

Original Article

Spectral Enhancement of GFDM Using Gaussian Pulse Shaping

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Abstract - Generalized Frequency Division Multiplexing GFDM is one of the multicarrier modulation techniques used in 5G that was derived from Orthogonal Frequency Division Multiplexing (OFDM). The main feature of GFDM is a pulse shaping the transmitted symbol to enhance the power spectral density of the transmitted signal and decrease the out-of-band (OOB) radiations, which will increase the spectral efficiency of the transmitted signal. In this paper, the Gaussian pulse shaping filter was introduced to shape the transmitted symbols of a simulated GFDM system. A comparison with the conventional shaping functions was also made to evaluate the Gaussian pulse shaping filter results. The simulated results show that the spectral efficiency was improved by (14.2) dB without effect on the overall system performance.

Keywords - GFDM, Gaussian filter, Pulse shaping, Spectral efficiency, 5G.

1. Introduction

5G mobile system started to be deployed in late 2019; this new generation of mobile communication significantly changed business and industry sectors. 5G technology is characterized by its coverage enhancement, high throughput, low latency and more efficient energy management [1]. Spectrum efficiency is one of the biggest challenges facing any wireless communication systems, including 5G, because of the limited spectrum and the growing radio communication demands [2]. Generalized Frequency Division Multiplexing (GFDM) is one of the waveforms used in 5G wireless communication because it has low latency, low (PAPR) and low (OOB) radiation and low Adjacent Channel Leakage Ratio (ACLR), and relaxed requirements of time and frequency synchronization [3]. GFDM is a block-based multicarrier modulation system, where its filter tails are reduced by using a cyclic pulse shaping filter. The resulting out-of-band (OOB) radiation of GFDM is reduced due to the introduction of that adjustable filter, which will significantly decrease the OOB interference. As a result, the dynamic spectrum access scenario will be enhanced [4]. GFDM can be constructed based on OFDM principles [3], which could make it an attractive candidate to replace OFDM for 5G mobile networks because it has better spectral efficiency than OFDM due to the use of the pulse shaping filter [5]. Several papers studied the influence of the pulse shaping filter on the GFDM system performance; for example, the authors in [4] proposed a new dual filter transmission scheme to eliminate the intrinsic self-interference, while a simple method to compute the shaping filter coefficients was introduced in [6] and its system

performance was evaluated. The optimum pulse shaping filter was introduced in [7]; this filter was used to eliminate the negative effects of intrinsic interferences and improve the spectrum efficiency. The design and the implementation of a pulse shaping filter were introduced in [5] to minimize the OOB radiation, and this filter was implemented using a quadratic programming approach. The derived study of the optimum cyclic tree-structure reconstruction-quadrature mirror filter bank and the implementation of this structure in the frequency domain was introduced in [2].

In [8], the authors derived a closed-form presentation of the noise enhancement factor based on the pulse shaping. This paper introduces the Gaussian pulse shaping filter, which is used to reduce the OOB of the GFDM system, and it has the following organization: section 2 discusses the GFDM modulator and demodulator, and the pulse shaping filter and its influence on the GFDM signal is introduced in section 3. Section 4 introduces the proposed GFDM system using a Gaussian filter. The simulated results are introduced in section 5, while the conclusion is introduced in section 6.

2. GFDM System

The GFDM system block diagram is shown in Figure 1 [5,9]. Each block uses K subcarriers, with each subcarrier transmitting M complex-valued sub-symbols, generating one block ($D = KM$) of transmitted symbols [10]. For any GFDM block, the m^{th} sub-symbol on the k^{th} subcarrier $[d]_{k+mK}$ will be shaped using the pulse shaping filter $g_{k,m}$, which has n^{th} entry $[g_{k,m}]$ equal to [9]:



$$g_{k,m}(n) = g((n - mK)_D) e^{-\frac{j2\pi nk}{K}} \quad n = 0, 1, \dots, D - 1$$

$$m = 0, 1, \dots, M - 1 \quad k = 0, 1, \dots, K - 1 \quad (1)$$

Where g is called the “prototype filter”. [10]

Let A be the $D \times D$ GFDM transmitted matrix; it can be described as [9]:

$$A = \begin{bmatrix} g_{0,0} & \dots & g_{K-1,0} & g_{0,1} & \dots & g_{K-1,1} & \dots & g_{0,M-1} & \dots & g_{K-1,M-1} \end{bmatrix} \quad (2)$$

Then, the transmit sample vector $x = Ad$, with n th entry is given by [6]:

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k+m} g[\langle n - mK \rangle_N] e^{-\frac{j2\pi nk}{K}} \quad (3)$$

These symbols will propagate through the last block in the transmitter, adding the Cyclic Prefix (CP) to each vector x to reduce the ISI. After that, the resulting signal will be converted to serial before transmitting it as a GFDM signal [10].

The CP is removed at the receiver side, and the received signal can be expressed by [11]:

$$r = Hx + w \quad (4)$$

Where H is a two-dimensional $D \times D$ circulant matrix, and the content of each row is the impulse response of the channel, which is circularly right-shifted by one element from the row below, and w represents the additive white Gaussian noise (AWGN) vector.

The estimated transmitted symbol vector is given by:

$$\hat{d} = Er \quad (5)$$

Where r is the received vector, and E is the receiver matrix.

The received sample will be detected using the next receiver blocks, the three main traditional receivers of GFDM, which are MF, ZF, and minimum mean square error (MMSE) [11, 12].

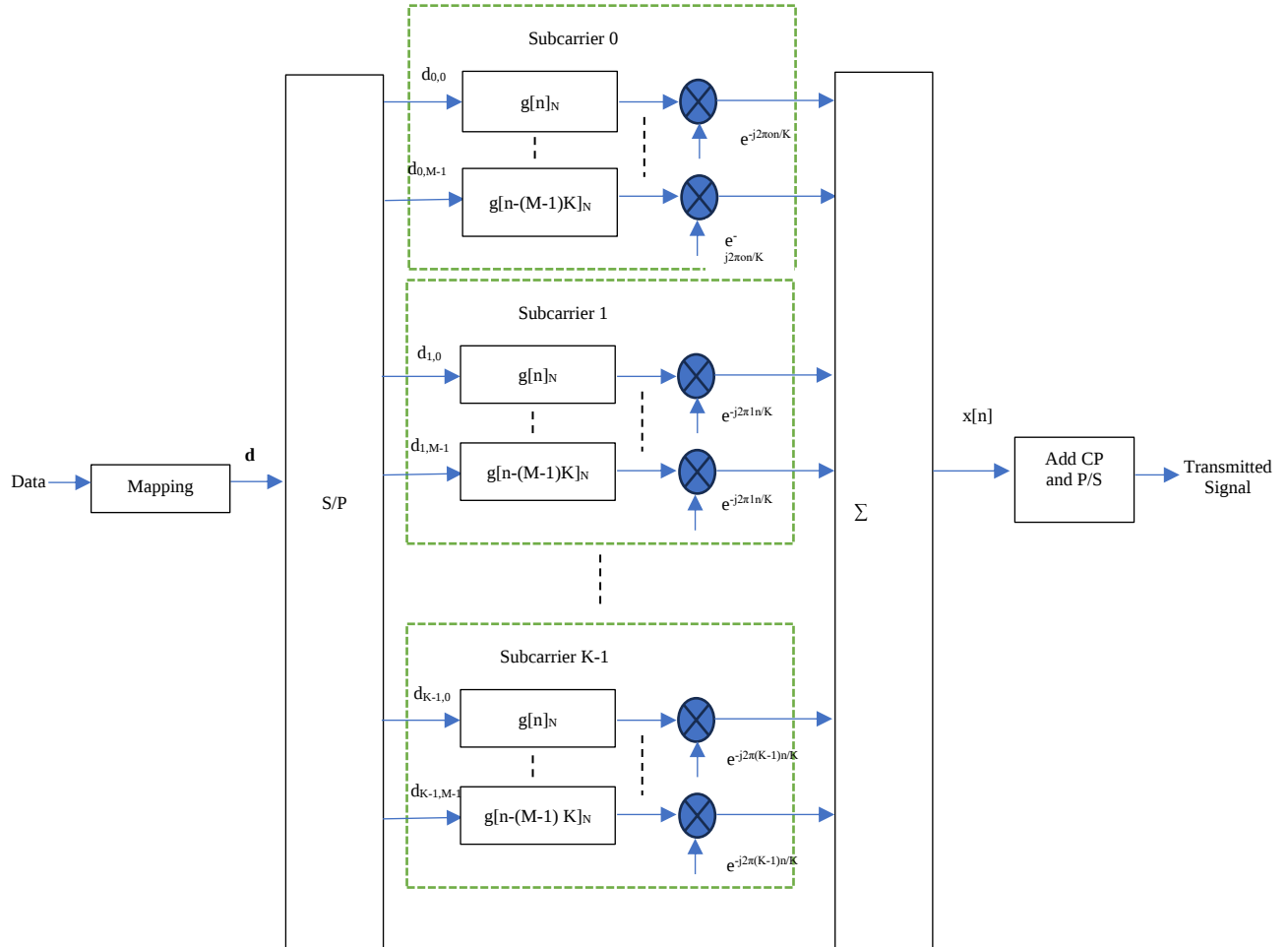


Fig. 1 GFDM transmitter

3. Pulse Shaping

As mentioned previously, the pulse shaping technique is used at the transmitter to eliminate the sharp transition edges of the transmitted rectangular pulses. This will limit the transmitted signal bandwidth, shape it to be more suitable for transmission along the communication channel, and minimize transmission power. The selection process of a pulse shaping technique should be considered so as not to make interference between pulses at the sampling point at the receiver. The resulting pulse shape should fade away rapidly in the boundary of the original rectangular pulse to ensure no interference may occur with the next pulse, and it must have zero crossing at that boundary [13]. OFDM uses rectangular pulse shapes, which cause spectral growth and high OOB radiation. Pulse shaping filter is one of the techniques used to reduce OOB radiation. The data symbols in GFDM are shaped using a prototype filter circularly shifted in time and frequency domain [5], and the pulse shaping filter determines the characteristics of the GFDM transmitted signal. Several types of shaping filters exist, such as raised cosine filters and Dirichlet filters [3,14] [15]. All these different filters try to reduce the OOB radiation and increase the spectral efficiency of the GFDM signal compared to OFDM. [6]

The pulse shaping filter impulse response $g(n)$ is a periodic signal due to the circular shift with period N , and it can be expressed as [7]:

$$g(n) = \sum_{i=0}^I a_j e^{j2\pi i n/D} \quad n = 0, 1, \dots, D-1 \quad (6)$$

Where a_j is the FS filter coefficient.

The normalized bandwidth of the pulse shaping filter is given by:

$$\Delta f = \frac{I}{D} \quad (7)$$

The GFDM bandwidth is then [7]

$$GFDM_{BW} = \Delta f \cdot K = \frac{I}{D} K = \frac{I}{MK} K = \frac{I}{M} \quad (8)$$

Where I limit the bandwidth of the pulse shaping filter, and this limitation reduces the OOB radiation.

4. Gaussian Pulse Shaping

Gaussian filter is a filter that has a Gaussian function impulse response and its response will cancel the overshoot of the step input and reduce the rise and fall time. The Gaussian filter is a linear filter, and its group delay is minimum and is normally used as a smoother [16]. The Gaussian pulse shaping, on the other hand, minimizes the OOB spectral energy [17]. A Gaussian-shaped impulse response filter generates a signal with low side lobes and a narrower main lobe than the rectangular pulse. This filter is used to reduce the

transmission bandwidth of the Minimum Shift Keying and produce the GMSK signal, which is used for voice calls over mobile communication networks. The rectangular pulse symbol can be filtered using a Gaussian filter with a Gaussian shape impulse response and Gaussian shape frequency response.

The impulse response of the Gaussian filter is given by [16,18]:

$$g(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2\right) \quad (9)$$

Its frequency response was given by [16,18]:

$$G(f) = \exp\left[-\left(\frac{f}{B}\right)^2 \left(\frac{\ln(2)}{2}\right)\right] = \exp(-\alpha^2 f^2) \quad (10)$$

$$\alpha = \frac{\sqrt{\ln(2)}}{\sqrt{2}B} = \frac{0.5887}{B} \quad (11)$$

Where B is the cutoff frequency of the filter.

As B decreases, the spectral side-lobes are reduced further, but inter-symbol-interference (ISI) increases. It can be shown that the degradation is not severe if the 3-dB bandwidth-bit duration product (BT) of the filter is greater than 0.5.

5. Simulation and Results

The GFDM system parameters simulated in this work are given in Table 1. The system was evaluated with respect to three commonly used pulse-shaping filtering techniques. These Different types of pulse shaping filters (including the proposed one) are simulated and tested, and their effect on the power spectral density PSD and system performance were compared.

The conventional types of pulse shaping techniques used here are raised cosine, root-raised cosine and Dirichlet. Figure 2 shows the PSD of the GFDM with a rectangular pulse (no pulse shaping applied), which is the same as OFDM. It's clear from the figure that the signal has high OOB radiation. The PSD of the transmitted GFDM signal with conventional pulse shaping is shown in Figures 3, 4 and 5. One can conclude from the figures that the pulse shaping reduces the OOB radiation of the transmitted signal and hence enhances the spectral efficiency.

Table 1. Simulation parameters

Description	Parameter	Value
Number of subcarriers	K	128
Number of Subsymbols	M	14
Pulse Shaping Filter	Raised Cosine, Root Raised Cosine, Dirichlet, Gaussian	Rolloff Factor 0.5,

Length of Cyclic Prefix	CP	20% from Symbol Length
Type of Modulation	QAM	
Channel Type	AWGN	
GFDM Demodulator	Matched Filter	

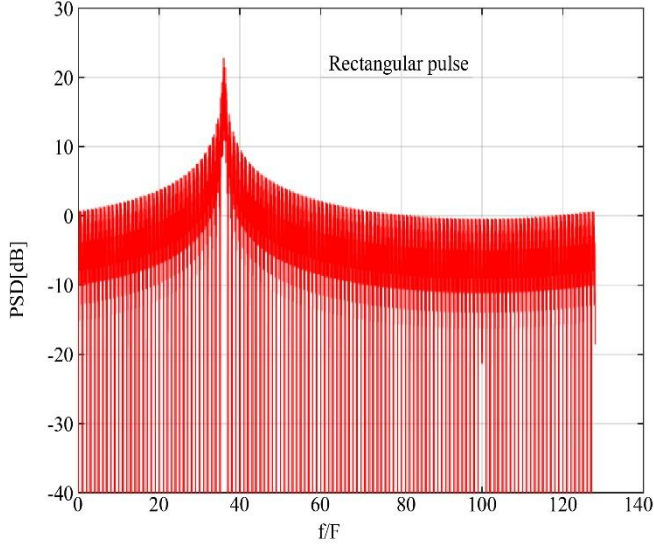


Fig. 2 PSD of the GFDM signal with a rectangular pulse (OFDM)

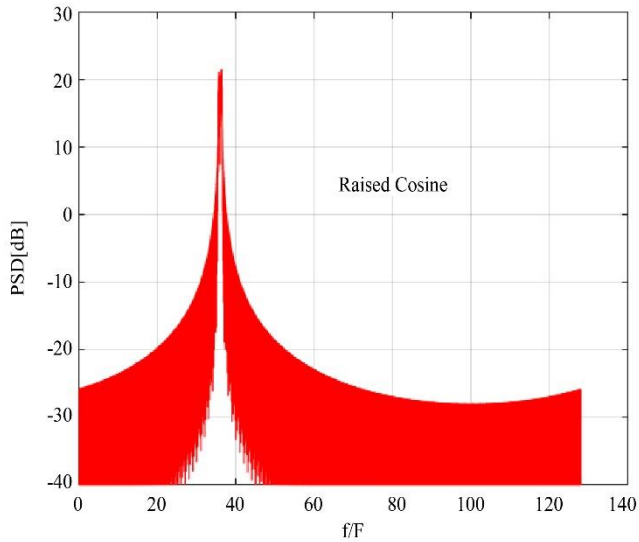


Fig. 3 PSD of GFDM with Raised Cosine Filter (roll-off factor 0.5)

The Gaussian pulse shaping filter is introduced in this work and evaluated with different BT products. The PSD of the GFDM signal with Gaussian pulse shaping is shown in Figures 6,7,8,9, and 10 for different BT values.

The simulation results show that as the BT product increases, the side lobes reduce further, and the OOB radiation decreases, but the ISI increases. From the figures, one can conclude that the best BT product of the Gaussian filter is 0.5 which gives PSD with low OOB radiation and acceptable ISI.

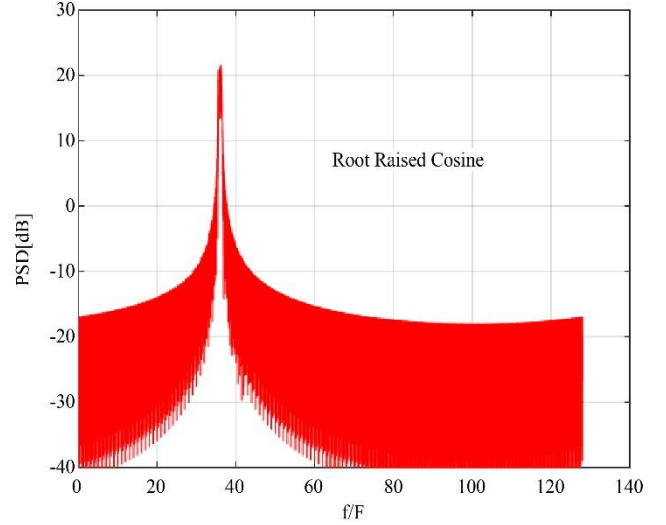


Fig. 4 PSD of GFDM with Root Raised Cosine Filter (roll-off factor 0.5)

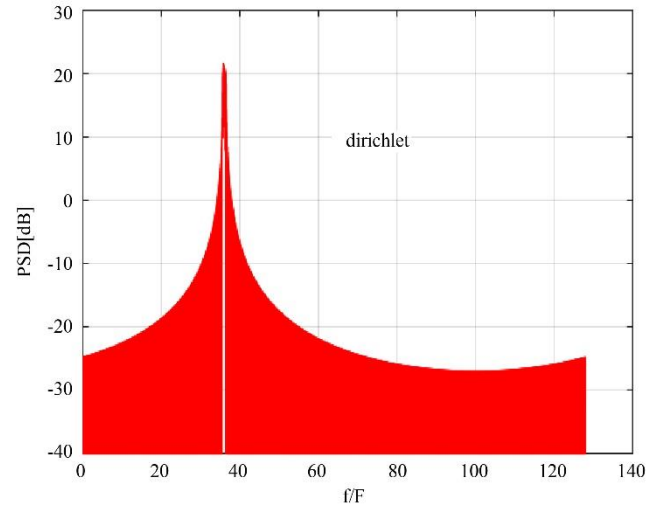


Fig. 5 PSD of GFDM with Dirichlet Filter

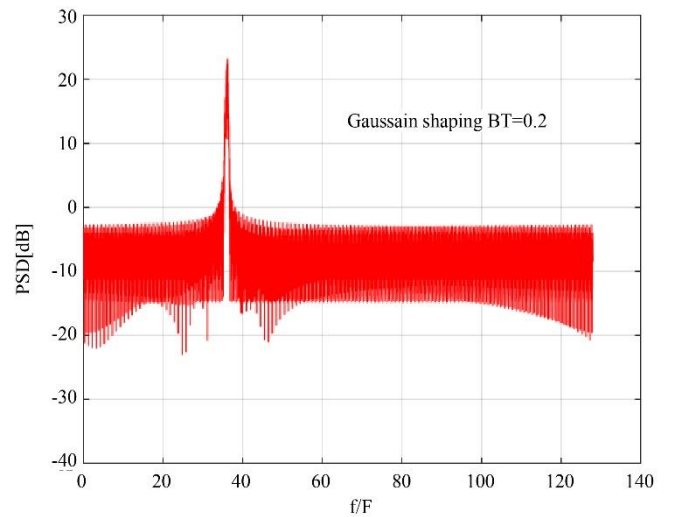


Fig. 6 PSD of GFDM with Gaussian pulse shaping BT = 0.2

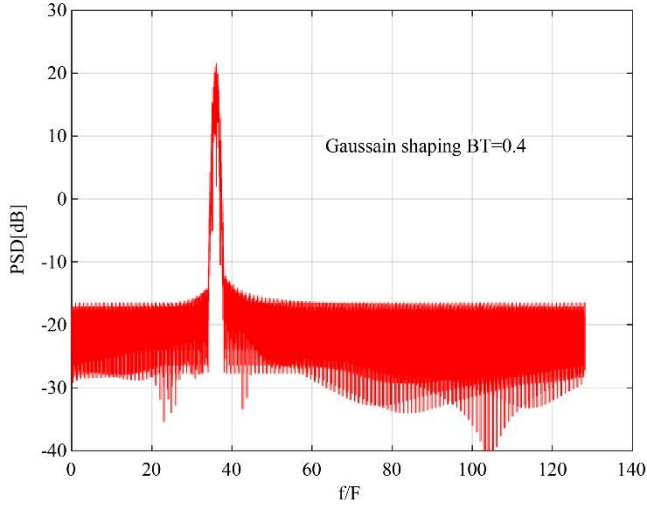


Fig. 7 PSD of GFDm with Gaussian pulse shaping BT = 0.4

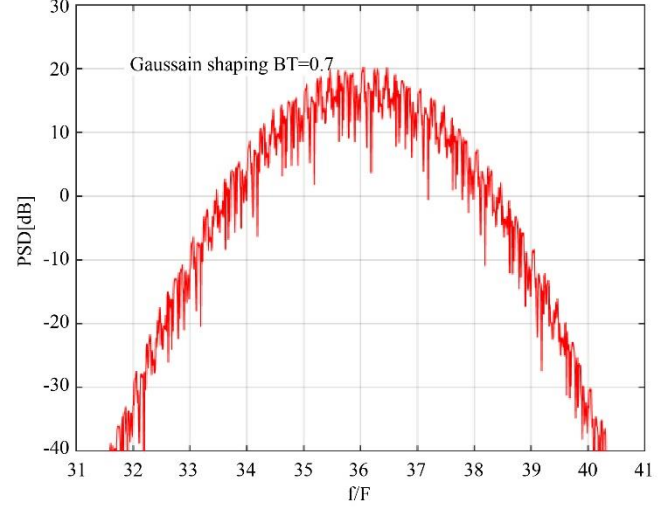


Fig. 10 PSD of GFDm with Gaussian pulse shaping BT = 0.7

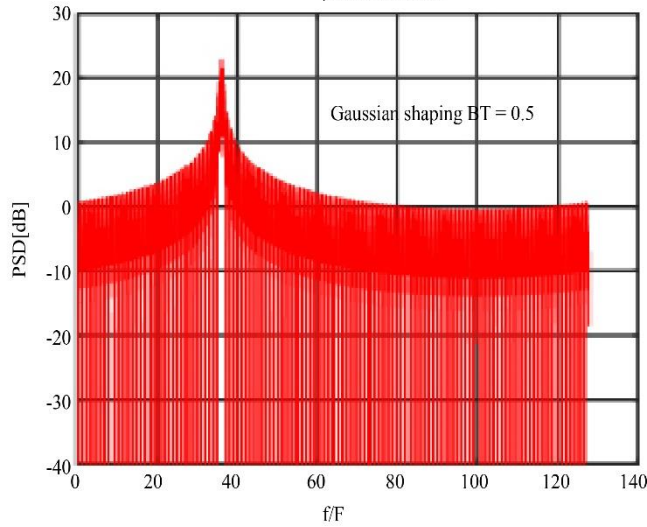


Fig. 8 PSD of GFDm with Gaussian pulse shaping BT = 0.5

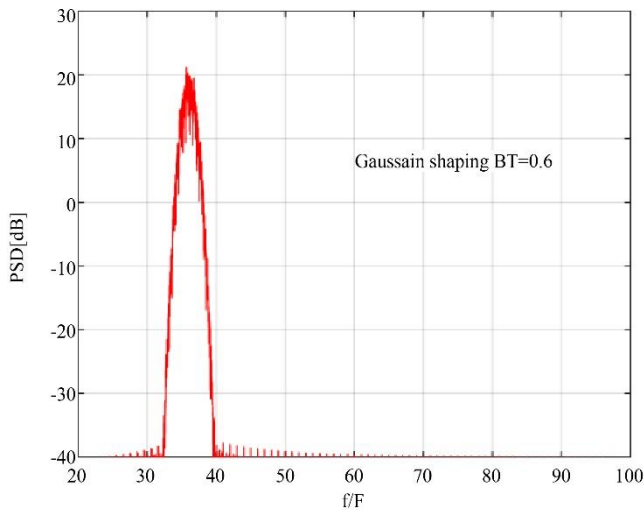


Fig. 9 PSD of GFDm with Gaussian pulse shaping BT = 0.6

The attenuation of the side lobes at the first null frequency for each PSD technique simulated in this work is shown in Table 2. From the table, it is clear that the Gaussian filter has a higher attenuation compared with other conventional filters. On the other hand, the side lobe attenuation is acceptable when the BT product is greater than 0.5.

The system performance was tested under the AWGN channel, the BER was measured with different SNR for different filter types, and the results are plotted in Figure 11. One can observe from the curve that the GFDm system with root-raised cosine pulse shaping technique gives performance identical to an OFDM system, and Dirichlet pulse shaping gives the worst-case performance. The performance of the proposed system with Gaussian pulse shaping is better than that of raised cosine and Dirichlet, and its approaches to the performance of root raised cosine will have less filter complexity. The SNR required to achieve a bit error rate of 10^{-4} for each pulse shaping is given in Table 3.

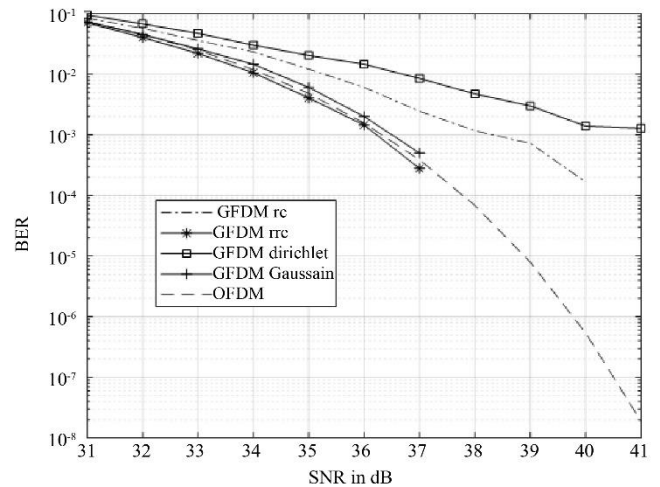


Fig. 11 BER VRS SNR for GFDm with different pulse shaping

Table 2. Side-lobe Attenuation for Different Filters at First Null Frequency

Filter Type	Side Lobe Attenuation in dB
Rectangles	8.96
Raised cosine (roll-off factor 0.5)	-10.26
Root Raised Cosine (roll-off factor 0.5)	-7.83
Dirichlet	-8.54
Gaussian BT = 0.2	-1.89
Gaussian BT = 0.4	-15.79
Gaussian BT = 0.5	-24.46
Gaussian BT = 0.6	-39.37
Gaussian BT = 0.7	-39.62

Table 3. SNR required to achieve BER of 10⁻⁴ for different pulse shaping

Type of Pulse Shaping	SNR in dB Required to Achieve BER of 10 ⁻⁴
Rectangular (OFDM)	11.7
Raised Cosine	14.2
Root Raised Cosine	11
Dirichlet	> 15
Gaussian	11

6. Conclusion

This work introduces a Gaussian filter to shape the pulses of a GFDM system. The simulation results show that the OOB radiation of the proposed system is reduced by a factor of 14.2 dB compared to the GFDM system with the raised cosine

shaping technique. Introducing the Gaussian filter enhanced the overall system performance compared to the other traditional techniques, and the SNR required to reach an acceptable performance (BER) is identical to the root-raised cosine shaping filter, which has the best performance compared to the other two traditional techniques.

References

- [1] Ivo Bizon Franco de Almeida et al., "5G Waveforms for IoT Applications," *IEEE Communications Surveys & Tutorials*, vol. 21, no. 3, pp. 2554-2567, 2019. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [2] Mohammad Hadi Abbaszadeh, Babak H. Khalaj, and Afroz Hagbini, "Optimum Low Complexity Filter Bank for Generalized Orthogonal Frequency Division Multiplexing," *EURASIP Journal on Wireless Communications and Networking*, vol. 2018, no. 1, pp. 1-14, 2018. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [3] Ching-Lun Tai, Tzu-Han Wang, and Yu-Hua Huang, "An Overview of Generalized Frequency Division Multiplexing (GFDM)," *Arxiv*, pp. 1-14, 2020. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [4] Fei Li et al., "Design and Performance of a Novel Interference-Free GFDM Transceiver with Dual Filter," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 5, pp. 4695-4706, 2019. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [5] Zee Ang Sim et al., "Performance of GFDM Systems Using Quadratic Programming Pulse Shaping Filter Design," *IEEE Access*, vol. 8, pp. 37134-37146, 2020. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [6] Randy Verdecia Peña, and Humberto Millán Vega, "Number of Subcarrier Filter Coefficients in GFDM System: Effect on Performance," *Ingenius*, vol. 23, pp. 53-61, 2020. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [7] Behzad Mozaffari Tazehkand, Mohammad Reza Ghavidel Aghdam, and Reza Abdoee, "Optimal Complex-Valued Prototype Filter Design for GFDM Systems," *Arxiv*, 2023. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [8] Mohammad Towliat, Morteza Rajabzadeh, and Seyyed Mohammad Javad Asgari Tabatabaee, "On the Noise Enhancement of GFDM," *IEEE Wireless Communications Letters*, vol. 9, no. 8, pp. 1160-1163, 2020. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [9] Peng Wei et al., "N-Continuous Signaling for GFDM," *IEEE Transactions on Communications*, vol. 68, no. 2, pp. 947-958, 2020. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [10] Nicola Michailow et al., "Generalized Frequency Division Multiplexing for 5th Generation Cellular Networks," *IEEE Transactions on Communications*, vol. 62, no. 9, pp. 3045-3061, 2014. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [11] Seyyed Mohammad Javad Asgari Tabatabaee, Morteza Rajabzadeh, and Mohammad Towliat, "A Novel Low-Complexity GFDM Relay Communication System: Relay Selection and Filter-and-Forward," *IEEE Transactions on Signal Processing*, vol. 69, pp. 5147-5158, 2021. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)
- [12] Ersin Ozturk, Ertugrul Basar, and Hakan Ali Cirpan, "Generalized Frequency Division Multiplexing with Index Modulation," *IEEE Globecom Workshops*, Washington, DC, USA, pp. 1-6, 2016. [\[CrossRef\]](#) [\[Google Scholar\]](#) [\[Publisher Link\]](#)

- [13] Ginne Rajesh Mehra, "FPGA Based Gaussian Pulse Shaping Filter Using Distributed Arithmetic Algorithm," *International Journal of Scientific & Engineering Research*, vol. 4, no. 8, pp. 711-715, 2013. [[Google Scholar](#)] [[Publisher Link](#)]
- [14] Maximilian Matthé et al., "Influence of Pulse Shaping on Bit Error Rate Performance and Out of Band Radiation of Generalized Frequency Division Multiplexing," *IEEE International Conference on Communications Workshops*, Sydney, NSW, Australia, pp. 43-48, 2014. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [15] Ching-Lun Tai, Borching Su, and Cai Jia, "Frequency-Domain Decoupling for MIMO-GFDM Spatial Multiplexing," *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing*, Brighton, UK, pp. 4799-4803, 2019. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [16] Shilpa Shukla, Puran Gour, "The Parametric Analysis of Gaussian Pulse Shaping Filter in WCDMA Network," *International Journal of Engineering Research & Technology*, vol. 2, no. 12, pp. 919-922, 2013. [[Google Scholar](#)] [[Publisher Link](#)]
- [17] N. Krishnapura et al., "A Baseband Pulse Shaping Filter for Gaussian Minimum Shift Keying," *1998 IEEE International Symposium on Circuits and Systems*, Monterey, CA, USA, pp. 249-252, 1998. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [18] Robert Bogdan Staszewski, and Poras T. Balsara, *All-Digital Frequency Synthesizer in Deep-Submicron CMOS*, John Wiley & Sons, pp. 1-280, 2006. [[Google Scholar](#)] [[Publisher Link](#)]