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Original Article

Diffraction of Acoustic Harmonic Waves in a Viscoelastic Cylinder

N.U. Kuldashov[1], B.Z. Nuriddinov[1*], M. Choriev[1], A. Sh.Ruzimov[1], Sh. F. Xalilov[1]

[1]*Tashkent Institute of Chemical Technology, Tashkent, Uzbekistan.*

[*]*Corresponding Author : baxtiyor.12.27.1985@gmail.com*

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Abstract *- The study is devoted to the study of the diffraction process of acoustic waves in a viscoelastic deformable cylinder. The study aims to study the diffraction of acoustic harmonic waves in a viscoelastic cylinder. The solution to the problem of diffraction of acoustic waves in a viscoelastic cylinder is obtained. This will allow us to determine the force factors in the form of the stress-strain state of the deformable cylinder. In the case of steady oscillations, the Helmholtz equation describes the propagation of small disturbances in an acoustic medium. And in a viscoelastic homogeneous isotropic cylinder - scalar and vector Helmholtz equations with complex coefficients. The stress and displacement of a point of a viscoelastic cylinder take on a maximum value in the region of long waves. The stresses and displacements of the points of the viscoelastic cylinder reach their maximum in the region of long waves. A method for solving and an algorithm for the problem of diffraction of acoustic waves in a viscoelastic cylinder have been developed.*

Keywords *- Viscoelastic cylinder, Wave, Wave equation, Hereditary integral, Bessel equation.*

1. Introduction

The problems of diffraction of plane harmonic acoustic waves in homogeneous isotropic elastic cylinders have been investigated [1,2,3,4]. When an acoustic wave interacts with an inhomogeneous medium, the wave field changes and it is necessary to consider the reflected waves that appear. Here the interaction of sound with bodies of finite dimensions or with bodies in which at least one of the characteristic dimensions is finite is considered. Drawing attention to the wide variety of such acoustic situations, it is noted that the interaction of the incident wave and obstacles, in general, affects the waveform, the direction of propagation and the distribution of amplitude along the front. The oblique incidence of incident waves is investigated in [5, 6]. The problems of acoustic wave scattering in inhomogeneous and anisotropic cylindrical bodies are investigated in [7, 8]. The incident acoustic wave is considered a plane; the wavefront is parallel to the longitudinal axis [9, 10]. From a mathematical point of view, problems of diffraction of acoustic waves in viscoelastic cylindrical bodies are much more complex than problems of diffraction in elastic cylindrical bodies [11,27]. The diffraction of elastic waves in inhomogeneous cylindrical elastic bodies placed in a deformable medium is discussed in [13,14]. In [15], the problem of elastic wave diffraction in a viscoelastic cylindrical body was considered. The problems of propagation of natural waves in extended viscoelastic plates of variable thickness are devoted to the works [16,17]. At the contact of the cylindrical body with the medium, the conditions of rigid contact are set. At infinity, the Sommerfeld radiation conditions are set.

In addition to the problem of wave propagation in an infinite cylinder, the problems of wave propagation in a sphere, an ellipsoid and other bodies have exact solutions.

This article considers the diffraction of acoustic waves in a viscoelastic cylinder. The methodology and algorithm of the solution were developed. Numerical results are obtained.

2. Methods

2.1. Problem Statement and Solution Technique

Let a harmonic wave with frequency ω propagate in a homogeneous medium characterized by density ρ and sound speed *c*. This is called an incident wave, and its complex pressure amplitude is denoted by *p*. The problem of diffraction of acoustic waves in a viscoelastic cylinder is considered. The task is set in cylindrical coordinates. Suppose that a plane wave is incident on this body. A cylindrical body is represented by equations Lame in displacements [18]:

$$\tilde{\mu}\nabla^2\vec{u} + (\tilde{\lambda} + \tilde{\mu})graddiv\vec{u} = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \qquad (1)$$

where $\vec{u}(u_1, u_2, u_3)$ is the displacement vector of the medium, ρ is the density of the cylinder,

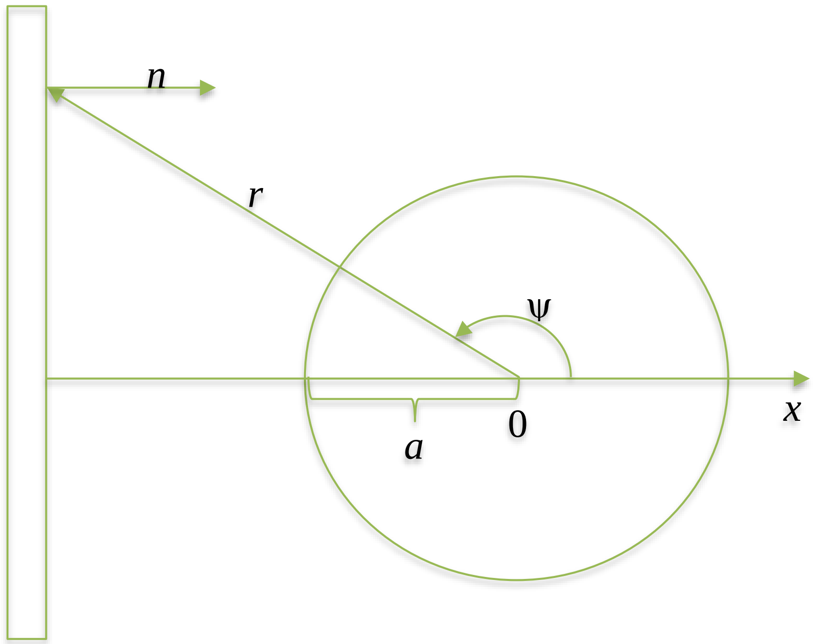

Fig. 1 An example of an incident plane wave on a cylinder

$$\tilde{\lambda} f(t) = \lambda_0 \left[f(t) - \int_{-\infty}^{t} R_\lambda(t-\tau) f(\tau) d\tau \right];$$

$$\tilde{\mu} f(t) = \mu_0 \left[f(t) - \int_{-\infty}^{t} R_\mu(t-\tau) f(\tau) d\tau \right],$$

$f(t)$ is an arbitrary function of time, $R_\lambda(t-\tau)$ and $R_\mu(t-\tau)$ are the relaxation kernels, $\lambda_0$and μ_0 are the instantaneous moduli of elasticity. Calculations used the Koltunov-Rzhanitsyn three-parameter relaxation kernel [19]: $R_k(t) = A_k e^{-\beta_k t}/t^{1-\alpha_k}$. The path that the plane wavefront takes is determined by the scalar product of the vectors ***nr***, where ***n*** is the unit vector ($|n| = 1$) of the normal to the surface of the wavefront, and r is the radius vector from the origin O to a point on the front surface. It is clear that $nr = rcos\psi$ ($r = |r|, \psi$ is the angle between n and r).

Let an infinitely long elastic cylinder with a radius, which is placed in an unlimited viscous medium, be affected by a plane harmonic wave of unit amplitude, the front of which is perpendicular to the z-axis of the cylinder [20]:

$$\varphi^{(p)} = e^{ik_{01}t} e^{-i\omega t} = e^{ik_{01} r \cos\theta} e^{-i\omega t} \quad (2)$$

where r, θ are cylindrical coordinates; ω - circular frequency; t - time.

Full acoustic field velocity potential

$$\varphi_c^{(1)} = \Phi_c^{(1)} e^{-i\omega t}.$$

In what follows, the exponential factor will be omitted.

The propagation of waves in an acoustic (in an ideal fluid) medium, in the case of steady oscillations, is described by the equation.

$$\Delta \Phi_c^{(1)} + k_{01}^2 \Phi_c^{(1)} = 0,$$

where $\Phi_c^{(1)} = \Phi_p^{(1)} + \Phi_s^{(1)}$ is the velocity potential of the full acoustic field in the outer region, $\Phi_s^{(1)}$ is the velocity potential of the scattered wave. In this case, the particle velocity and acoustic pressure $\vec{v}$ in the liquid are determined by the formulas.

$$\vec{v} = grad\Phi_c^{(1)}, p = i\rho_0 \omega \Phi_c^{(1)}$$

where $\rho_0 -$ is the density of the material of the acoustic medium, ω is the frequency, and λ_{01} are the elastic characteristics of the acoustic medium. The mixing of a cylindrical body is described using the Green-Lemb expansion. Then the longitudinal and transverse displacements satisfy the following relations.

$$\varphi^{(2)} = \Phi^{(2)} e^{-i\omega t}, \psi^{(2)} = \Psi^{(2)} e^{-i\omega t}.$$

Vibrations of a viscoelastic homogeneous isotropic cylinder, in the case of harmonic motion, are described by the scalar and vector Helmholtz equations [28]:

$$\frac{\partial^2 \Phi_0^{(2)}}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi_0^{(2)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi_0^{(2)}}{\partial \theta^2} + k_1^2 \Phi = 0\ ,$$

$$\frac{\partial^2 \Psi_0^{(2)}}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi_0^{(2)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi_0^{(2)}}{\partial \theta^2} + k_2^2 \Psi_0^{(2)} = 0. \quad (3)$$

The problem of determining the diffraction pattern leads to the solution of the Helmholtz equations.

$$U_r = \frac{\partial \varphi_0^{(2)}}{\partial r} + \frac{1}{r}\frac{\partial \Psi_0^{(2)}}{\partial \theta}, U_\theta = \frac{1}{r}\frac{\partial \varphi_0^{(2)}}{\partial \theta} - \frac{\partial \Psi_0^{(2)}}{\partial r}, \quad (4)$$

Where $\kappa_1^2 = \frac{\omega^2}{c_{p1}^2 \Gamma_{\lambda\mu}}, \kappa_2^2 = \frac{\omega^2}{c_{s1}^2 \Gamma_\mu}, c_{p1}^2 = (\lambda_0 + 2\mu_0)/\rho_1, c_{s1}^2 = \mu_0/\rho_1$ is the propagation velocity of longitudinal and transverse waves;

$$\Gamma^\bullet_{\lambda\mu} = 1 - \Gamma^C_{\lambda\mu}(\omega_R) - i\Gamma^S_{\lambda\mu}(\omega_R), \Gamma^\bullet_\mu = 1 - \Gamma^C_\mu(\omega_R) - i\Gamma^S_\mu(\omega_R),$$

$$\Gamma^c_\lambda(\omega_R) = \int_0^\infty R_\lambda(\tau)\cos\omega_R\tau d\tau, \Gamma^c_\mu(\omega_R) = \int_0^\infty R_\mu(\tau)\cos\omega_R\tau d\tau,$$

$$\Gamma^s_\lambda(\omega_R) = \int_0^\infty R_\lambda(\tau)\sin\omega_R\tau d\tau, \Gamma^s_\mu(\omega_R) = \int_0^\infty R_\mu(\tau)\sin\omega_R\tau d\tau,$$

$\Gamma^C_{\lambda\mu}(\omega_R)$,$\Gamma^S_{\lambda\mu}(\omega_R)$,$\Gamma^C_\mu(\omega_R)$,$\Gamma^S_\mu(\omega_R)$ are the cosine and sine of the Fourier image of the relaxation kernels, respectively, are defined in a similar way, $\varphi = \varphi_i + \varphi_s$ are the scalar potential of longitudinal waves in a liquid, which is summed from the potential of φ_i- longitudinal incident waves and the potential of φ_s - longitudinal reflected waves, Φ is the scalar potential of longitudinal waves in a liquid, $\Psi$and ψ are, respectively, scalar potentials of transverse waves in a viscoelastic cylinder and liquid.

On the surface of the cylinder at $r=a$, the following boundary conditions must be met:

$$p_{rr} = -\sigma_{rr}, V_r = -iwU_r, \quad p_{r\theta} = -\sigma_{r\theta}, V_\theta = -iwU_\theta, \quad (5)$$

where V_r, V_θ are the normal and tangent to fluid particles; U_r, U_θ- normal and circumferential mixing of an elastic medium; $p_{rr}, p_{r\theta}$- normal and tangential components of the stress tensor in the fluid; $\sigma_{rr}, \sigma_{r\theta}$ - normal and tangential components of the stress tensor in the cylinder. For a rigid cylinder, the radial velocity on its surface is zero.

At infinity must be satisfied the conditions [22,23]:

$$r\left(\frac{\partial \varphi_c}{\partial n} + ik_1\varphi_c\right)_{r\to\infty} = 0\left(\frac{1}{r}\right), (\varphi_c)_{r\to\infty} = 0\left(\frac{1}{r}\right). \quad (6)$$

We are looking for a solution to the problem in the form of a series. To do this, we expand the function corresponding to the incident plane wave in a Fourier series:

$$\Phi_s^{(1)} = e^{ik_{01}r\cos\theta} = \sum_{n=-\infty}^{\infty} J_n(k_{01}r)e^{in\theta}.$$

We represent the velocity potential of the reflected wave as a superposition of cylindrical waves emanating from points on the axis of the cylinder:

$$\Phi_s^{(1)} = \sum_{n=-\infty}^{\infty} A_n H_n^1(k_1 r)e^{in\theta}$$

The potential of the common field

$$\Phi_c^{(1)} = \Phi_p^{(1)} + \Phi_s^{(1)} = \sum_{n=-\infty}^{\infty}(J_n(k_1 r) + A_n H_n^1(k_1 r))e^{in\theta} \quad (7)$$

And the potentials in a viscoelastic solid cylinder have the form

$$\Phi_0^{(2)} = \sum_{n=-\infty}^{\infty} C_n H_n^{(1)}(k_1 r)e^{in\theta}, \quad (8)$$

$$\Psi_0^{(2)} = \sum_{n=-\infty}^{\infty} D_n H_n^{(1)}(k_2 r)e^{in\theta}. \quad (9)$$

The displacements of a viscoelastic cylinder are related to the scalar potential and the only nonzero component of the vector potential by the relations [24]:

$$U_r = \frac{\partial \psi}{\partial r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}; \quad U_\theta = \frac{1}{r}\frac{\partial \psi}{\partial \theta} - \frac{\partial \psi}{\partial r}$$

then

$$U_r = -\left(k_1 C_n H_n^{(1)}(k_1 r) + \frac{in}{r} D_n H_n^{(1)}(k_2 r)\right)e^{in\theta}, \quad (10)$$

$$U_\theta = -\left(\frac{in}{r} C_n H_n{}^{(1)}(k_1 r) - k_2 D_n H_n^{(1)}(k_2 r)\right)e^{in\theta}.$$

The radial and tangential components of the stress tensor in a viscous fluid are determined by the relations:

$$p_{rr} = p_0\frac{\partial \varphi}{\partial t} - 2\mu_0\left(\frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial \phi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \phi}{\partial r \partial \theta}\right),$$

$$p_{r\theta} = 2\mu_0\left(\frac{1}{r}\frac{\partial^2 \varphi}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial \varphi}{\partial \theta} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} - \frac{1}{2\nu_0}\frac{\partial \phi}{\partial t}\right).$$

then

$$p_{rr} = \sum_{n=-\infty}^{\infty}\left(\left(i\omega\rho_0 - \lambda_0 k_1^2 - 2\mu_0\frac{n^2}{r^2}\right)J_n(k_1 r) - 2\mu_0\frac{k_1}{r}J_n(k_1 r)\right)e^{in\theta} +$$
$$+ \sum_{n=-\infty}^{\infty}\left(\left(i\omega\rho_0 - \lambda_0 k_1^2 - 2\mu_0 k_1^2 + 2\mu_0\frac{n^2}{r^2}\right)H_n^{(1)}(k_1 r) - 2\mu_0\frac{k_1}{r}H_n^{(1)}(k_1 r)\right)A_n e^{in\theta} +$$
$$+2\mu_0\sum_{n=-\infty}^{\infty}\left(\frac{in}{r}k_2 H_n^{(1)}(k_2 r) - \frac{in}{r^2}H_n^{(1)}(k_2 r)\right)B_{n^{e^{in\theta}}}; \quad (11)$$

$$p_{r\theta} = 2\mu_0\sum_{n=-\infty}^{\infty}\left(\frac{ni}{r}k_1 J_n(k_1 r) - \frac{ni}{r^2}J_n(k_1 r)\right)e^{in\theta} +$$

$$+2\mu_0 \sum_{n=-\infty}^{\infty} \left(\frac{ni}{r} k_1 H_n^{(1)}(k_1 r) - \frac{ni}{r^2} H_n^{(1)}(k_1 r)\right) A_n e^{in\theta} +$$
$$+2\mu_0 \sum_{n=-\infty}^{\infty} \left(\frac{k_2^2}{2} H_n^{(1)}(k_1 r) - \frac{n^2}{r^2} H_n^{(1)}(k_2 r) + \frac{k_2}{r} H_n^{(1)}(k_2 r)\right) B_n e^{in\theta}.$$

The radial and tangential components of the stress tensor in a viscoelastic cylinder are determined by the relations.

$$-\frac{\sigma_{rr}}{2\mu_{01}} = \frac{\bar{\lambda}+2\bar{\mu}}{2\mu_{01}} k_1^2 \psi + \frac{1}{r}\frac{\partial \psi}{\partial \theta} + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta^2} - \frac{1}{r}\frac{\partial^2 \psi}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta};$$
$$\frac{\sigma_{\theta\theta}}{2\mu_{01}} = -\frac{\bar{\lambda}}{2\mu_{01}} k_1^2 \psi + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \psi}{\partial r \partial \theta},$$
$$\frac{\sigma_{r\theta}}{2\mu_{01}} = \frac{1}{2} k_2^2 \Gamma_\mu \ \psi + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \psi}{\partial r \partial \theta}.$$

then

$$\sigma_{rr} = -2\bar{\mu} \sum_{-\infty}^{+\infty} \left(\frac{\lambda+2\mu}{2\mu} k_1^2 H_n^{(1)}(k_1 r) + \frac{k_1}{r} H_n^{(1)}(k_1 r) - \frac{n^2}{r^2} H_n^{(1)}(k_1 r)\right) C_n e^{in\theta} -$$
$$-2\bar{\mu} \sum_{-\infty}^{+\infty} \left(\frac{in}{r} k_2 H_n^{(1)}(k_2 r) - \frac{in}{r^2} H_n^{(1)}(k_2 r)\right) D_n e^{in\theta},$$
$$\sigma_{r\theta} = 2\mu \sum_{-\infty}^{+\infty} \left(\frac{k_4^2+2\mu}{2} H_n^{(1)}(k_4 r) + \frac{k_4}{r} H_n^{(1)}(k_4 r) - \frac{n^2}{r^2} H_n^{(1)}(k_4 r)\right) D_n e^{in\theta} -$$
$$-2\mu \sum_{-\infty}^{+\infty} \left(\frac{in}{r^2} H_n^{(1)}(k_3 r) - \frac{in}{r} k_3 H_n^{(1)}(k_3 r)\right) C_n e^{in\theta}. \quad (12)$$

Satisfying the boundary conditions on the surface of the cylinder (2), a system of equations for finding arbitrary constants is obtained A_n, B_n, C_n, D_n

$$a_{k1} A_n + a_{k2} B_n + a_{k3} C_n + a_{k4} D_n = b_k \quad (13)$$

then

$$a_{11} = k_{01} a H_n^{(1)}(k_{01} a), a_{12} = in H_n^{(1)}(k_{02} a), a_{13} = iwak_1 H_n^{(1)}(k_1 a), a_{14} = -\omega n H_n^{(1)}(k_2 a), a_{21} = in H_n^{(1)}(k_{01} a), a_{22} = -k_{02} a H_n^{(1)}(k_{02} a),$$
$$a_{23} = -\omega n H_n^{(1)}(k_1 a), a_{24} = -i\omega a k_2 H_n^{(1)}(k_2 a), \quad (14)$$

$$a_{31} = (i\omega\rho_0 a^2 + 2\mu_0 n^2) H_n^{(1)}(k_{01} a) - 2\mu_0 k_{01} a H_n^{(1)}(k_{01} a),$$
$$a_{32} = 2\mu_0 (ink_{02} a H_n^{(1)}(k_{02} a) - in H_n^{(1)}(k_{02} a) = 2\mu_0 in (k_{02} a H_n^{(1)}(k_{02} a) - H_n^{(1)}(k_{02} a),$$

$$a_{33} = 2\bar{\mu} \left(\frac{\bar{\lambda}+2\bar{\mu}}{2\bar{\mu}} k_1^2 a^2 H_n^{(1)}(k_1 a) + k_1 a H_n^{(1)}(k_1 a) - n^2 H_n^{(1)}(k_1 a)\right),$$

$$a_{34} = -2\bar{\mu} in \left(k_2 a H_n^{(1)}(k_2 a) - H_n^{(1)}(k_2 a)\right),$$
$$a_{41} = 2\mu_0 (ink_{01} a H_n^{(1)}(k_{01} a) - in H_n^{(1)}(k_{01} a)) = 2\mu_0 ni (k_{01} a H_n^{(1)}(k_{01} a) - H_n^{(1)}(k_{01} a)),$$

$$a_{42} = 2\mu_0 \left(\frac{k_{02}^2 a^2}{2} H_n^{(1)}(k_{02} a) + k_{02} a H_n^{(1)}(k_{02} a) - n^2 H_n^{(1)}(k_{02} a)\right),$$

$$a_{43} = 2\mu in \left(H_n^{(1)}(k_1 a) - k_1 a H_n^{(1)}(k_1 a)\right),$$
$$a_{44} = -\mu_0 \left(\frac{k_2^2 a^2}{2} H_n^{(1)}(k_2 a) + k_2 a H_n^{(1)}(k_2 a) - n^2 H_n^{(1)}(k_2 a)\right),$$

$$b_1 = -k_{01} a J_n(k_{01} a), b_2 = -in J_n(k_{01} a),$$
$$b_3 = (i\omega\rho_0 a^2 - 2\mu_0 n^2) J_n(k_{01} a) + 2\mu_0 k_{01} a J_n(k_{01} a),$$
$$b_4 = 2\mu_0 (J_n(k_{01} a) - k_{01} a J_n(k_{01} a)).$$

The pressure in a viscous medium consists of the sum of the pressures of incident p_i and reflected p_s waves [23]:

$$p = p_i + p_s \quad , \quad (15)$$

where in accordance with the formula

$$p = (i\omega\rho_0 - (\lambda_0 + 2\mu_0) k_{01}^2)\varphi$$

and solutions (4) and (5)

$$p_i = (iw\rho_0 - (\lambda_0 + 2\mu_0) k_{01}^2) \sum_{n=-\infty}^{\infty} J_n(k_{01} r) e^{in\theta}, \quad (16)$$

$$p_s = (iw\rho_0 - (\lambda_0 + 2\mu_0) k_{01}^2) \sum_{n=-\infty}^{\infty} A_n H_n(k_{01} r) e^{in\theta}. \quad (17)$$

3. Results and Analysis

Numerical results were obtained using the Matlab software package. On Figure 2 shows the positional scattering cross-section for the corresponding values of the cylinder wave radius ka.

The scattering of sound by a cylinder at low frequencies is considered, i.e. at ka < 1. In calculations, this corresponds to ka ≤ 0.3. On the one hand, attention should be paid to the character of the curve and, on the other hand, to the magnitude of the positional scattering cross-section.

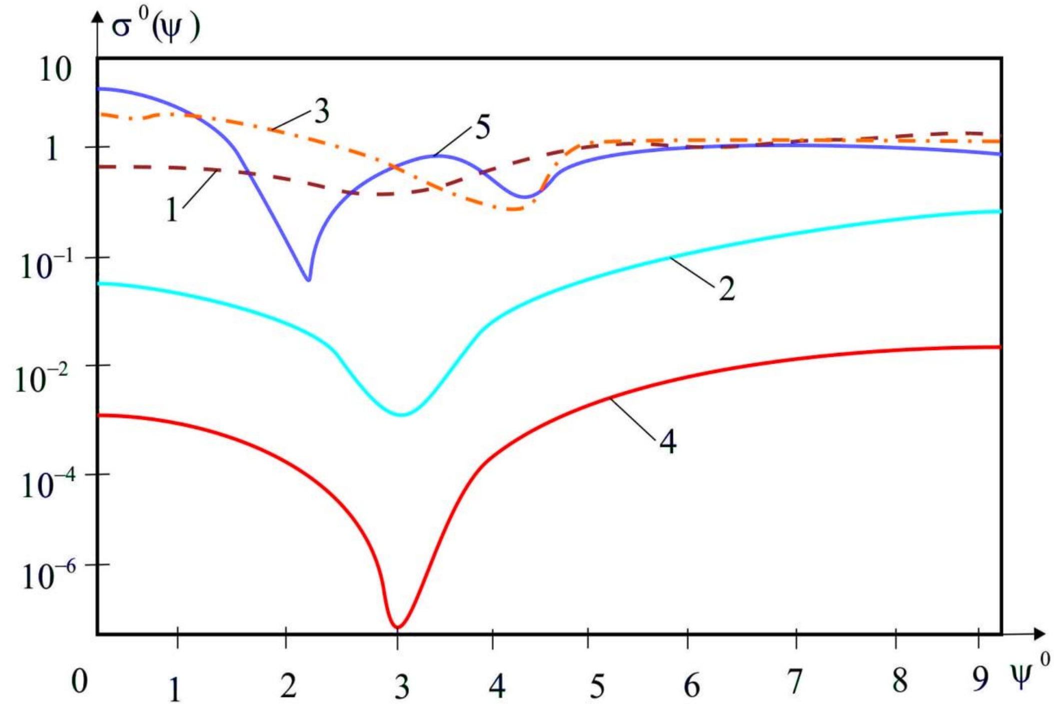

Fig. 2 Dependences σ(ψ)/πa for different values of the wave radius of a rigid cylinder:
1. $ka = 0.9$; 2. $ka = 0.3$; 3. $ka = 2.9$; 4. $ka = 0.1$; 5. $ka = 4.5$.

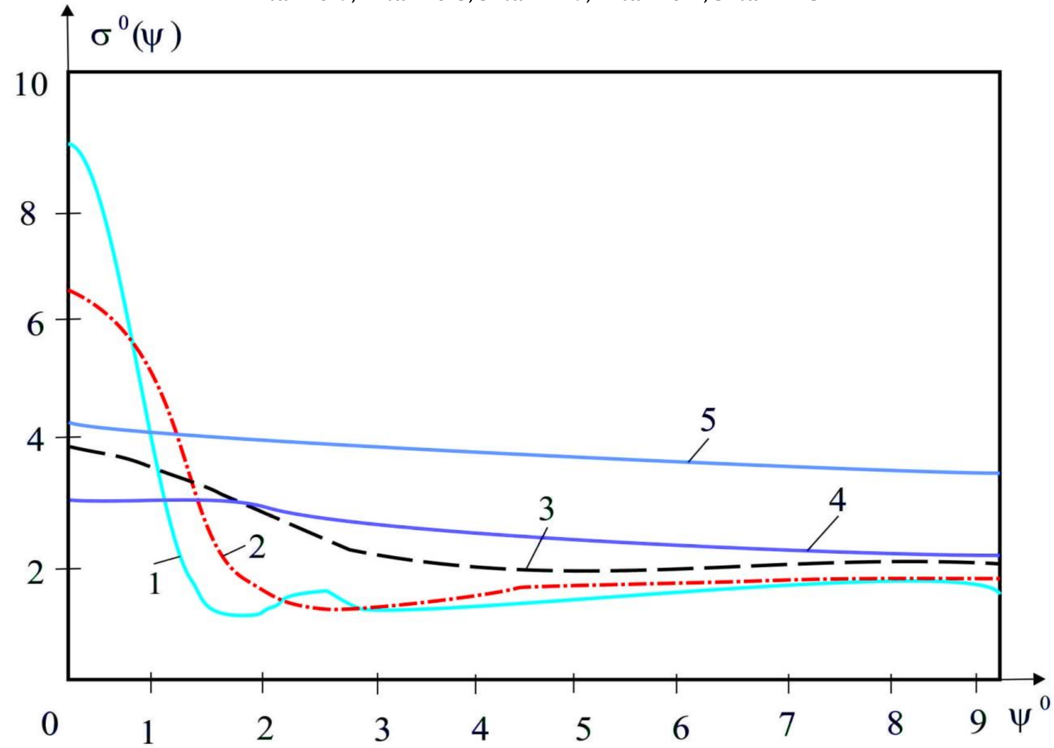

Fig. 3 Dependences σ(ψ)/πa for different values of the wave radius of the soft cylinder:
1. $ka = 2.9$; 2. $ka = 3.0$; 3. $ka = 1.0$; 4. $ka = 0.3$; 5. $ka = 0.1$.

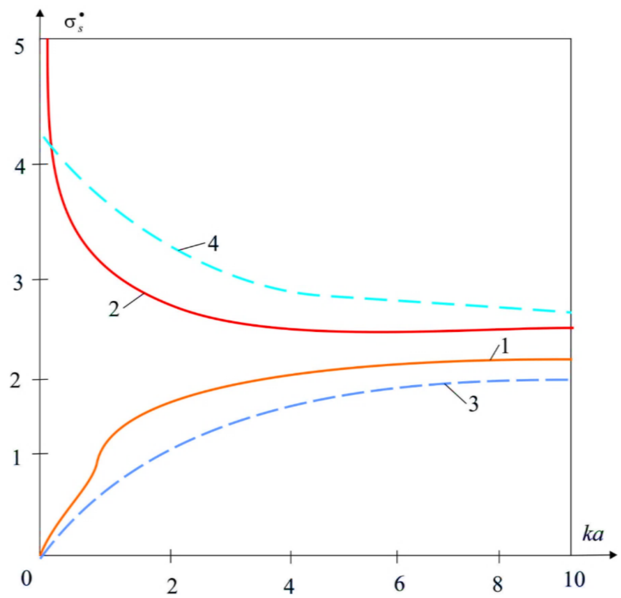

Fig. 4 Dependence of the total scattering cross-section of an ideal cylinder and ideal sphere on the wave radius ka: 1 - rigid cylinder; 2 - soft cylinder; 3 - hard-sphere; 4 - soft sphere

As you can see, in the case of a soft cylinder (Fig. 3, $k_a \leq 0.3$), the scattering diagram is similar to a circular diagram, which is typical for monopole scattering. A rigid cylinder (Fig. 2, a, $k_a \leq 0.3$) is similar to a cardioid diagram, formed as a superposition of monopole and dipole scattering.

Monopole scattering by a small obstacle, as already noted, is due only to the difference in the compressibility of the obstacle and the medium.

For a rigid cylinder, the compressibility is zero. Dipole scattering is associated with such an obstacle, which differs from the medium only in density.

The following physical considerations can comment on this result. In a sound wave, the processes of compression and displacement of particles of the medium occur.

A small obstacle in the sound field, in fact, a small particle in the form of a rigid cylinder, changes the nature of the compression and displacement of the particles of the medium near the cylinder. This causes the appearance of a scattered wave such that the boundary conditions must be satisfied on the surface of the cylinder: the radial velocity is zero.

Thus, a change in the nature of compression leads to monopole scattering, and a change in the nature of the motion of the particles of the medium leads to dipole scattering, and the contribution of both types of scattering turns out to be approximately the same, which is easy to verify.

Fig 4 shows numerical calculations of the value σ_s /2a using the exact formula (12). As can be seen, if $k_a >> 1$, then for both cylinders, the value σ_s /2a → 2; if $k_a << 1$, then the value σ_s /2a tends to zero for a rigid cylinder and increases for a soft cylinder. Once again, we note that an unlimited increase in the value of σ_s at $k_a \rightarrow 0$ is related to the infinite length of the cylinder. For a cylinder of finite length, there is no such result.

The results obtained were compared with the results obtained in [18]. The results differ by up to 9% with the same medium and cylinder data.

4. Conclusion

1. A method for solving and an algorithm for the problems of diffraction of acoustic waves in a viscoelastic cylinder has been developed.

2. It is established that if $k_a >> 1$, then for both cylinders, the value $\sigma_s/2a \rightarrow 2$; if $k_a << 1$, then the quantity $\sigma_s/2a$ tends to zero for a rigid cylinder and increases for a soft cylinder. We also note that an unlimited increase in the value of σ_s at $k_a \rightarrow 0$ is related to the infinite length of the cylinder. For a cylinder of finite length, there is no such result.

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