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Original Article

Leveraging the Gaussian Q-Function Approximation for Error Metrics Assessment of Digital Modulation Schemes in $\alpha-\kappa-\mu$ Fading Channel

Jyoti Gupta[1], Ashish Goel[2]

[1,2]*Department of ECE, Jaypee Institute of Information Technology, Noida, Uttar Pradesh, India.*

[2]*Corresponding Author : jyotiguptas21@gmail.com*

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Abstract *- This research includes the application of a Gaussian Q-function approximation for the error metrics analysis of communication systems. The Bit Error Rate (BER) and Symbol Error Probability (SEP) are paramount metrics for assessing wireless communication systems. The inherent fluctuation in signal intensity induced by fading effects necessitates a thorough analysis of error performance. The Gaussian Q-function appears to be an effective mathematical tool for calculating error probability in the context of random changes in channel strength. The Gaussian Q-function approximation is crucial for dealing with fading channels in communication systems. Leveraging the Gaussian Q-function approximations simplifies computations, boosting the utility of the proposed methodology in real-world communication scenarios. The present work generates a more accurate and simple approximate solution for error rate analysis for numerous modulation techniques. In this paper, we used popular digital modulation techniques for the application of Gaussian Q-function approximation in $\alpha-\kappa-\mu$ fading distribution. Monte-Carlo simulations validated the analytical results and accuracy of the proposed closed-form expression for various digital modulation schemes.*

Keywords *- Error metrics, $\alpha-\kappa-\mu$ fading, Gaussian Q-function and Digital modulation schemes.*

1. Introduction

Wireless communication is an ever-growing field with numerous applications, including cellular communication, Wi-Fi, Bluetooth, etc. In wireless communication, ensuring reliable and error-free communication is of utmost importance, but it is also quite difficult due to the impact of fading during the transmission of a signal. Data in wireless communication flows across an unpredictable channel that may be affected by shadowing, fading, or both. The dynamic and intricate nature of this communication medium creates issues that necessitate a thorough examination and understanding of these ramifications. One of the primary challenges in these scenarios is the combined impact of fading and shadowing. A thorough grasp of the subtleties of signal propagation under such difficult circumstances is essential to the successful design and implementation of wireless communication systems. To ensure error-free reception of signals over a wireless medium, current research intends to limit the influence of fading during the transmission of signals. There are various types of fading channels. In [1], two generalized fading distributions, the $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ were introduced by G. Fraidenraich and M. D. Yacoub. The other important fading channels, such as Rician, Nakagami-m, Rayleigh, one-sided Gaussian, Weibull, $\alpha-\mu$ and $\kappa-\mu$, are derived from this generalized fading distribution $\alpha-\kappa-\mu$ as their special case [2]. Moreover, computing important metrics like bit error rate and symbol error rate for numerous modulation techniques across intricate fading distributions becomes unfeasible in any case. Studying the accuracy and feasibility of the aforementioned methodologies is a subject of interest. Several researchers have proposed various methodologies to evaluate the error rate analysis. To demonstrate the legitimacy of the proposed work, we have chosen the $\alpha-\kappa-\mu$ fading distribution for the error rate analysis. In this paper, the authors used the well-known Moment-Generating Function (MGF) approach [3] to solve this problem. Furthermore, in the performance evolution of wireless communication systems, the Q-function has played a crucial role because the expression of error rate analysis has an intractable integral in the form of the Q-function. The need for Gaussian Q-function approximation arises from the complexity of the integral in its definition. In many practical scenarios, the integral cannot be expressed in closed form, and numerical methods or approximations are used. These approximations allow for more efficient analysis and optimization of communication systems over fading channels. The literature [4-14] includes numerous Gaussian Q-function approximations and bounds. In the previous study of this work, we compared a thorough literature survey on Q-function approximations [4] to [14] and examined their techniques, types of proposed approximations, and probable limits of each method. As a result, we discovered that some of the approximations are invalid across the full range of x, while others are mathematically difficult, making them unsuitable for algebraic manipulations in statistical performance analysis. Figure 2 and Table 2 show that the approximation provided in the previous study is more accurate and a simple combination of exponential functions that is valid across the whole range of x. It also possesses a generalized expression for any value of n. Owing to its simplicity and accurate representation, we adopted the Q-function approximation presented in the same.

Furthermore, we propose a new closed-form solution by leveraging this Q-function approximation for various digital modulation schemes in the $\alpha-\kappa-\mu$ fading channel. A. Goel and J. Gupta have proposed a closed-form solution for $\alpha = 2$, and in this paper, we propose a novel closed-form solution that is generalized for any values of α.

There are four sections in this paper. In Section II, We used the "Moment Generating Function (MGF)" to establish a unique, closed-form solution for "BPSK" and "TQAM 16" in the "α–κ–μ fading channel". Section III illustrates the outcomes obtained using the proposed method, as well as their validity over the fading channels. Finally, in Section IV, the contributions offered in this work are completed.

2. Mathematical Expression For Error Metrics Evaluation

In this paper, we have used BPSK and TQAM-16 modulation schemes to validate the proposed work over the $\alpha-\kappa-\mu$ fading channel. The main goal of this paper is to evaluate standardized and versatile equations for the error rate metrics of "BPSK" and "TQAM-16" over fading scenarios utilizing the MGF approach. The definition of the pdf for $\alpha-\kappa-\mu$ fading is characterized by a generalization as follows [2]:

$$f(y) = \sum_{j=0}^{\infty} \frac{0.5\,\alpha\,\mu^{\mu+2j}\kappa^{j}(1+\kappa)^{\mu+j}}{\Gamma(\mu+j)\,j!\,e^{\kappa\mu}\bar{\gamma}^{0.5\alpha(\mu+j)}}\, y^{0.5\alpha(\mu+j)-1} e^{-\frac{\mu(1+\kappa)}{\bar{\gamma}^{0.5\alpha}} y^{0.5\alpha}} \quad (1)$$

Where y represents the "instantaneous Signal-to-Noise Ratio (SNR)", $\bar{\gamma}$ is the "average SNR", "the parameter α specifies the "fading channel's nonlinearity", $\kappa > 0$ signifies the "proportion of total power attributed to the dominant component in relation to the scattered component", and $\mu > 0$ is the "count of multipath clusters" [15]. The "ABER" of the "BPSK" over any fading channel, whose distribution function $f(y)$ is given [15]:

$$ABER_{BPSK} = \int_0^{\infty} Q\left(\sqrt{2\gamma}\right) f(y)\, d\gamma \quad (2)$$

Where $Q(\cdot)$ is the Gaussian Q-function. The formula for Q-function relating $erfc()$ is denoted by [16]:

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \quad (3)$$

The newly proposed approximation of $erfc(x)$ for $n = 3$ is:

$$erfc(x) \approx \frac{1}{3}\left[e^{-44.1103x^2}\cosh(37.9011x^2) + e^{-2.1945x^2}\cosh(0.6533x^2) + e^{-1.1022x^2}\cosh(0.0898x^2)\right] \quad (4)$$

The MGF is very useful for assessing the efficacy of wireless communication systems, particularly when dealing with distribution functions that show the small-scale fading of multipath channel models [17]. It is essential in many applications, including the determination of bit error rate. In some cases, knowing the MGF function precisely makes it easier to evaluate these applications [17]. The generalized moment-generating Function (MGF) expression can be written in the following form:

$$M(s) = \int_0^{\infty} f(y) e^{-s\gamma}\, dy \quad (5)$$

Equation (1) can be formulated as:

$$f(y) = \Theta_1 \gamma^{\Theta_2 - 1} e^{-\Theta_3 \gamma^{0.5\alpha}} I_v(\Theta_4 \gamma^{0.25\alpha}) \quad (6)$$

where $I_v(.)$ signifies the "modified Bessel function of the first kind, order v". We obtain it using (5) and (6).

$$M(s) = \Theta_1 \int_0^{\infty} e^{-sy}\, \gamma^{\Theta_2-1} e^{-\Theta_3\gamma^{0.5\alpha}} I_v(\Theta_4\gamma^{0.25\alpha})\mathrm{d}y \quad (7)$$

[18] states that,

$$I_v(z) = \sum_{j=0}^{\infty} \frac{(0.5z)^{2j+v}}{j!\,\Gamma(j+v+1)} \quad (8)$$

and [19], $e^{-\tilde{\beta}z} = G_{0,1}^{1,0}\left(\tilde{\beta}z \middle|\, {}^{-}_{0}\right)$ (9)

with some adjustments, by using (6) and (8), the MGF expression can be written as

$$M(s) = \sum_{i=0}^{\infty} F \int_0^{\infty} e^{-\acute{s}z^{2/\alpha}} z^{R-1}\, e^{-\tilde{\beta}z}\, \mathrm{d}z \quad (10)$$

where $F = \frac{\Theta_1}{i!\,2^{2i+v}\,\Gamma(i+v+1)\,\bar{\alpha}\Theta_4{}^{2\Theta_2/\bar{\alpha}}}$, $R = \frac{\Theta_2}{\bar{\alpha}} + i + 0.5v$, $\acute{s} = \frac{s}{\Theta_4{}^{2/\alpha}}$ and $\acute{\beta} = \Theta_3/(\Theta_4)^2$

The final MGF expression is determined by applying [17, eqn. (2.24.1.1)] and the Meijer-G representation of the two exponentials, which is given in the form of (9).

$$M(s) = \sum_{i=0}^{\infty} F \left[\frac{k^{0.5}\, l^{0.5+R-1}}{\acute{\beta}^{R}(2\pi)^{0.5(l+k)-1}}\right] G_{l,k}^{k,l}\left(\frac{\acute{s}^k\, l^l}{k^k\,\acute{\beta}^l} \middle|\, {}^{\epsilon(l,1-R)}_{\epsilon(k,0)}\right) \quad (11)$$

Where $\eth(k,h) = \frac{h}{k}, \frac{h+1}{k}, \cdots, \frac{h+k-1}{k}$ and we set $2/\alpha = l/k$ to ensure that the greatest common divisor (gcd) of l and k is 1, allowing for non-integer values of α.

Where, $\Theta_1 = 0.5\frac{\alpha\mu(1+\kappa)^{0.5(1+\mu)}}{\kappa^{0.5(\mu-1)}\, e^{\mu\kappa}\, \bar{\gamma}^{0.25\alpha(\mu+1)}}$,

$\Theta_2 = 0.25\alpha(\mu+1)$, $\Theta_3 = \frac{\mu(1+\kappa)}{\bar{\gamma}^{0.5\alpha}}$, $\Theta_4 = \frac{2\mu\sqrt{\kappa(1+\kappa)}}{\bar{\gamma}^{0.25\alpha}}$,

$\bar{\alpha} = 0.5\,\alpha$, $v = \mu - 1$.

Using (3), the "ABER" for "BPSK" in a fading channel (2) can alternatively be expressed as follows.

$$ABER_{BPSK} = \frac{1}{2}\int_0^{\infty} erfc(\sqrt{\gamma})\, f(y) dy \quad (12)$$

Substituting (4) into (12), the expression of the ABER of BPSK is

$$ABER_{BPSK} = \frac{1}{12}\int_0^{\infty}\left[\sum_{r=1}^{6} e^{-U_m y}\right] f(y) dy \quad (13)$$

Now, using (5), we can transform (13) as

$$ABER_{BPSK_e} = \frac{1}{12}\left[\sum_{m=1}^{6} M(U_m)\right] \quad (14)$$

Where,

$$[U_m]_{m=1}^{6} = [6.2092,\ 82.0116,\ 1.5412,\ 2.848,\ 1.0124,\ 1.192]$$

The following is a general expression of "SEP" for numerous modulation methods in an additive white Gaussian noise channel [20]:

$$P_{AWGN} = KQ\left(\sqrt{\psi y}\right) + \frac{2}{3}K_C Q^2\left(\sqrt{\frac{2\psi y}{3}}\right) - 2K_C Q\left(\sqrt{\psi y}\right) Q\left(\sqrt{\frac{\psi y}{3}}\right) \quad (15)$$

Where,

$y =$ "SNR", $\psi =$ "Modulation techniques parameter"s,

$K =$ "Average number of nearest-neighbours",

$K_C =$ "Average number of nearest-neighbors and average number of couples of adjacent nearest-neighbors" [20]

The following $\psi = 2/9$, $K = 33/8$ and $K_C = 27/8$ are the defined symbol error probability parameters for TQAM-16 constellations [20].

Any digital modulation technique's symbol error probability is commonly expressed as a linear combination of the integrals below or as one of their special instances [16]:

$$I_1 = \int_0^{\infty} Q\left(a_1\sqrt{y}\right) Q\left(a_2\sqrt{y}\right) f(y)\, dy \quad (16a)$$

$$I_2 = \int_0^{\infty} Q^g\left(a_1\sqrt{y}\right) f(y)\, dy \quad (16b)$$

Where g is the "order of $Q(.)$" and $f(y)$ is the "probability density function of fading distribution". a_1 & a_2 are the "real positive constants that vary depending on the specific digital modulation technique" [1]. The "symbol error probability" of "triangular quadrature amplitude modulation-16" for fading channels is:

$$SEP_{fading} = P_{fading} = \int_0^{\infty} P_{AWGN}.f(y) dy \quad (17)$$

A novel equation for "symbol error probability" over the fading channel in "TQAM-16" can be obtained by using both (15) and (17).

$$SEP_{fading} = \int_0^{\infty}\left\{\frac{33}{8}Q\left(\sqrt{\frac{2}{9}y}\right) + \frac{9}{4}Q^2\left(\sqrt{\frac{4}{27}y}\right) - \frac{27}{4}Q\left(\sqrt{\frac{2}{9}y}\right)Q\left(\sqrt{\frac{2}{27}y}\right)\right\}.f(y)dy \quad (18)$$

The ultimate expression for "symbol error probability" over the "$\alpha-\kappa-\mu$ fading channel" for "TQAM-16" is derived by utilizing the complementary error function $erfc(x)$ approximation (4), within (18).

$$SEP_{fading} = \frac{33}{96}\left[\sum_{i=1}^{6} M(T_{1i})\right] + \frac{1}{64}\left[\sum_{i=1}^{6} M(T_{2i}) + \left\{2\sum_{i=1}^{15} M(T_{3i})\right\}\right] - \frac{3}{64}\left[\sum_{i=1}^{36} M(T_{4i})\right] \quad (19)$$

Where,

$$[T_{1i}]_{i=1}^{6} = [0.6899, 9.1124, 0.1712, 0.3164, 0.1125, 0.1324]$$

$$[T_{2i}]_{i=1}^{6} = [0.9198, 12.1498, 0.2284, 0.422, 0.15, 0.1776]$$

$$[T_{3i}]_{i=1}^{15} = \begin{bmatrix} 6.5348, 0.5741, 0.6709, 0.5349, 0.5482, \\ 6.1891, 6.2859, 6.1499, 6.1632, 0.3252, \\ 0.1892, 0.2025, 0.286, 0.2993, 0.1633 \end{bmatrix};$$

$$[T_{4i}]_{i=1}^{36} = \begin{bmatrix} 0.9199, 9.3424, 0.4012, 0.5464, 0.3425, 0.3624, \\ 3.7274, 12.1499, 3.2087, 3.3539, 3.15, 3.1699, \\ 0.747, 9.1695, 0.2283, 0.3735, 0.1696, 0.1895, \\ 0.7954, 9.2179, 0.2767, 0.4219, 0.218, 0.2379, \\ 0.7274, 9.1499, 0.2087, 0.3539, 0.15, 0.1699, \\ 0.734, 9.1565, 0.2153, 0.3605, 0.1566, 0.1765 \end{bmatrix}$$

3. Simulation Results and Discussion

The proposed closed-form expression tightness is examined in this section for various fading parameter values. Consequently, in Figure 1, it can be observed that across all $\alpha-\kappa-\mu$ fading channel scenarios, the average bit error rate graph generated by the approximated Q-function approximation aligns with the precise outcomes, the new suggested analytical, and the outcomes procured through simulations. Figure 1 illustrates the impact of BPSK modulation schemes for various fading factors, highlighting the effect of fading parameters on the average bit error rate. The impact of different fading parameters on the binary phase shift keying modulation technique is depicted in Figure 1, highlighting the effect of fading factors on error rate metrics. A discernible pattern is evident in Figure 1: ABER of the BPSK modulation technique falls as any parameter of the $\alpha-\kappa-\mu$ fading channel increases. The illustrated graphs show that the newly developed mathematical expressions and computer simulations agree perfectly.

Furthermore, Table 1 furnishes a comprehensive numerical juxtaposition of the exact, analytical, and proposed results for ABER of BPSK over $\alpha-\kappa-\mu$ fading channel. The findings acquired using the proposed ABER closed-form solution and computer simulations are 0.185936022912857 and 0.185899625000000, and Table 1 shows that the actual value of the ABER of BPSK for the combination of $\alpha = 0.5, \kappa = 1.5, \mu = 1.5$ at 0 dB SNR is 0.185784680695068. It is obvious that the proposed analytical and simulation outcomes are most accurate at capturing the true value of the bit error rate. Additionally, Table 1 also leads to similar conclusions for $\alpha = 0.5, \kappa = 2.5, \mu = 2$ at 40 dB, 0.000762442202935077, 0.000762330720370597 and 0.000768375000000000 for exact, analytical, and simulation, respectively. Therefore, the proposed work exhibits accuracy for values of SNRs.

Table 1. Comparing the accuracy of the "Average BER" of "BPSK" across the "α–κ–μ fading channel"

BPSK			
SNR (dB)	**Exact**	**Analytical**	**Simulation**
$\boldsymbol{\alpha = 0.5, \kappa = 1.5, \mu = 1.5}$			
0	0.185784680695068	0.185936022912857	0.185899625000000
20	0.0368876393986615	0.0369117862504333	0.0369708750000000
40	0.00605575727125999	0.00606029393215632	0.00607825000000000
$\boldsymbol{\alpha = 0.5, \kappa = 1.5, \mu = 2}$			
0	0.170310564440707	0.170269726597826	0.170336750000000
20	0.0217714397101806	0.0217684171938324	0.0218221250000000
40	0.00197408270317223	0.00197394650032718	0.00199400000000000
$\boldsymbol{\alpha = 0.5, \kappa = 2.5, \mu = 2}$			
0	0.156901233541226	0.156779134004927	0.157167500000000
20	0.0130262317347009	0.0130227389523395	0.0130463750000000
40	0.000762442202935077	0.000762330720370597	0.000768375000000000
$\boldsymbol{\alpha = 1.5, \kappa = 1.5, \mu = 1.5}$			
0	0.123400252318754	0.123459358425260	0.123597250000000
20	0.000854724465217732	0.000854793697329846	0.000862125000000000
40	4.61203803419728e-06	4.61243044402525e-06	4.00000000000000e-06

Table 2. Comparing TQAM-16's accuracy in SEP across the α–κ–μ fading

TQAM-16			
SNR (dB)	**Exact**	**Analytical**	**Simulation**
$\boldsymbol{\alpha = 0.5, \kappa = 1.5, \mu = 1.5}$			
9	0.435662497174856	0.434947979694228	0.432758000000000
18	0.231086316145937	0.230859354452760	0.229436000000000
27	0.108801263979374	0.108716793845646	0.107748000000000
$\boldsymbol{\alpha = 0.5, \kappa = 1.5, \mu = 2.5}$			
9	0.398977035806779	0.398968053779999	0.396336000000000
18	0.160657054895342	0.160783765803531	0.159480000000000
27	0.0497942231651174	0.0498384976335870	0.0490460000000000
$\boldsymbol{\alpha = 0.75, \kappa = 1.5, \mu = 2.5}$			
9	0.371610835963793	0.372635878048654	0.367618000000000
18	0.0932450942297051	0.0933960076895923	0.0932720000000000
27	0.0144340843463457	0.0144513486531934	0.0140300000000000

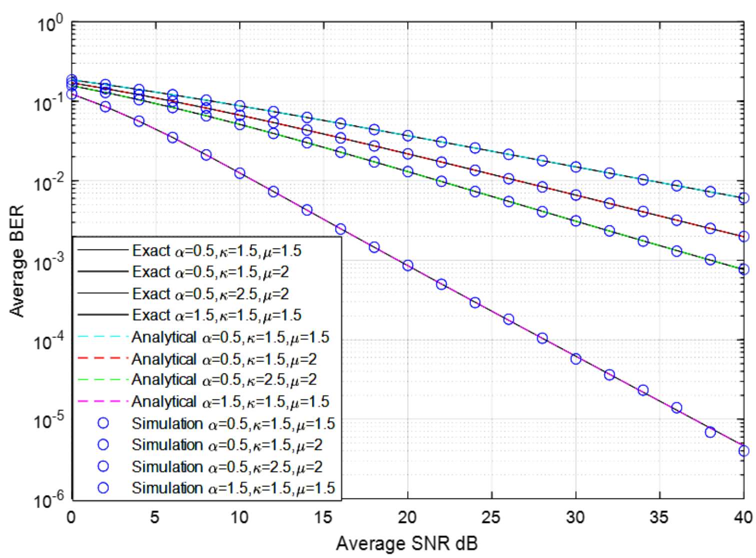

Fig. 1 Illustration of "ABER" of "BPSK" over $\alpha - \kappa - \mu$ fading

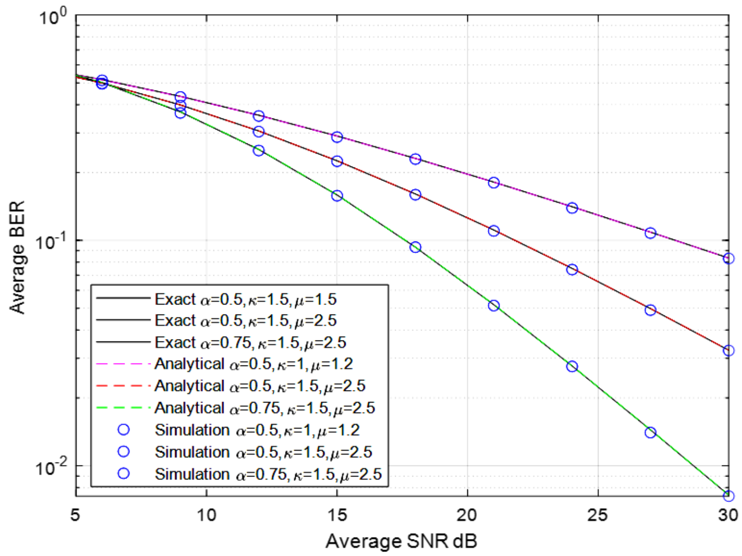

Fig. 2 Illustration of Symbol error probability of TQAM-16 over α– κ– μ fading channel

Figure 2 further shows that the symbol error probability results of the TQAM-16 across fading channels are better for $\alpha = 0.5,\ \kappa = 1.5$ and $\mu = 2.5$ than for $\alpha = 0.5,\ \kappa = 1.5$ and $\mu = 1.5$, and it is much better when $\alpha = 0.5,\ \kappa = 1.5$ and $\mu = 2.5$ are used. Based on the observed trends in the SEP results, it can be concluded that for a prescribed value of α, the "SEP performance" of "TQAM-16" modulation techniques exhibits improvement with the increase of either κ or μ or increase in both of them, and further improvement can be achieved by increasing the fading parameter α.Moreover, to illustrate the accuracy of the new closed-form expression, we examined several combinations of the α–κ–μ fading factors listed in Table 2. As stated in Table 2, the actual value of the average SEP of TQAM-16 for the values $\alpha = 0.5, \kappa = 1.5$ and $\mu = 2.5$, is 0.398977035806779 at SNR 9 dB, and the analytical and simulation findings that correlate to these values are 0.398968053779999 and 0.396336000000000, respectively. Similar to this, for $\alpha = 0.75, \kappa = 1.5$ and $\mu = 2.5$, the actual, analytical, and simulation values of the SEP of TQAM-16 at 27 dB are, respectively, 0.0144340843463457, 0.0144513486531934, and 0.0140300000000. These results show that the results of analytical expression, Monte Carlo simulations, and exact expression using Q-function are also almost identical for all SNR levels.

4. Conclusion

The average BER and SEP closed-form solutions for BPSK and TQAM-16 across the α–κ–μ fading channel are derived in this work utilizing the moment-generating function (MGF). How closely the precise and simulated findings match the analytical outcomes serves as proof of the expression's validity. The exponential-based approximation of the Q-function is utilized to deduce the proposed closed-form expression for BPSK and TQAM-16. The newly developed MGF expressions provide more effective computational evaluation and simplified analytical manipulation. In order to demonstrate the applicability and correctness of the novel moment-generating function expressions, the error rates for BPSK and TQAM-16 are examined.

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