^{-6} - \frac{a^2}{2D}t\right] \ldots\ldots\ldots\ldots (31)$$

In this case it is observed that one scale increase with time other two are decrease with time. Pressure and density also decrease with time. It is a contradiction .So only some specific value of n gives a reliable result for present Universe.

IV. THE COSMOLOGICAL PARAMETER AND THE CLASSICAL POTENTIAL

$$H = \frac{1}{nt}$$

The first observational parameter for the expansion of the universe is Hubble parameter.

$$\theta = 3H = \frac{3}{nt}$$

$$v = (nDt)^{\frac{3}{n}}$$

$$q = n - 1$$

$$\sigma = \frac{nx_1}{6}(ndt)^{\frac{n-6}{3}}$$

For accelerating Universe, we impose the restriction on the value of $n (0 < n < 1)$. If we take $n = 0.27$,then value of DP is -0.73 which exactly matches with the observed value of DP at present epoch. It is known that the age of the universe in connection with is given by

$T = \frac{1}{q-1}H^{-1}$

This shows that the value of q in the range $-1 < q < 0$ increase the age of Universe.

V. KASNER SOLUTION

We have assumed the kasner metric for Bianchi-V model is given by

$$ds^2 = dt^2 - (c_1 t^p)^2 dx^2 - e^{-2ax}[(c_2 t^q)^2 dy^2 - (c_3 t^r)^2 dz^2] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

where $c_1 t^p = b_1$, $e^{-ax} c_2 t^q = b_2, e^{-ax} c_3 t^r = b_3$ for $a_i = 0$

We choose the following diagonal tetrad

$$e_\mu^A = diag(1, b_1(t), b_2(t), b_3(t)) \ldots\ldots\ldots\ldots (33)$$

The torsion scalar for the choosen tetrad (11),

$$T = \frac{-2}{b_1 b_2 b_3}\left(b_3 \dot{b_1} \dot{b_2} + b_2 \dot{b_1} \dot{b_3} + b_1 \dot{b_2} \dot{b_3}\right) \ldots (34)$$

We can write the expression for T in (12) using anisotropic Hubble parameter

$$T = -2\left(H_{b_1} H_{b_2} + H_{b_1} H_{b_3 +} H_{b_2} H_{b_3}\right) \ldots\ldots\ldots (35)$$

We also calculate the dimensionless parameter

$$p_1(t) = -\frac{{H_{b`_1}}^2}{\dot{H_{b_1}}}, p_2(t) = -\frac{{H_{b`_2}}^2}{\dot{H_{b_2}}},$$

$$p_2(t) = -\frac{{H_{b`_3}}^2}{\dot{H_{b_3}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

The torsion scalar for the Bianchi v cosmological model is given by

$$T = R_{avg} t^{\frac{3-2n}{n}} + R_1 t^{\frac{n^2-9n+9}{3n}} + R_2 t^{\frac{3-2n}{n}} - R_3 t^{\frac{n^2-7n+5}{3n}} + R_4 \exp\left[-\frac{x_1}{D(n-3)}(nDt)^{\frac{n-3}{3}}\right] t^{\frac{n^2-9n+9}{3n}} \ldots\ldots (37)$$

Where

$$R_2 = D^{\frac{3}{n}}.n^{\frac{3-2n}{n}}$$

$$R_3 = \frac{\sqrt{A}nx_1}{6} D^{\frac{n^2-9n+9}{3n}}.n^{\frac{n^2+9-9n}{n}}$$

$$R_4 = \frac{A_* Lnx_1}{6A} D^{\frac{n-3}{n}}.n^{\frac{n^2+9-9n}{3n}}$$

$\boldsymbol{Case 1}: When\, n > 3, let\, n = 5$

$$T = Rt^{\frac{-7}{5}} + R_1 t^{\frac{29}{15}} + R_2 t^{\frac{-7}{5}} - R_3 t + R_4 \exp\left[-\frac{x_1}{2D}(5Dt)^{\frac{2}{3}}\right] t^{\frac{29}{15}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

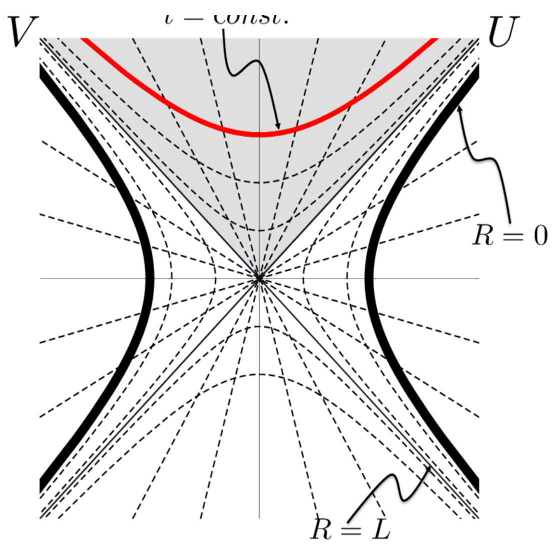
$\boldsymbol{Case 2}: When\, n < 3, let\, n = 2$

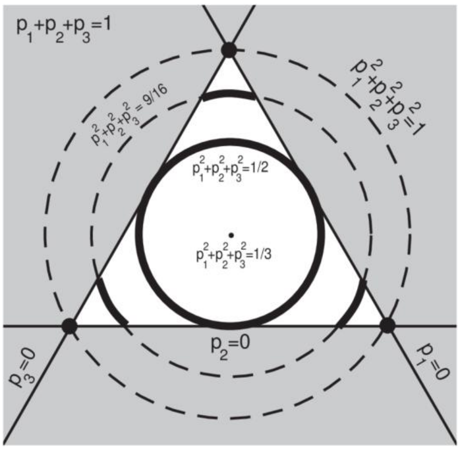
$$T = Rt^{\frac{-1}{2}} + R_1 t^{\frac{-5}{6}} + R_2 t^{\frac{-1}{2}} - R_3 t^{\frac{-1}{2}} + R_4 \exp[\frac{x_1}{D}(2Dt)^{\frac{-1}{3}}] t^{\frac{-5}{6}} \ldots\ldots\ldots (39)$$

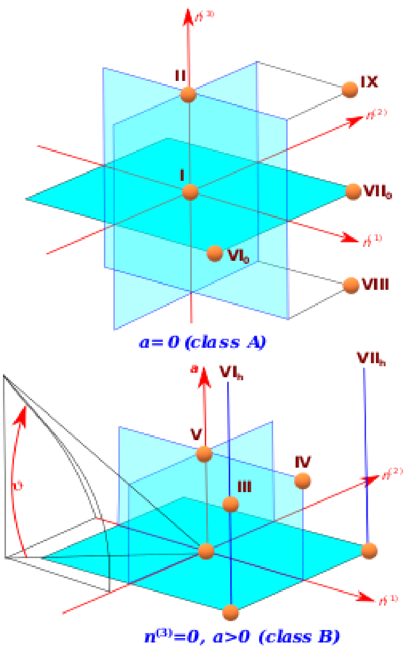
Case 3: when n=0.27

$$T = Rt + R_1 t^{-1.54} + R_2 t^{-1.36} - R_3 t^{-1.36} + R_4 \exp[\frac{x_1}{2.73D}(.27Dt)^{\frac{-2.73}{3}}] t^{-1.54} \ldots\ldots\ldots (40)$$

VI. SOME PICTURES OF KASNER SOLUTION







VII. CONCLUSION

In this paper we have studied Bianchi Type V cosmological model for equation of state and kasner solution using power law expansion. It is observed that the constant n has some special value. For different value of $n$the metric gives different physical interpretation of the universe. It is examined the deceleration parameter gives a satisfactory data for a specific value of n.Also we derived kasner metric for Bianchi V cosmological model. It is examined that torsion scalar and Hubble parameter decreases with the increase of time in case of kasner solution,which means that it describes an anisotropic universe without matter.

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