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Original Article

Distance Antimagic Labeling of Special Classes of Graphs

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Abstract *- Labeled graphs serve as a versatile mathematical model with diverse applications in engineering fields. A bijective function* $\vartheta: V(G) \rightarrow \{1, 2, \ldots, n\}$ *is a distance antimagic labeling of a graph G with n vertices such that the vertex weights determined by* $\omega(a) = \sum_{b \epsilon N(a)} \vartheta(b)$ *Are unique i.e.* $\omega(a) \neq \omega(b)$ *for distinct vertices* $a, b \,\epsilon\, V(G)$ *where N(a) is the open neighbourhood of vertex a in G. This paper demonstrates that distance antimagic labeling and inclusive distance antimagic labeling exist for certain special graph constructions- specifically Mycielskian graphs, splitting graphs, and shadow graphs when applied to fundamental graph classes like cycles, paths, crown graph and star graphs. Furthermore, the uniqueness of the calculated vertex weights is confirmed through a Python-Based Computational Algorithm.*

Keywords *- Inclusive Distance Antimagic Labeling, Distance antimagic labeling, Splitting graph, Shadow graph, Mycielski Graph.*

1. Introduction

Graph labeling introduced by Rosa [1] in 1967 is the process of allocating integers to vertices, edges, or both under specified conditions. Labeled graphs act as a mathematical tool, finding applications in fields like astronomy, cryptography, communication networks, and various optimization problems.

Radio labeling provides an effective approach for reducing computational time in sensor network applications [2]. Such networks are commonly modeled as chain graphs, where sensors are sequentially interconnected. In contrast, antimagic labeling represents a distinct class of graph labeling techniques that is primarily employed to strengthen data transmission security by supporting different encryption mechanisms [3].

The distance antimagic labeling schemes developed in this work offer a practical and adaptable approach for industrial and engineering applications that involve constructing networks or interconnections analogous to the graph structures analyzed in this study.

The idea of distance magic labeling emerged as a result of the study of magic squares. Distance magic labeling was introduced by Vilfred [4] and is a bijection $\vartheta: V(G) \rightarrow \{1, 2, \ldots, n\}$ such that there exists a positive integer *m* and the vertex weight $\omega(p) = \sum_{r \epsilon N(p)} \vartheta(r)$ For any vertex *p* in $V(G)$ with *n* vertices, where $N(p)$ is the set consisting of vertices in the open neighbourhood of vertex *p* in $V(G)$. Distance magic labeling naturally leads to distance antimagic labeling given by Kamatchi and Arumugam [5], in which the vertex weights $\omega(a) \neq \omega(b)$ for any pair of distinct vertices *a* and *b* in $V(G)$. Further, Dafik [6] initiated the notion of inclusive Distance antimagic labeling.

Definition 1.1 ([5]) Let $\vartheta: V(G) \rightarrow \{1, 2, \ldots, n\}$ be a bijection for a graph G with n vertices. Define vertex weight $\omega(p) = \sum_{r \epsilon N(p)} \vartheta(r)$ for any vertex p in V(G). If $\omega(p) \neq \omega(q)$ for every pair of unique vertices p, q in V(G), then ϑ is said to be Distance Antimagic. Any graph G that allows this type of labeling is referred to as a Distance Antimagic graph.

Definition 1.2 ([6]) Let $\vartheta: V(G) \rightarrow \{1, 2, \ldots, n\}$ be a bijection for a graph G with n vertices. Define vertex weight $\varphi(p) = \vartheta(p) + \sum_{r \epsilon N(p)} \vartheta(r)$ for any vertex p in V(G). If $\varphi(p) \neq \varphi(q)$ for every pair of unique vertices p, q in V(G), then ϑ is said to be Inclusive Distance Antimagic. Any graph G that allows this type of labeling is referred to as an Inclusive Distance Antimagic graph.

A Mycielskian graph is obtained by iteratively applying the Mycielski construction to an initial triangle-free graph. This construction, introduced by Mycielski [5], established the:

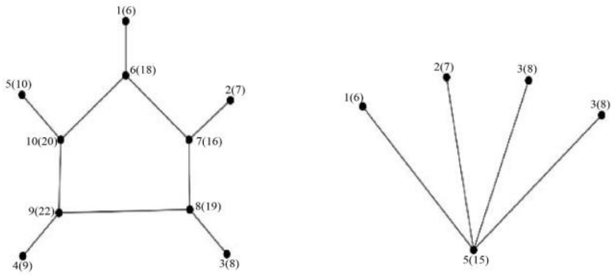

Fig. 1 Example for distance antimagic labeling of a Sun graph and inclusive distance antimagic labeling of a Star graph, where vertex labels are written as usual, and vertex weights are given within brackets.

Existence of graphs that are free of triangles yet possess arbitrarily large chromatic numbers. A key feature of the construction is that it preserves the triangle-free nature of the original graph while strictly increasing its chromatic number at each iteration.

A Mycielski graph obtained from a simple, finite, triangle-free graph inherits several important structural and coloring properties. The Mycielski construction preserves the triangle-free nature of the original graph while increasing its chromatic number by exactly one; that is, if G is a graph with chromatic number $\chi(G)$, then its Mycielskian $M(G)$ satisfies $\chi(M(G)) = \chi(G)+1$. The construction also maintains simplicity, introducing neither loops nor multiple edges, and preserves connectivity whenever the initial graph is connected. Structurally, the Mycielskian $M(G)$ contains $2|V(G)|+1$ vertices, with an edge set expanded in a systematic manner from that of G. Moreover, the clique number remains unchanged under the construction; in particular, starting from a triangle-free graph ensures that the resulting Mycielski graph has maximum clique size two. Repeated application of this construction therefore produces an infinite class of graphs that are triangle-free yet exhibit unbounded chromatic number, highlighting the significance of Mycielski graphs in graph coloring theory and extremal graph theory.

Definition 1.3 Let G be a graph with vertex set $\{a_1, a_2, \dots, a_n\}$. The Mycielski graph associated with G, denoted by M(G), is constructed by enlarging the vertex set to include an additional copy of each original vertex together with one extra vertex. Thus, $V(M(G)) = \{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n, a\}$ so that M(G) has 2n+1 vertices. The edge set of M(G) consists of all edges of the original graph G, along with edges joining each new vertex a'_i to all neighbours of a_i in G, and edges connecting the additional vertex a to every vertex a'_i, for $1 \leq i \leq n$.

E. Sampathkumar and Walikar [9] gave the notion of the splitting graph of a graph.

Definition 1.4 ([9]) For every vertex a of a graph G, take another point a′ and join a′ To all vertices of G adjacent to a. The graph Sl(G) thus obtained is called the splitting graph of G.

Definition 1.5 The shadow graph of a graph G, denoted by Sh(G), is constructed by taking a duplicate copy G′ of G and then connecting each vertex of G′ to all those vertices in G that are adjacent to its corresponding vertex in the original graph.

Distance antimagic labeling is a relatively recent extension of classical antimagic labeling, in which vertices are assigned distinct integers such that the induced vertex weights-defined as the sum of labels of vertices at a specified distance-are pairwise distinct. This labeling paradigm has attracted attention due to its applicability in modeling interference-free communication networks, frequency assignment, and distributed system design. Although distance magic and distance antimagic labelings have been investigated for several standard graph families, the impact of graph transformations on Distance antimagic properties remains insufficiently understood, particularly for constructions such as Mycielskian graphs, splitting graphs, and shadow graphs. Existing research on Mycielskian graphs has primarily concentrated on chromatic properties and, more recently, on distance magic labeling, with only limited results available for Distance antimagic labeling [9, 10]. Similarly, antimagic labeling of splitting and shadow graphs has been addressed mainly for specific graph classes or under restricted distance conditions [11, 12]. For instance, distance-2 antimagic labeling of shadow graphs of cycles and complete bipartite graphs has been established, demonstrating that shadow graph operations can preserve antimagic behavior under certain constraints [11]. However, these studies are fragmented and do not provide a unified analysis across different graph operations or for fundamental graph families such as paths, stars, and crown graphs. Moreover, comparative results linking Mycielskian, splitting, and shadow constructions within the same Distance antimagic framework are largely absent from the literature.

In view of these gaps, this paper presents a systematic investigation of Distance antimagic labeling for Mycielskian graphs, splitting graphs, and shadow graphs derived from basic classes of graphs, including paths, cycles, star graphs, and crown graphs. The novelty of this work lies in extending distance antimagic labeling results simultaneously to multiple graph transformations and previously underexplored graph families. In addition, a Python-based computer-aided procedure is employed at the beginning of each section to verify the uniqueness of vertex weights, thereby providing computational validation alongside theoretical proofs. This integrated approach not only enhances the reliability of the results but also aligns with the applied and algorithmic orientation, offering a scalable methodology for future studies in graph labeling and network modeling.

2. Results and Discussion

2.1. Distance Antimagic Labelling of Mycielski and Splitting Graph of Graphs

This section examines the existence of Distance antimagic labelings for Mycielskian and splitting graph constructions derived from basic graph classes, namely paths, cycles, star graphs, crown graphs, and friendship graphs.

Theorem 2.1.1 Mycielskian graph of the path P_n Is Distance antimagic when n is even.

Proof Consider a path $G = P_n$ where n is even. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of P_n. Let $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n, a\}$ be the vertex set of Mycielskian graph M(G) of G such that a'_j is connected to all the vertices adjacent to a_j and a is connected to a'_j for $1 \le j \le n$. If n = 2, then M(G)) is a cycle C_5 Which is Distance antimagic [3]. For $n \neq 2$, let $\vartheta: V(M(G)) \to \{1, 2, \dots, 2n+1\}$ be a labeling function such that

$$\vartheta(a_j) = 2j$$

$$\vartheta(a'_j) = 2j - 1$$

$$\vartheta(a) = 2n + 1$$

where $1 \le j \le n$. It is evident that ϑ is a bijective mapping, and the corresponding vertex weights are defined as

$$\omega(a_j) = \begin{cases} 7 & : j = 1 \\ 8j - 6 & : 2 \le j \le n-1 \\ 4n - 5 & : j = n \end{cases}$$

$$\omega(a'_j) = \begin{cases} 2n + 5 & : j = 1 \\ 4j + n + 1 & : 2 \le j \le n-1 \\ 4n - 1 & : j = n \end{cases}$$

$$\omega(a) = n^2$$

As unique vertex weights are obtained, so the Mycielskian graph of path $M(G)$ is Distance antimagic when n is even.

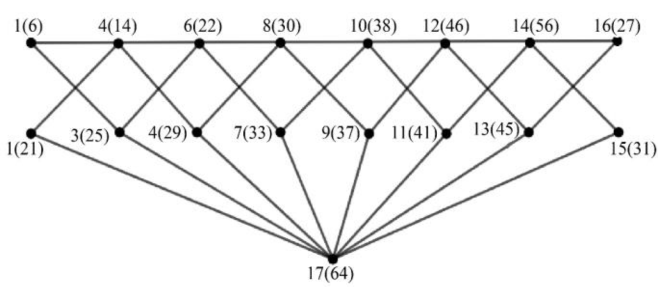

Fig. 2 Example for distance antimagic labeling of the mycielskian graph of P_8. The labels are written as usual, and vertex weights are given in brackets.

The algorithm described below is validated using a Python-based computational approach to confirm the distinctness of vertex weights.

ALGORITHM 7: ALGORITHM TO VERIFY DISTINCTNESS OF VERTEX WEIGHTS OF *G*

```
Input: Positive integer n
Output: Weight matrix W
If (n == 2) or (n % 2 != 0) then
    Print (" The result holds for even values of n > 2.")
Else
    Number of vertices = 2n + 1
    V_a = [ ]
    V_{a_i} = [ ]
    V_{a'_i} = [ ]
For i ← 1 to (2 * n + 1) do
    If (i % 2 == 1) then
        V_{a'_i}.append(i)
    Else
        V_{a_i}.append(i)
End
V_a = 2 * n + 1
W_a = [ ]
W_{a_i} = [ ]
W_{a'_i} = [ ]
def find_ W_{a_i}()
For i in range (1 to (n + 1)) do
    If (i == 1) then
        W_{a_i}.append(7)
    Else if (i == n) then
        W_{a_i}.append(4 * n − 5)
    Else
        W_{a_i}.append(8 * i − 2)
End
Return W_{a_i}
def find_ W_{a'_i}()
For i in range (1 to (n + 1)) do
    If (i == 1) then
        W_{a'_i}.append(2 * n + 5)
    Else if (i == n) then
        W_{a'_i}.append(4 * n - 1)
    Else
        W_{a'_i}.append(4 * i + 2 * n + 1)
End
Return W_{a'_i}
W_a = n^2
If (len(W_{a'_i}) == len(set(W_{a'_i}))) and (len(W_{a_i}) == len(set(W_{a_i}))) then
    Distinct elements
Else
    Similar elements
```

Theorem 2.1.2 Mycielskian graph of the cycle C_n Is Distance antimagic when n is odd.

Proof Consider a path $G = C_n$ where n is odd. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of C_n.

Let $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n, a\}$ be the vertex set of Mycielskian graph M(G) of G such that a'_j is connected to all the vertices adjacent to a_j and a is connected to a'_j for $1 \le j \le n$. Let $\vartheta: V(M(G)) \to \{1, 2, \dots, 2n+1\}$ be a labeling function such that:

$$\vartheta(a_j) = 2j$$

$$\vartheta(a_j') = 2j + 1$$

$$\vartheta(a) = 1$$

Where $1 \le j \le n$. It is evident that ϑ is a bijective mapping, and the corresponding vertex weights are defined as

$$\omega(a_j) = \begin{cases} 4n + 10 & : j = 1 \\ 8j + 2 & : 2 \le j \le n - 1 \\ 4n + 2 & : j = n \end{cases}$$

The flowchart for the description of the algorithm validating the uniqueness of vertex weights of Distance antimagic labelling of Mycielskian graph is described below:

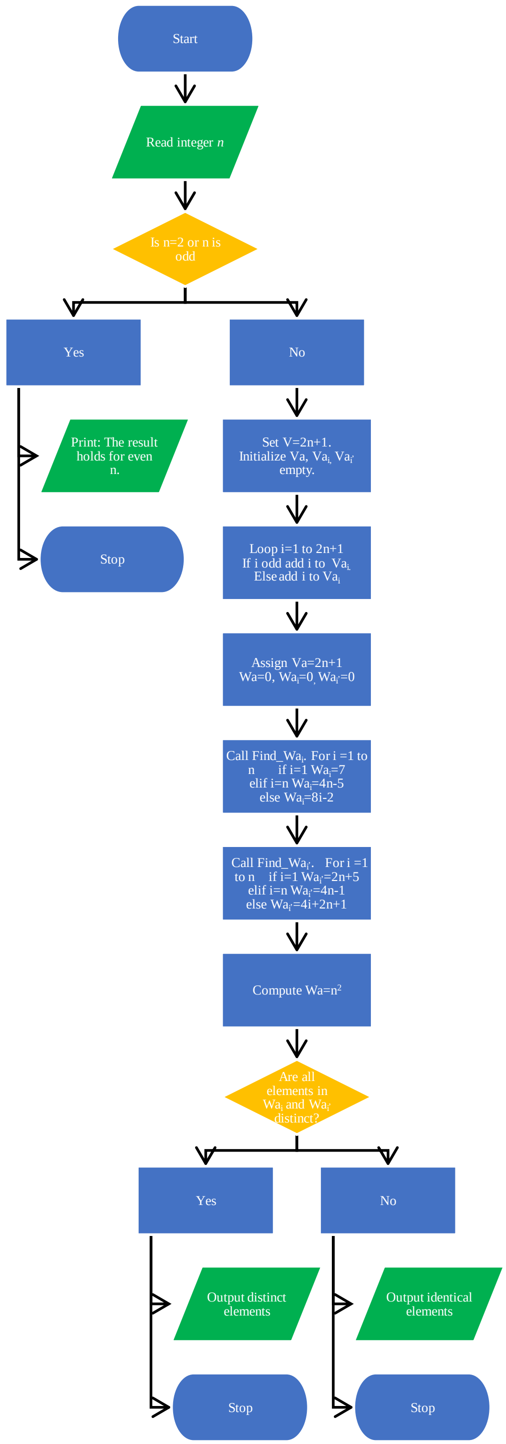

Fig. 3 Flow chart

$$\omega(a'_j) = \begin{cases} 2n+5 \ : \ j=1 \\ 4j+1 \ : 2 \le j \le n-1 \\ 2n+1 \ : j=n \end{cases}$$

$$\omega(a) = (n+1)^2 - 1$$

As unique vertex weights are obtained, so the Mycielskian graph of cycle *M*(*G*) is Distance antimagic when *n* is odd.

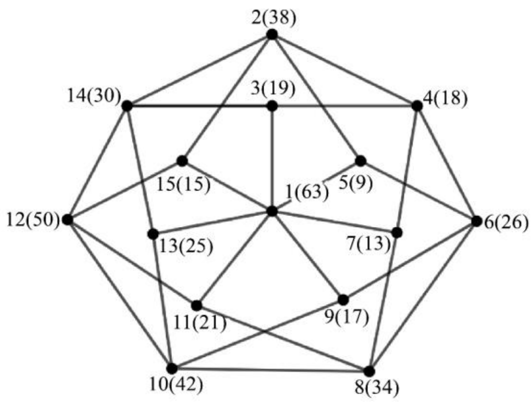

Fig. 4 Example for distance antimagic labeling of mycielskian graph of cycle C_7.

Theorem 2.1.3 Splitting graph of the path P_n is Distance antimagic when n is even.

Proof Consider a path $G = P_n$ where n is even. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of P_n. Let $\{a'_1, a'_2, \dots, a'_n\}$ be the corresponding vertex set such that a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$. Let Sl(G) be the splitting graph of path $G = P_n$. Define a labelling function $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2n\}$ such that:

$$\vartheta(a_j) = 2j$$

$$\vartheta(a'_j) = 2j - 1$$

where $1 \le j \le n$. Clearly, the function ϑ establishes a one-to-one correspondence, and the associated vertex weights are determined by:

$$\omega(a_j) = \begin{cases} 7 & : \ j=1 \\ 8j-2 & : 2 \le j \le n-1 \\ 4n-5 & : j=n \end{cases}$$

$$\omega(a'_j) = \begin{cases} 4 & : \ j=1 \\ 4j & : 2 \le j \le n-1 \\ 2n-2 & : j=n \end{cases}$$

As each vertex is assigned a distinct weight, it follows that the splitting graph of the path admits a distance antimagic labeling.

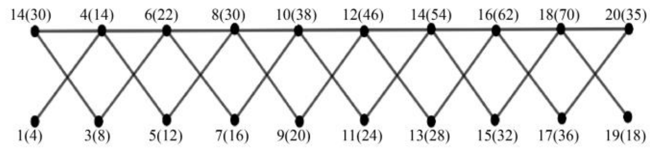

Fig. 5 Example for distance antimagic labeling of the splitting graph of path P_{10}

Theorem 2.1.4 Splitting graph of the cycle C_n is Distance antimagic when n is odd.

Proof Consider a cycle $G = C_n$ where n is odd. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of C_n. Let $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n\}$ be the vertex set of Splitting graph Sl(G) of G such that a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$. Let $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2n\}$ be a function such that:

$$\vartheta(a_j) = 2j$$

$$\vartheta(a'_j) = j + n$$

Where $1 \le j \le n$. The function ϑ establishes a one-to-one correspondence, and the associated vertex weights are determined by:

$$\omega(a_j) = \begin{cases} 4n+4 & : \ j=1 \\ 2n+4j & : 2 \le j \le n-1 \\ 4n & : j=n \end{cases}$$

$$\omega(a'_j) = \begin{cases} n+2 & : \ j=1 \\ 2j & : 2 \le j \le n-1 \\ 4n & : j=n \end{cases}$$

As unique vertex weights are obtained, the splitting graph of cycle Sl(G) is Distance antimagic when n is odd.

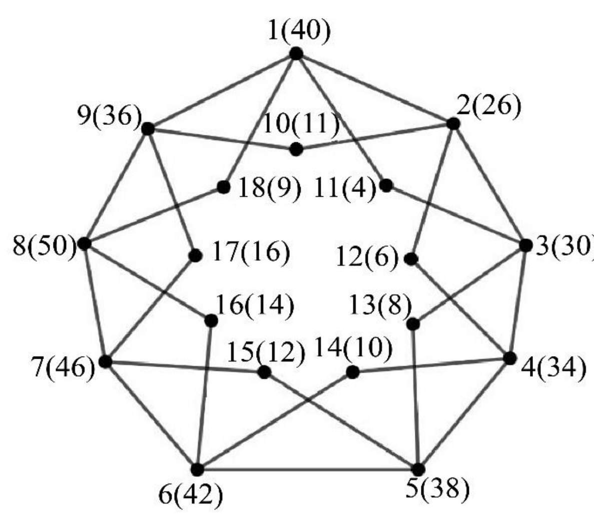

Fig. 6 Example for distance antimagic labeling of the splitting graph of cycle C_9

Theorem 2.1.5 Splitting graph of the complete graph K_n is Distance Antimagic.

Proof Consider a complete graph $G = K_n$. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of K_n. Let $\{a'_1, a'_2, \dots, a'_n\}$ be the corresponding vertex set such that a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$. Let Sl(G) be the splitting graph of K_n. Let $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2n\}$ be a function such that

$$\vartheta(a_j) = j$$

$$\vartheta(a'_j) = j + n$$

where $1 \le j \le n$. It is evident that ϑ is a bijective function, and the vertex weights are defined as:

$$\omega(a_j) = 2(n^2 - j)$$

$$\omega(a'_j) = \frac{n(n+1)}{2} - j$$

As the vertex weights are monotonically decreasing, they are distinct. Hence, the Splitting graph Sl(G) of a complete graph is Distance antimagic.

Crown graph of order n for $n \ge 3$ is a graph G with vertex set $\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n\}$ and edge set $\{(a_r b_s): 1 \le r, s \le n,\ r \ne s\}$.

Theorem 2.1.6 Splitting Graph of the Crown Graph is Distance Antimagic.

Proof Consider a crown graph $G = H_{n,n}$. Let $\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n\}$ be the vertex set of G. Let $\{a'_1, a'_2, \dots, a'_n, b'_1, b'_2, \dots, b'_n\}$ be the corresponding vertex set such that a'_j is connected to all the vertices adjacent to a_j and b'_j is connected to all vertices adjacent to b_j for $1 \le j \le n$. Let $Sl(G)$ be the splitting graph of G. Define a function $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 4n\}$ such that

$$\vartheta(a_j) = j$$

$$\vartheta(a'_j) = j + 2n$$

$$\vartheta(b_j) = j + n$$

$$\vartheta(b'_j) = j + 3n$$

where $1 \le j \le n$. Clearly, ϑ establishes a one-to-one correspondence, and the vertex weights are determined by

$$\omega(a_j) = n(5n - 3) - 2j$$

$$\omega(a'_j) = \frac{n(3n-1)}{2} - j$$

$$\omega(b_j) = n(3n - 1) - 2j$$

$$\omega(b'_j) = \frac{n(n+1)}{2} - j$$

Since the vertex weights form a strictly decreasing sequence, they are all distinct. Hence, the Splitting graph $Sl(G)$ of the crown graph is Distance antimagic.

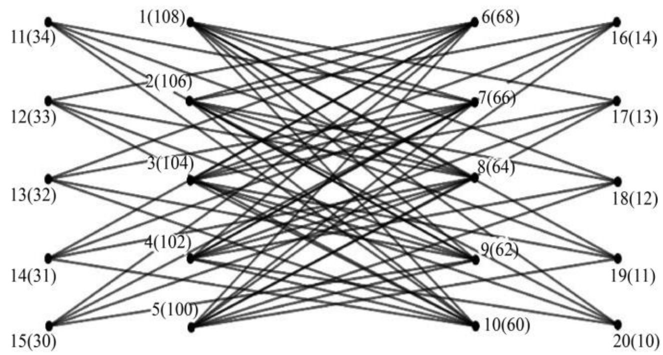

Fig. 7 Example for distance antimagic labeling of the splitting graph of the crown graph $H_{5,5}$

2.2. Inclusive Distance Antimagic Labeling of Splitting and Shadow Graph of Graphs

This section is devoted to the study of inclusive Distance antimagic labeling of the splitting and shadow graphs associated with basic graph classes, namely paths, cycles, and star graphs.

Theorem 2.2.1 Splitting graph of the path P_n Is inclusive Distance antimagic.

Proof Consider a path $G = P_n$. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of P_n. Let $\{a'_1, a'_2, \dots, a'_n\}$ be the corresponding vertex set such that a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$. Let $Sl(G)$ be the splitting graph of path $G = P_n$. Let $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2n\}$ be a labelling function such that

$$\vartheta(a_j) = 1 + 2n - j$$

$$\vartheta(a'_j) = j$$

Where $1 \le j \le n$. It is evident that ϑ is a bijection, and the vertex weights are computed as

$$\varphi(a_j) = \begin{cases} 4n + 1 & : j = 1 \\ 6n + 3 - j & : 2 \le j \le n - 1 \\ 3n + 2 & : j = n \end{cases}$$

$$\varphi(a'_j) = \begin{cases} 2n & : j = 1 \\ 4n - j + 2 & : 2 \le j \le n - 1 \\ 2n + 2 & : j = n \end{cases}$$

As we obtain unique vertex weights, the Splitting graph of the path $Sl(G)$ is inclusive Distance antimagic.

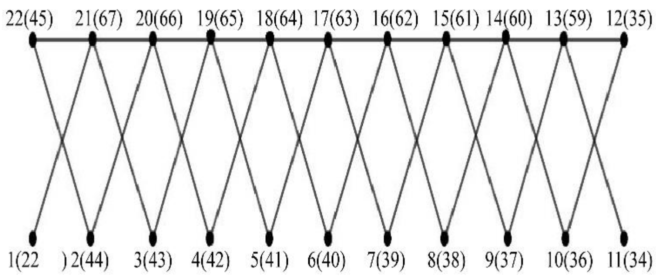

Fig. 8 Example for inclusive distance antimagic labeling of the splitting graph of P_{11}

Theorem 2.2.2 Splitting Graph of the Cycle $\mathbf{C_n}$ Is inclusive Distance Antimagic.

ALGORITHM 8: ALGORITHM TO VERIFY DISTINCTNESS OF VERTEX WEIGHTS OF *G*

```
Input: Positive integer n
Output: Weight matrix W
Number of vertices = 2n
V_ai = []
V_ai' = []
For i in range (1 to (n + 1)) do
    V_ai'.append(i)
    V_ai.append(2 * n + 1 - i)
End
W_ai = []
W_ai' = []
def find_W_ai()
For i in range (1 to (n + 1)) do
    If (i == 1) then
        W_ai.append(4 * n + 1)
    Else If (i == n) then
        W_ai.append(3 * n + 2)
    Else
        W_ai.append (6 * n + 3 - i)
End
Return W_ai
def find_W_ai'()
For i in range (1 to (n + 1)) do
    If (i == 1) then
        W_ai'.append(2 * n)
    Else if (i == n) then
        W_ai'.append(2 * n +2)
    Else
        W_ai'.append(4 * n - i + 2)
End
Return W_ai'
If (len(W_ai') == len(set(W_ai'))) and (len(W_ai) == len(set(W_ai))) then
    Distinct elements
Else
    Similar elements
```

Proof Consider a cycle $G = C_n$. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of C_n. Let $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n\}$ be the vertex set of Splitting graph $Sl(G)$ of G such that a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$. Consider a labelling function $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2n\}$ such that

$$\vartheta(a_j) = j$$

$$\vartheta(a'_j) = 2n + 1 - j$$

where $1 \le j \le n$. ϑ establishes a one-to-one correspondence, and the associated vertex weights are determined by

$$\varphi(a_j) = 4n + 2 + j$$

$$\varphi(a'_j) = \begin{cases} 3n + 2 & : j = 1 \\ 2n + 1 + j & : 2 \le j \le n - 1 \\ 2n + 1 & : j = n \end{cases}$$

As unique vertex weights are obtained, the splitting graph of cycle $Sl(G)$ is inclusive Distance Antimagic.

The algorithm is outlined below for confirming the distinctness of the vertex weights of the Splitting graph of a path.

The star graph denoted by $K_{1,n}$ Is a graph with a central vertex a and connected to $(n + 1)$ vertices of degree 1 each.

Theorem 2.2.3 Splitting Graph of the Star Graph $K_{1,n}$ Is inclusive Distance Antimagic.

Proof Consider a star graph $G = K_{1,n}$. Let a be the central vertex connected to $(n + 1)$ vertices $\{a_1, a_2, \dots, a_{n+1}\}$. Let a' and $\{a'_1, a'_2, \dots, a'_{n+1}\}$ be the corresponding vertices such that a' is connected to all vertices adjacent to a and a'_j is connected to all the vertices adjacent to a_j for $1 \le j \le n$ respectively. Let $Sl(G)$ be the splitting graph of the star graph $G = K_{1,n}$. Consider a labelling function $\vartheta: V(Sl(G)) \to \{1, 2, \dots, 2(n + 2)\}$ such that:

$$\vartheta(a) = 1$$

$$\vartheta(a_j) = n + 3 + j$$

$$\vartheta(a') = n + 3$$

$$\vartheta(a'_j) = 1 + j$$

where $1 \le j \le n$. Clearly, ϑ establishes a one-to-one correspondence, and the associated vertex weights are determined by:

$$\varphi(a) = 2n^2 + 8n + 7$$

$$\varphi(a'_j) = 2 + j$$

$$\varphi(a') = \frac{(n+2)(3n+7)}{2}$$

$$\varphi(a_j) = 2n + 7 + j$$

As we obtain unique vertex weights, the Splitting graph $Sl(G)$ of the star graph is inclusive Distance antimagic.

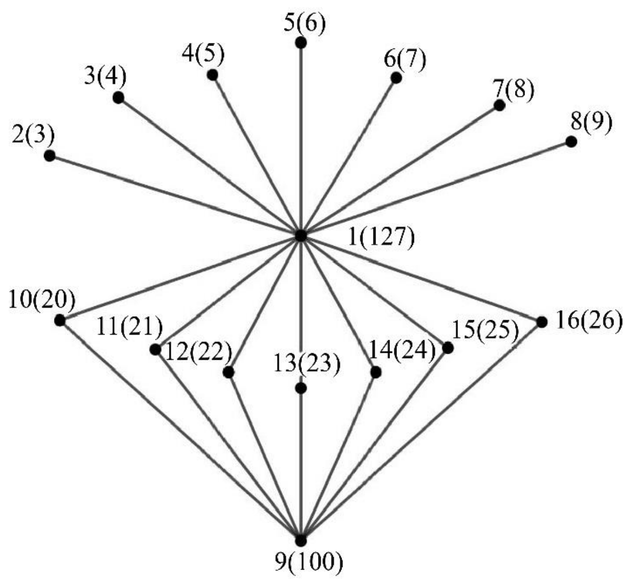

Fig. 9 Example for inclusive distance antimagic labeling of the splitting graph of the star graph $K_{1,6}$

Theorem 2.2.4 Shadow graph of the path P_n Is inclusive Distance antimagic.

Proof Consider a path $G = P_n$. Let $\{a_1, a_2, \dots, a_n\}$ be the vertex set of P_n. Let $\{a'_1, a'_2, \dots, a'_n\}$ be the vertex set of a copy of P_n.

Let $Sh(G)$ be the shadow graph of path $G = P_n$ with vertex set $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n\}$. Consider a labeling function $\vartheta: V(Sh(G)) \to \{1, 2, \dots, 2n\}$ such that

$$\vartheta(a_j) = j$$

$$\vartheta(a'_j) = 2n + 1 - j$$

where $1 \le j \le n$. It is evident that ϑ is a bijection, and the vertex weights are computed as

$$\varphi(a_j) = \begin{cases} 2n + 2 & : j = 1 \\ 4n + 2 + j & : 2 \le j \le n - 1 \\ 3n + 1 & : j = n \end{cases}$$

$$\varphi(a'_j) = \begin{cases} 1 + 4n & : j = 1 \\ 3 + 6n - j & : 2 \le j \le n - 1 \\ 3n + 2 & : j = n \end{cases}$$

Since each vertex in the shadow graph is assigned a unique weight, the path shadow graph $Sh(G)$ admits inclusive Distance antimagic.

Theorem 2.2.5 Shadow Graph of the Cycle C_n Is Inclusive Distance Antimagic.

Proof Consider a cycle $G = C_n$ with vertex set $\{a_1, a_2, \dots, a_n\}$. Let $\{a'_1, a'_2, \dots, a'_n\}$ be the vertex set of a copy of C_n. Let $Sh(G)$ be the shadow graph of G with vertex set. $\{a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n\}$. Let $\vartheta: V(Sh(G)) \to \{1, 2, \dots, 2n\}$ be a labelling function such that

$$\vartheta(a_j) = j$$

$$\vartheta(a'_j) = 2n + 1 - j$$

where $1 \le j \le n$. It is evident that ϑ is a bijective function, and the vertex weights are determined as

$$\varphi(a_j) = \begin{cases} 4n + 3 & : j = 1 \\ 4n + 2 + j & : 2 \le j \le n - 1 \\ 5n + 2 & : j = n \end{cases}$$

$$\varphi(a'_j) = \begin{cases} 2 + 6n & : j = 1 \\ 3 + 6n - j & : 2 \le j \le n - 1 \\ 5n + 3 & : j = n \end{cases}$$

As we obtain unique vertex weights, the Shadow graph of the cycle $Sh(G)$ is inclusive Distance antimagic.

2.3. Practical Applications

Distance antimagic labeling of graphs, where distinct vertex labels from 1 to $|V(G)|$ produce unique sums of labels on neighbours for every vertex pair, has garnered interest for its structural properties and potential applications, particularly in constructions like splitting graphs, shadow graphs, and Mycielskian graphs .

This labeling ensures vertex distinguishability via distance weights, with research extending to practical uses in cryptography and graph coloring. Practical applications of Distance antimagic labeling include data encryption schemes, where splitting graphs of paths generates prime-based vertex weights for RSA integration. Local variants induce proper vertex colorings, bounding chromatic numbers for generalized Mycielskian graphs of Regular graphs, while D-distance antimagic properties in shadow graphs classify labelings over specific distance sets like {1} or {0,2}, supporting broader conjectures.

Splitting graphs demonstrate robust practicality for cryptographic protocols due to consistent labelings, whereas Mycielskian graphs underscore non-preservation under construction, contrasting shadow graphs' adaptability in distance-specific contexts.

3. Conclusion

This paper investigates Distance antimagic and inclusive distance antimagic labelings for specific graph classes, including Mycielskian graphs, splitting graphs, and shadow graphs derived from paths, cycles, complete graphs, crown graphs, and stars. Key findings establish the existence of Distance antimagic labelings for Mycielskian graphs of paths and cycles, as well as splitting graphs of paths, cycles, complete graphs, crown graphs, and friendship graphs. Additionally, inclusive Distance antimagic labelings are proven for splitting and shadow graphs of paths, cycles, and stars, with vertex weight uniqueness rigorously verified through Python computations. These results highlight the robustness of Distance antimagic properties under Mycielskian and splitting constructions for basic graphs, contrasting with selective applicability in shadow graphs, thereby resolving open conjectures on labeling preservation. The computational validation via Python underscores practical verifiability, enhancing methodological rigor in combinatorial graph theory.

Implications extend to cryptographic protocols, where splitting graph labelings generates distinct prime-based weights for secure encryption schemes, and to bounding chromatic numbers in local variants for generalized Mycielskians. This work advances graph labeling theory by classifying antimagic behaviors across constructions, informing future research on supermagic generalizations and algorithmic optimizations in network design and coding theory.

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