International Journal of Engineering Trends and Technology
ISSN: 2231–5381 / https://doi.org/10.14445/22315381/IJETT-V71I12P217

Volume 71 Issue 12, 168-175, December 2023

Original Article

Effects of Combined Thermo-Mechanical Loading on Stress Concentration in High-Pressure Vessels with a Cross-Bore

Patrick Kiola Nziu

Department of Industrial Engineering, Operation Management and Mechanical Engineering, Vaal University of Technology, South Africa.

Corresponding Author : pnziu@yahoo.com

Received: 18 July 2023 Revised: 11 September 2023 Accepted: 20 November 2023 Published: 06 December 2023

Abstract *- The purpose of this study was to determine the effects of combined thermo-mechanical loading on Stress Concentration Factor (SCF) in cross-bored thick-walled cylinders while considering the varying temperatures. The combined thermo-mechanical stress analysis was performed only on the reported geometrically optimised cross bores. A total of 14 different part models were created and analysed using three-dimensional finite element modelling software. The modelling was done under transient conditions to simulate the start-up conditions of pressure vessels until steady-state conditions were reached. Throughout the analyses, the fluid pressure was assumed to be constant at 1* MN/m^2*. The resulting stresses were recorded at 17 different temperature distribution intervals, ranging from 20ºC to 300ºC according to the thickness ratio. The hoop stress due to internally applied combined thermo-mechanical loading increased gradually with an increase in temperature until it reached a maximum value, after which it began to fall sharply. In contrast, the corresponding SCF reduced gradually with an increase in temperature until it reached a uniform steady state. After which, any further increase in temperature had an insignificant change in the stress concentration factor. The optimal SCF magnitude due to combined thermo-mechanical loading was 1.43. This SCF magnitude was slightly lower due to the pressure load acting alone.*

Keywords *- Cross bores, Pressure vessels, Stress concentration, Thermomechanical loading.*

1. Introduction

Pressure vessels are loaded with working fluid at high pressures and temperatures [1]. This loading induces dynamic and thermal stresses on the cylinder wall due to the variation in pressure and temperature, respectively [2]. However, due to the geometric discontinuities in the cylinder, such as the drilling of hole openings (also referred to as cross bores), the stress distribution along the cylinder wall is not uniform.

Henceforth, the cylinders cannot be analysed using Lame's theory. Any form of geometric discontinuities creates regions of high stress referred to as stress concentrations. The stress concentrations due to static stresses are calculated using dimensionless factors called the Stress Concentration Factor (SCF). While thermal stresses are determined using the Thermal Stress Concentration Factor (TSCF) [3]. High magnitude of stress concentration factors is among some common sources of pressure vessel failures [1] and is associated with reduced operating life [2].

Several studies have been conducted on high pressure vessels with a cross bore subjected to internal pressure [4-9]. Most of these studies research the effects of various geometric configurations of a cross bore, such as size, shape (circular or elliptical), location (radial or offset) and obliquity.

However, the aforementioned studies failed to analyse the effects of stress concentration in cross-bored high-pressure vessels subjected to both pressure and thermal loading, commonly referred to as thermomechanical loading, despite this occurrence being common in most industrial applications where the working fluid is in high-pressure and temperature.

In addition, there is very scanty information in the literature on the effects of thermo mechanical loading in cross-bored thick cylinders.

Therefore, this study evaluates the effects of combined thermo-mechanical loading on stress concentration in thick-walled cylinders with a cross bore. Particularly, considering the effects of varying fluid temperature on thick cylinders having different thickness ratios.

2. Literature Review
2.1. Thermal Stresses
Thermal stresses occur whenever a part of any solid body is prevented from attaining the size and shape it could freely attain due to temperature changes. Thermal stresses are classified under localised stresses, such as fatigue since they cause minimal distortion on the overall shape of the body [10]. Harvey [11] cited thermal stress distribution σ of a bar restricted to expand freely upon temperature change as;

$$\sigma = \mathrm{E}\alpha\Delta\mathrm{t} \quad (1)$$

Where:
E is the Young's modulus of elasticity
α is the coefficient of thermal expansion
Δt is the change in temperature

Thus, thermal stress is a function of the material property (Young's modulus of elasticity and coefficient of thermal expansion), together with the change in temperature across the thickness of the bar.

The rapid increase of the working fluid temperature in the pressure vessel induces thermal and dynamic stresses on the walls of cylinders. Thermal stress is induced in the wall of the pressure vessel whenever there is any temperature gradient or non-uniform temperature distribution across the wall [12]. Large thermal stresses may cause susceptible component failure, reduce operating life [2], or limit operational flexibility. The temperature gradient in the pressure vessel is usually oscillating and therefore, there is a need for accurate thermal stress analysis.

The thermal stress distribution in thick pressure vessels made of a single material layer is determined by Timoshenko and Goodier [13] as follows;

$$\text{Hoop stress } \sigma_\theta = \frac{\alpha E}{(1-\nu)}\frac{1}{r^2}\left[\frac{r^2+r_i^2}{r_0^2-r_i^2}\int_{r_i}^{r_o}\mathrm{Trdr} - \int_{ri}^{r}\mathrm{rdr}\right] \quad (2)$$

$$\text{Radial stress } \sigma_r = \frac{\alpha E}{(1-\nu)}\frac{1}{r^2}\left[\frac{r^2-r_i^2}{r_0^2-r_i^2}\int_{r_i}^{r_o}\mathrm{Trdr} - \int_{r_i}^{r}\mathrm{Trdr}\right] \quad (3)$$

$$\text{Longitudinal stress } \sigma_Z = \frac{E\alpha}{1-\nu}\left[\frac{2}{r_0^2-r_i^2}\int_{r_i}^{r_0}\mathrm{Trdr} - \mathrm{T}\right] \quad (4)$$

Where:
α is the coefficient of thermal expansion
E is the Young's modulus of elasticity
ν is the Poisson's ratio
r is the radius
r_i is the inside radius
r_o is the outside radius
T is the temperature change

From these three stress equations, it can be seen that thermal stress is a function of the material properties (Poisson's ratio, Young's Modulus of elasticity and coefficient of thermal expansions), the cylinder size and the temperature gradient across the wall of the cylinder.

The majority of thermal stress analyses in pressure vessels have been investigated under steady-state conditions using Von Mises theory to determine thermal stress [14].

Kandil *et al.* [15] studied the effect of thermal stresses on thick-walled cylinders and reported that the maximum thermal stress occurred at the inside surface of the cylinder. The peak of the thermal stress occurred at the beginning of the operating temperature. The study recommended gradual preheating of the cylinder wall up to the operating temperature since it reduces thermal stress by 50 – 60% for a short period of time when the normalising heating time was equal to 1.0. Moreover, the study also reported that long-term heating, when the normalising heating time was ≥ 3.0, had insignificant effects on reducing thermal effective stress. The study concluded that the time required for a thick-walled cylinder to attain steady-state operating conditions depended on the heating time and the diameter ratio.

Segall [16] derived a lengthy analytical polynomial equation expressing temperature as a function of a hollow cylinder's radius and time for an arbitrary internal thermal boundary. The study recommended using Segall's polynomial equation in the design and manufacturing processes of pressure vessels. In addition, the authors recommended using Segall's polynomial equation as a calibration tool in FEA modelling. However, it was noted that the input data was limited to a particular sequence for the equation to give accurate results.

Marie [17] developed another solution for determining hoop, radial and axial thermal stresses in high-pressure vessels due to temperature variation. Marie's solution took into consideration the effects of the inner surface layer of the cylinder. However, the solution ignored other thermal parameters such as material conduction, thermal diffusivities and the coefficient of heat exchange. Despite these assumptions, the study recommended using Marie's equations to solve thermal shock and fatigue problems in pressure vessels and piping elements.

Radu *et al.* [10] derived another analytical solution for determining radial, hoop and axial elastic thermal stresses in the wall of a long hollow cylinder under sinusoidal transient thermal loading. The equations were developed using finite Hankel transform, and their subsequent solutions were solved by MATLAB software package. Radu's equation was found to be independent of any temperature field and more applicable to both steady and transient conditions. However, to obtain a stable response using Radu's equations, the authors recommended the use of at least a total of 100 transcendental roots having radial steps in order of thousands.

2.2. Thermo Mechanical Loading

Pressure vessels operate in extreme conditions such as high temperature, pressure and corrosive environments. Therefore, it is not easy to have one single material satisfy all the requirements. To overcome this problem, multi-layered composite materials and, more recently, Functionally Graded Materials (FGMs) are being used to design high-pressure vessels. Multi-layered composites consist of different layers of material, with the inner layer being of higher performance alloy than the outer layer [12]. FGMs consist of two or more different materials, with their material volume fraction varying smoothly along the desired directions—the most common examples of FGMs materials are the combination of Ceramics and Metals [12].

Choudhhury *et al.* [12] researched the rate of the heat flow across the wall of a multi-layered pressure vessel consisting of Titanium and Steel layers under thermo-mechanical loading. The study considered two different experimental setups to investigate the effects of centrifugal and centripetal thermal flux in the cylinder wall. The latter is where the temperature at the inner surface of the cylinder is higher than the outside ambient temperature. Whereas the former was vice versa. It was found that the rate of centripetal flux was higher than centrifugal flux.

Okrajni and Twardawa [18] performed heat transfer modelling of a superheater in a steam power plant operating under thermo-mechanical loading using FEA. The study established that using time-dependent heat transfer coefficients in heat transfer problems eliminates disparities in temperature measurements.

Zhang *et al.* [19] derived analytical solutions for determining hoop σ_θ, radial σ_r and axial σ_z stresses in a multi-layered composite pressure vessel under thermo-mechanical loading. The analysis took into consideration the effects of the closed ends of the cylinder. The three Zhang's equations are presented here as follows;

$$\sigma_\theta = \frac{E}{(1+\mu)(1-2\mu)}[(1-\mu)\varepsilon_\theta + \mu(\varepsilon_r + \varepsilon_z)] - \frac{E\alpha T}{1-2\mu} \quad (5)$$

$$\sigma_r = \frac{E}{(1+\mu)(1-2\mu)}[(1-\mu)\varepsilon_r + \mu(\varepsilon_\theta + \varepsilon_z)] - \frac{E\alpha T}{1-2\mu} \quad (6)$$

$$\sigma_z = \frac{E}{(1+\mu)(1-2\mu)}[(1-\mu)\varepsilon_z + \mu(\varepsilon_\theta + \varepsilon_r)] - \frac{E\alpha T}{1-2\mu} \quad (7)$$

Where:

E is the Young's modulus of elasticity
μ is the Poisson's ratio
ε_θ is the hoop strain
ε_r is the radial strain
ε_z is the axial strain
A is the coefficient of thermal expansion
T is the temperature change

The stresses due to thermo-mechanical loading were shown to depend on the vessel size and length, the variation of pressure and temperature and the material properties of the cylinder wall. Moreover, the authors validated the three equations using 3D FEA on a six-layered composite pressure vessel. Geometric and material properties considered for each composite layer in this FEA modelling include wall thickness, Young's modulus of elasticity, Poisson's ratio, thermal conductivity, coefficient of thermal expansion, density and specific heat. The study reported good correlations between the analytical and the 3D FEA solution. In addition, the authors recommended using these equations in designing multi-layered pressure vessels to be subjected to thermal and mechanical loading.

Chaudhry *et al.* [20] studied the behaviour of hoop stress across the wall of multi-layered pressure vessels during normal start-up and shutdown conditions subjected under thermo-mechanical loading.

They reported that at the inner surface of the pressure vessel wall, the hoop stress was found to be compressive during normal start-up and tensile during normal shutdown. However, the study did not investigate the effects of hoop stress in pressure vessels during emergency shutdown under thermo-mechanical loading conditions. Emergency shutdown occurs when pressure is suddenly cut off.

In general, the reviewed studies did not analyse the effects of stress concentration in cross-bored high-pressure vessels subjected under the combination of thermal and mechanical loading, which arises from working fluids at high pressures and temperatures.

3. Materials and Methods

We conducted an extensive study on the effects of the geometric configuration of a cross bore subjected to internal pressure only using three-dimensional finite element modelling, Abaqus software, to determine the optimal conditions. A detailed step-by-step modelling procedure is presented in the aforementioned study. The study examined various cylinders with different thickness ratios ranging from 1.4 to 3.0.

The cross bores were of different sizes of cross-sectional areas of 1.963×10^(-5) 〖m〗^2 and 3.926×10^(-3) 〖m〗^2, having circular and elliptical shapes, respectively, located both in radial and offset positions (offset ratio 0 – 0.9). It is worth noting that the major diameters of the two cross-bore shapes were made equal. Whereas the diameter ratio of the elliptical cross bore was kept at 2 as suggested by Harvey [11].

The study revealed various optimal SCF magnitudes due to mechanical loading only for each thickness ratio and cross-bore location, as tabulated in Tables 1 and 2.

Table 1. Optimum thickness ratio

K	1.4	1.5	1.75	2.0	2.25	2.5	3.0
SCF	2.312	2.392	2.319	1.898	2.05	1.733	1.794
Location	0.9	0.685	0.685	0	0	0	0
Shape	Circular	Circular	Elliptical	Elliptical	Elliptical	Elliptical	Elliptical

Table 2. Optimal offset locations

Location	0	0.24	0.48	0.685	0.9
SCF	1.733	1.971	2.128	2.319	2.312
K	2.5	2.25	3.0	1.75	1.4
Shape	Elliptical	Elliptical	Elliptical	Elliptical	Circular

Henceforth, these reported optimal conditions were further used in analyses of the effects of thermomechanical loading in this work.

3.1. Combined Thermo-Mechanical Stress Analysis

The combined thermo-mechanical stress analysis was performed only on the geometrically optimised cross-bore sizes tabulated in Tables 1 and 2. Combined thermo-mechanical stresses on the geometrically optimised cross bore for each vessel thickness were determined using thermo-couple analysis. The internal fluid pressure and temperature were taken as 1 MN/m^2 and 300ºC, respectively, as recommended in technical literature by Zhang et al.[19], Chaudhry et al.[20] and Choudhury et al. [12]. The ambient temperature was taken as 20ºC. The thermal properties of the material used are listed in Table 3.

A total of 14 different part models were created and analysed in this work using a commercial finite element modelling software, Abaqus. Because three cylinder sizes, namely K=1.4, 1.75 and 2.5, had the same optimal SCF magnitude, satisfying the optimal design requirements for both the cross-bore location and shape.

The modelling was done under linear transient conditions to simulate the start-up conditions of pressure vessels until steady-state conditions were reached. Throughout the analyses, the fluid pressure was assumed to be constant at 1 MN/m^2. The stresses due to thermo-mechanical loading were recorded at 17 different nodal temperatures ranging from 20ºC to 300ºC for each thickness ratio. Details of thermomechanical modelling procedures are presented elsewhere.

Table 3. Material properties for the thermal analysis [20]

Parameter	Value
Thermal conductivity	17 W/m K
Thermal expansion coefficient	11E-6 mm/mm/º C
Specific heat	0.48J/kg K

4. Results and Discussion

4.1. Effects of the Combined Thermo-Mechanical Loading on Hoop Stress

Figures 1 – 7 show the variation of hoop stresses with temperature distribution for thickness ratios, K = 1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0:

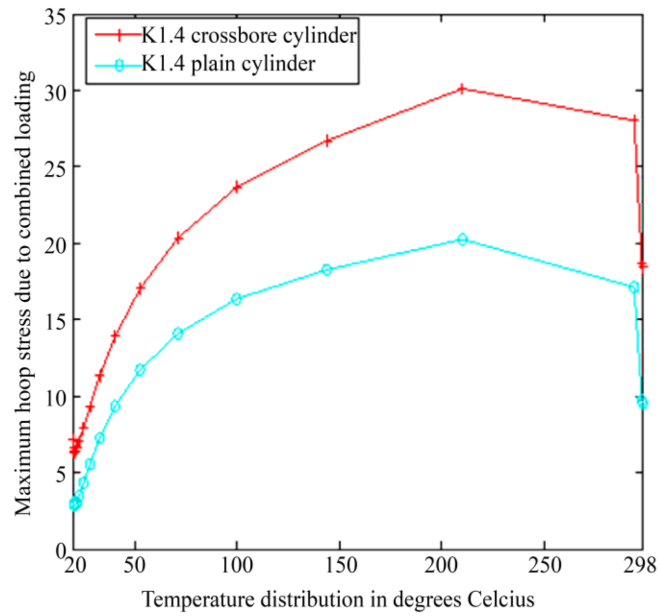

Fig. 1 Optimum cross bore for K=1.4

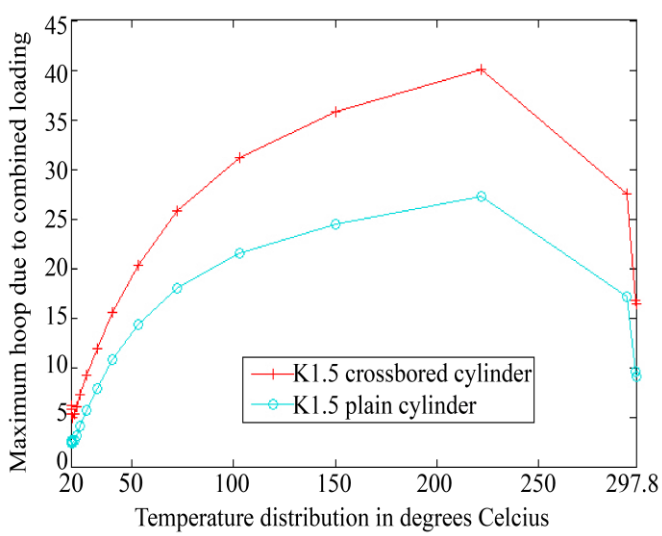

Fig. 2 Optimum cross bore for K=1.5

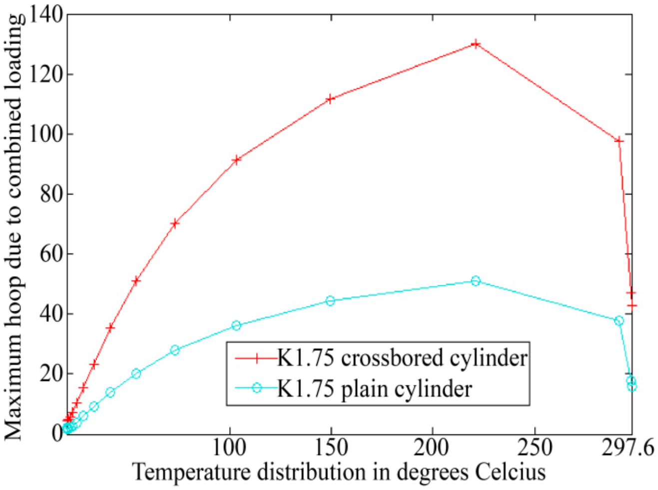

Fig. 3 Optimum cross bore for K=1.75

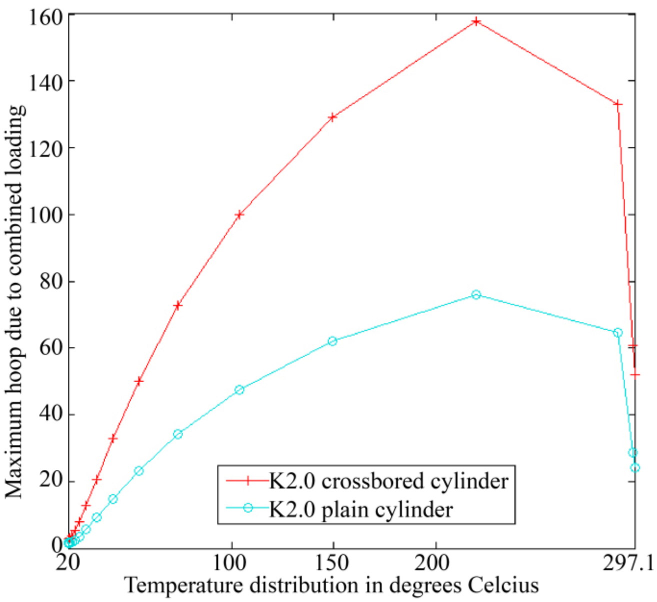

Fig. 4 Optimum cross bore for K=2.0

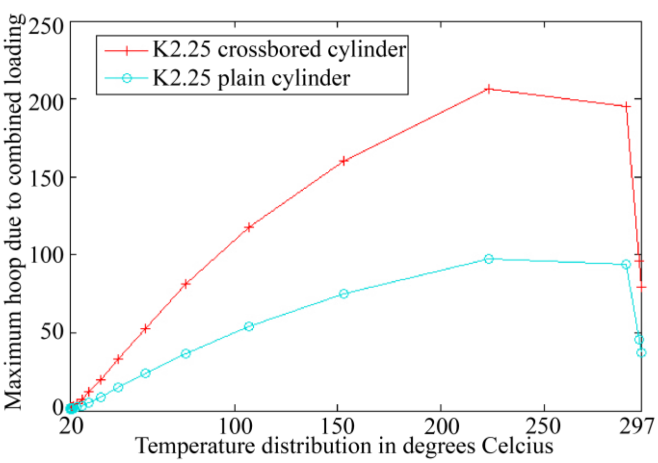

Fig. 5 Optimum cross bore for K=2.25

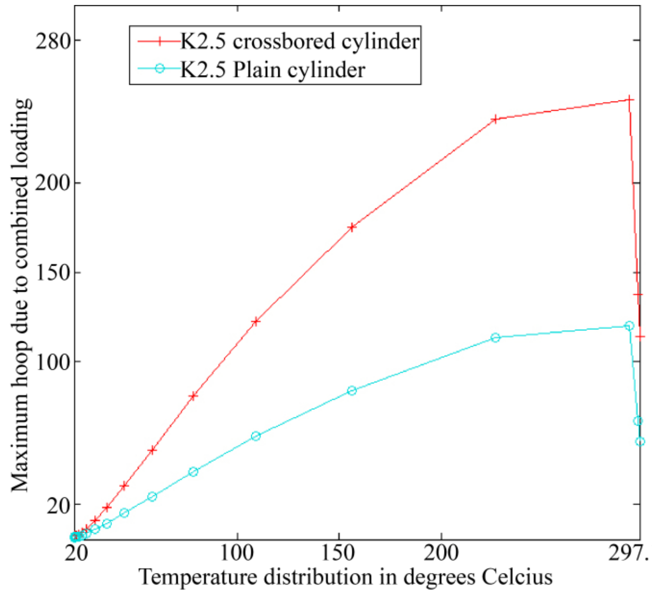

Fig. 6 Optimum cross bore for K=2.5

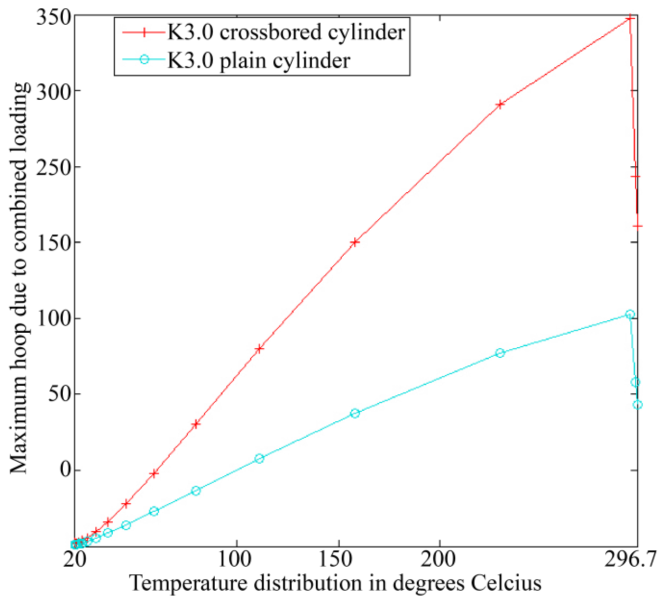

Fig. 7 Optimum cross bore for K=3.0

A similar stress distribution pattern was exhibited between the cross-bored cylinder and the plain cylinder, as illustrated in Figures 1 – 7. However, the hoop stresses present in the cross-bored cylinder were higher than those in the plain cylinder. Generally, the hoop stresses increased gradually with an increase in temperature until they reached a maximum, after which they began to fall sharply. The temperatures in the cylinder which corresponded to the stress maxima for K =1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0 were 210.3, 221.8, 220.9, 219.7, 223.3, 292.0 and 293.4°C, respectively. At these mentioned temperatures, the increase in hoop stress magnitudes between the cross-bored cylinders and those of plain cylinders was 48.57%, 47.23%, 153.96%, 74.78%, 111.67%, 105.79% and 127.33 %, respectively.

From the preceding paragraph, it was noted that the lowest maximum temperature point occurred at K = 1.5. Similarly, the minimum increase in stress magnitude between the cross-bored and plain cylinders occurred at K = 1.5. In contrast, these parameters' highest magnitudes were recorded at K = 3.0.

4.2. General Discussion of Temperature Effects on Hoop Stress

Usually, thermal stress in cylinders mainly depends on the temperature variation between the inner and outer surfaces, among other factors. During the starting up of the pressure vessel, the inner surface is at a higher temperature, causing the inner fibres to undergo compression. On the other hand, at the outer surface, the temperature is low, leading to the stretching of the outer fibres of the cylinder. As the operating time increases, the difference in the temperature gradient across the cylinder wall reduces, and this reduction in the temperature gradient between the inner and outer surfaces results in a reduction of the hoop stress. Probably, at this maximum stress point, the operating conditions of the cylinder begin to change from transient to steady-state conditions. As reported in the study by Kandil et al. [15], the magnitude of the maximum stress in the cylinder can be reduced by up to 60 % when the cylinder walls are warmed up to operating temperature before start-up.

4.3. Effects of Combined Thermo-Mechanical Loading on Stress Concentration Factors

The stress concentration factor due to thermo-mechanical loading was computed based on the ratio of localised maximum principal stresses in a cross-bore cylinder to the corresponding ones present in a similar plain cylinder.

4.3.1. Effects of Temperature on Cylinder Thickness Ratio

Figures 8 and 9 show the variation of stress concentration factors with temperature due to thermo-mechanical loading at the selected optimal thickness ratios and offset positions, respectively.

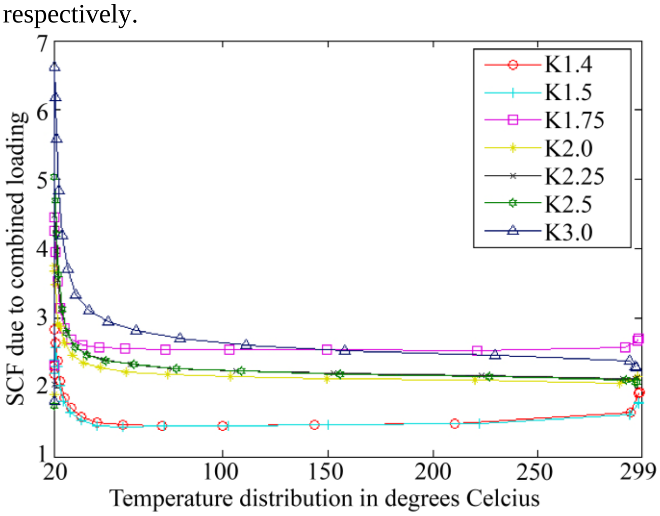

Fig. 8 Variation of stress concentration factors with temperature on optimal thickness ratios

As illustrated in Figure 8, as the temperature increased, the corresponding stress concentration factor reduced gradually until it reached a uniform steady state. After which, any further increase in temperature caused an insignificant change in the stress concentration factor. A similar observation has been cited by Kandil et al. [15].

The lowest stress concentration factors occurred at the thickness ratios of 1.4 and 1.5, reaching a minimum magnitude of 1.433. This SCF magnitude indicated a reduction of pressure carrying capacity by 30.2% compared to a similar plain cylinder without a cross bore.

On the other hand, the highest stress concentration factors occurred at thickness ratios of 1.75 and 3.0, having a SCF magnitude of approximately 2.50. These stress concentration magnitudes due to combined thermo-mechanical loading were lower than those presented in Table 1, arising from mechanical loading only. This occurrence was attributed to the compressive nature of thermal stresses during the starting-up of the vessel, which acts as a relief to tensile mechanical stresses.

As cited by Harvey [11], it is worthwhile to note that thermal stresses do not cause failures or ruptures on a ductile material upon their first application, irrespective of the magnitude. Infact, failures or ruptures of ductile material occur due to repeated cycling loading over a period of time attributed to fatigue.

4.3.2. Effects of Temperature on the Cross Bore Geometry

Variations in stress concentration factors with temperature, as exhibited in Figure 8, were also replicated in Figure 9. As illustrated in Figure 9, the minimum stress concentration factor occurred at the offset position ratio of 0.9 at a thickness ratio of 1.4 of a circular cross bore. However, the highest corresponding stress concentration factor was recorded at an offset ratio of 0.24 in K = 2.25 cylinder.

In conclusion, therefore, the optimal cylinder size due to combined thermo-mechanical loading was K = 1.4, having a circularly shaped cross bore at 0.9 offset position ratio. The corresponding optimal magnitude of SCF generated at these conditions was 1.433.

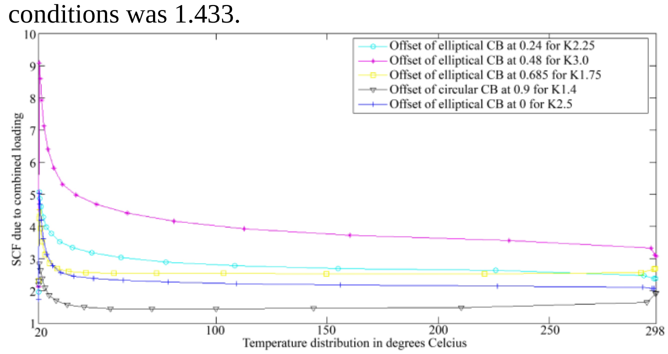

Fig. 9 Variation of stress concentration factors with temperature at offset position

5. Conclusion

- The hoop stresses due to combined thermo-mechanical loading increased gradually with an increase in temperature until it reached a maximum, after which it began to fall sharply.
- The stress concentration factor due to the combined thermo-mechanical loading reduced gradually with an increase in temperature until it reached a uniform steady state. After which, any further increase in temperature had an insignificant change in the stress concentration factor.
- The optimal cylinder size due to combined thermo-mechanical loading was K = 1.4, having a circularly shaped cross bore at 0.9 offset position ratio. The corresponding magnitude of SCF generated was 1.433. This SCF magnitude indicated a reduction of pressure carrying capacity of 30.2% in comparison to a similar plain cylinder without a cross bore.

Funding Statement

This research work was supported by the Vaal University of Technology.

Acknowledgments

The author wishes to thank the department of Industrial Engineering, Operation Management and Mechanical Engineering at Vaal University of Technology for facilitating this work.

References

[1] A. Nayebi, and S. Ansari Sadrabadi, "FGM Elastoplastic Analysis under Thermomechanical Loading," *International Journal of Pressure Vessels and Piping*, vol. 111-112, pp. 12-20, 2013. [CrossRef] [Google Scholar] [Publisher Link]

[2] Woosung Choi et al., "Development of Thermal Stress Concentration Factors for Life Assessment of Turbine Casing," *International Journal of Pressure Vessels and Piping*, vol. 98, pp. 1-7, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[3] Babu R. Nagendra, K.V. Ramana, and K. Mallikarjuna Rao, "Prediction of Stress Concentration Effect under Thermal and Dynamic Loads on a High Pressure Turbine Rotor," *Journal of Mechanical Engineering Research*, vol. 2, no. 6, pp.106-115, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[4] Leonard M. Masu, "*The Effect of Cross Bore Geometry on the Strength of Pressure Vessels*," PhD Thesis, University of Nairobi, 1989. [Google Scholar] [Publisher Link]

[5] Kihiu John Muniu "*Numerical Stress Characterization in Cross Bored Thick Walled Cylinders under Internal Pressure*," PhD Thesis, The University of Nairobi, 2002. [Google Scholar] [Publisher Link]

[6] T. Comlekci et al., "Elastic Stress Concentration at Radial Crossholes in Pressurized Thick Cylinders," *The Journal of Strain Analysis for Engineering Design*, vol. 42, no. 6, pp. 461-468, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[7] P. Makulsawatudom, D. Mackenzie, and R. Hamilton, "Stress Concentration at Crossholes in Thick Cylindrical Vessels," *The Journal of Strain Analysis for Engineering Design*, vol. 39, no. 5, pp. 471-481, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[8] C.A. Adenya, and J.M. Kihiu, "*Stress Concentration Factors in Thick Walled Cylinders with Elliptical Cross Bore*," Master's Thesis, JOMO Kenyatta University of Science and Technology, 2010. [Google Scholar] [Publisher Link]

[9] N. Kiplagat, "*Optimal Geometric Configuration of a Cross Bore in Thick Compound Cylinders*," Master's Dissertation, Vaal University of Technology. [Google Scholar] [Publisher Link]

[10] V. Radu, N. Taylor, and E. Paffumi, "Development of New Analytical Solutions for Elastic Thermal Stress Components in a Hollow Cylinder under Sinusoidal Transient Thermal Loading," *International Journal of Pressure Vessels and Piping*, vol. 85, no. 12, pp. 885-893, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[11] John F. Harvey, *Theory and Design of Pressure Vessels*, Van Nostrand Reinhold, 1985. [Google Scholar] [Publisher Link]

[12] Arnab Choudhury, Samar Chandra Mondol, and Susenjit Sarkar, "Finite Element Analysis of Thermos Mechanical Stresses of Two Layered Composite Cylindrical Pressure Vessel," *International Journal of Research in Applied Science & Engineering Technology*, vol. 2, no. XI, pp. 341-349, 2014. [Google Scholar] [Publisher Link]

[13] Stephen Timoshenko, *Strength of Materials part II: Advanced Theory and Problem*, D. Van Nostrand Company, 1945. [Google Scholar] [Publisher Link]

[14] D.C. Gonyea, "Method for Low-Cycle Fatigue Design Including Biaxial Stress and Notch Effects," *Fatigue at Elevated Temperatures, ASTM – STP38879S*, pp. 678–687, 1973. [CrossRef] [Google Scholar] [Publisher Link]

[15] A Kandil, A.A. El-Kady, and A. El-Kafrawy, "Transient Thermal Stress Analysis of Thick-Walled Cylinders," *International Journal of Mechanical Science*, vol. 37, no. 7, pp. 721-732, 1995. [CrossRef] [Google Scholar] [Publisher Link]

[16] A.E. Segall, "Transient Analysis of Thick-Walled Piping under Polynomial Thermal Loading," *Nuclear Engineering and Design*, vol. 226, no. 3, pp. 183-191, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[17] Stéphane Marie, "Analytical Expression of Thermal Stresses in a Vessel or Pipe with Cladding Submitted to any Thermal Transient," *International Journal of Pressure Vessels and Piping*, vol. 81, no. 4, pp. 303-312, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[18] Jerzy Okrajni, and Mariusz Twardawa, "Description of the Thermo-Mechanical Fatigue of Thick-Walled Pressure Vessels," *Procedia Materials Science*, vol. 3, pp. 918-923, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[19] Q. Zhang et al., "Analytical Solution of the Thermo-Mechanical Stresses in a Multilayered Composite Pressure Vessel Considering the Influence of the Closed Ends," *International Journal of Pressure Vessels and Piping*, vol. 98, pp. 102-110, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[20] V. Chaudhry et al., "Thermo-Mechanical Transient Analysis of Reactor Pressure Vessel," *Procedia Engineering*, vol. 86, pp. 809-817, 2014. [CrossRef] [Google Scholar] [Publisher Link]