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Original Article

Propagation of Proper Waves in a Viscoelastic Timoshenko Plate of Variable Thickness

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Abstract *- Elastic distribution waves in lamellar bodies have been studied for more than a hundred years, but work in this direction continues, which indicates a continuing interest in this problem. In this paper, the wave propagation problem in a viscoelastic thin film is considered—a plate of variable cross-section subordinating to the Timoshenko hypothesis. The article's main purpose is to study the distribution of natural waves in an infinite viscoelastic lamellar waveguide with a wedge cross-section based on the Timoshenko hypothesis. The variation of complex natural frequencies and waveforms depending on various waveguide parameters (wavenumber and geometric parameters) is investigated. Integro-differential equations of waveguide motion are obtained based on the variational principle. After applying the freezing method, a system of differential equations with complex coefficients is obtained, which is further solved using the orthogonal run method with a combination of the Muller method on complex arithmetic. Based on the obtained results, it is established that with an increase in the wave number, the real and imaginary parts of normal modes in the Timoshenko wedge-shaped plate tend to have constant values. In this case, the localization of motion is observed near the sharp edge of the waveguide. It is also found that comparing the results obtained according to the Kirchhoff and Timoshenko theories for small wedge angles differs by up to 10%. Real and imaginary parts of a complex function of the propagation velocity of the Azov wave, the propagation velocity of the first mode in a wedge-shaped plate practically does not depend on the Poisson's ratio (a change within 0.5 %). In general, the numerical analysis of edge waves in Kirchhoff and Timoshenko plates allows us to conclude that Kirchhoff's hypotheses are quite suitable for calculating wave processes in wedge-shaped plates, including at frequencies with a wavelength of the order of plate thickness. This discrepancy with the classical results of the theory of Kirchhoff plates of constant thickness is explained by the phenomenon of localization of the waveform with increasing frequency, which occurs only in plates of variable thickness. At the same time, the relative simplicity of the mathematical apparatus of the Kirchhoff plate theory makes it possible to study the dispersion characteristics of waveguides with a more complex cross-section configuration, which is very difficult to construct in the framework of a three-dimensional theory. The results of this work can be applied in calculations of engineering structures consisting of extended plates of constant and variable cross-sections.*

Keywords *- Damped wave, Viscoelastic plate sector cross-section, Navier equation, Spectral boundary value problem, Orthogonal run.*

1. Introduction

To date, no general methods have been developed for calculating a plate of variable cross-section, considering the material's rheological properties. Results of investigations of elastic free wave propagation in plateschat rooms (and rectangular elastic rods) are given in the articles [1, 2, 3]. The paper [21] describes obtaining a system of independent differential equations for an unbounded axisymmetric rod. The solution is sought in the form of a traveling wave. As a result, the Bessel equation is obtained, the resolution of which is expressed in terms of the Bessel and Neumann functions. The problem is reduced to the propagation of plane waves in an equivalent plate. Dispersion curves are constructed for different angles of helical wave propagation.

The article [5] discusses the applicability of Kirchhoff's hypotheses in the study of problems of propagation of magneto-elastic waves in a plate. A comparative analysis of approximate and exact solutions is given. The acceptability of Kirchhoff's hypotheses in the study of magneto-elastic wave propagation problems is proved [6].

The existence of surface waves in elastic wedges was first discovered in [7-9]. Thus, the conclusion of y came based on the results obtained, which were obtained numerically and by the am I method. It was found that the waves have an oscillating character along the edge of the wedge and exponentially decrease with distance from the edge. The propagation speed of surface waves in elastic wedges depends on the angle of the edge. Therefore, depending on the angle, their speed can be significantly less than the speed of Rayleigh waves. This circumstance creates a very attractive opportunity for practice, expressed in the use of delay lines. At the same time, the question of proving, from a physical and mathematical point of view, the existence of such waves, taking into account physical, mechanical and geometric parameters, remained open.

In [22], surface waves traveling along the edge of an elastic wedge were considered. In the last paper, the existence of surface waves in acute-angled wedges is mathematically proved for an isotropic elastic wedge. The proof is based on the variational approach of elasticity theory.

The paper [11] deals with the propagation of elastic waves in a wedge. The proposed approach to solving the problem was developed, and the propagation velocity of surface waves was determined for symmetric and antisymmetric modes of an isotropic wedge.

The articles [12, 23] deal with new acoustic effects during wave propagation in wedge-shaped media. Studies of wave propagation in an elastic wedge-shaped plate have revealed new acoustic effects near the rib that are unknown in the technical literature. When a surface wave moves in this region, the acoustic field is continuously rearranged, and the volume waves are emitted. As we approach the edge, the surface wave velocity drops to zero, but in the opposite direction, the wave gains speed.

Researchers selected viscoelastic infinite plates or strips of variable thickness and viscoelastic cylindrical bodies with radial cracks in the current job as objects. The relations between the parameters of harmonic wave processes are investigated [14,15,24]. On the other hand, knowledge of the propagation patterns of harmonic wave processes is of great interest in practice. Also, the study of harmonic vibrations is the shortest way to quantify the features of non-stationary processes occurring in deformable bodies.

2. Methods

2.1. Problem Statement and Solution Methods

The wave propagation problem in a viscoelastic thin film is considered—a plate of variable cross-section subordinating to the Timoshenko hypothesis. Let in an infinite viscoelastic lamellar waveguide with a wedge cross-section that obeys the Timoshenko hypothesis, distributed by its waves. It is necessary to study the change in complex natural frequencies and waveforms depending on various waveguide parameters (wavenumber and geometric parameters). To obtain differential equations of vibrations of a plate of variable thickness, we use the principle of possible displacements [6]:

$$\delta(N - T) = 0, \qquad (1)$$

where δN is the variation of potential energy; δT is the variation of kinetic energy.

Geometric and physical-mechanical parameters, thickness h, *density* p, *and operator Young's modulus are given* for a plate of variable thickness [2]:

$$\tilde{E}_n[\phi(t)] = E_{0n}\left[\phi(t) - \int_0^t R_{En}(t-\tau)\phi(t)d\tau\right], \qquad (2)$$

Where $\phi(t)$ is an arbitrary time function $R_{En}(t-\tau)$– is the relaxation kernel, $E_{0n}$and is the instantaneous elastic modulus.

We assume that the integral terms in (1) are small. Then the functions $\phi(t)$ can be represented in the form $\phi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$-is a slowly changing function of time, ω_R -is a real constant. Then [13], we replace relations (1) with approximations of the form

$$\bar{E}_n\phi = E_{0j}\left[1 - \Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)\right]\phi,$$

Where

$$\Gamma_n{}^C(\omega_R) = \int_0^\infty R_n(\tau)\cos\omega_R\,\tau d\tau,$$
$$\Gamma_n{}^S(\omega_R) = \int_0^\infty R_n(\tau)\sin\omega_R\,\tau d\tau-$$

Here, the cosine and sine are Fourier images of the relaxation kernel of the material. As an example of a viscoelastic material, we take a three-parameter relaxation kernel$R_n(t) = A_n e^{-\beta_n t}/t^{1-\alpha_{jn}}$.

Let us apply the principle of possible displacements (1) and (2), taking into account the Timoshenko hypotheses:

$$\sigma_{33} = 0;\ \sigma_{3i} = \frac{\chi\bar{E}}{2(1+v)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right)$$

$$u_i^{(x_3)} = x_3\theta_i;\ W^{(x_3)} = W;\ i = 1,2, \qquad (3)$$

where θ_i– angles of rotation of the normal (fig. 1), χ– correction factor that considers the distribution of shear stresses over the thickness. In this case, the components of the strain and stress tensors will take the form:

$$\varepsilon_{ij} = -\frac{1}{2}x_3\left(\frac{\partial\theta_i}{\partial x_j} + \frac{\partial\theta_j}{\partial x_i}\right);\varepsilon_{3i} = \frac{1}{2}\left(\frac{\partial W}{\partial x_i} - \theta_i\right);$$

1. $$\sigma_{11} = -\frac{E\Gamma_k}{1-v^2}x_3\left(\frac{\partial\theta_1}{\partial x_1} + \nu\frac{\partial\theta_2}{\partial x_2}\right);$$

$$\sigma_{22} = -\frac{E\Gamma_k}{1-v^2}x_3\left(\frac{\partial\theta_2}{\partial x_2} + \nu\frac{\partial\theta_1}{\partial x_1}\right);$$

$$\sigma_{12} = -\frac{E\Gamma_k}{2(1+v)}x_3\left(\frac{\partial\theta_1}{\partial x_2} + \nu\frac{\partial\theta_2}{\partial x_1}\right);$$

$$\sigma_{3i} = \frac{\chi E\Gamma_k}{2(1+v)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right), \quad i, j=1, \qquad (4)$$

Substituting the expression for working on virtual movements, we get:

$$\delta A=\int_{-h/s}^{h/2}\int_s\left[\begin{array}{l}-\sigma_{ij}\dfrac{x^3}{2}\left(\dfrac{\partial\delta\theta_i}{\partial x_j}+\dfrac{\partial\delta\theta_j}{\partial x_i}\right)+\\ +\sigma_{3i}\left(\dfrac{\partial\delta W}{\partial x_i}-\delta\theta_i\right)+\end{array}\right.$$
$$\left.+\rho\ddot{W}\delta W+\rho x_3^2\ddot{\theta}_i\delta\theta_i\right]dSdx_3=0. \quad (5)$$

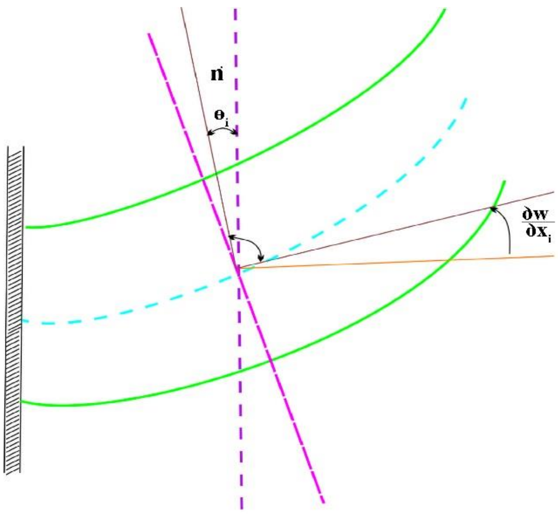

Fig. 1 Normal rotation angle

Or, by introducing notation for the corresponding moments:

$$\bar{M}_{11}=D_1\Gamma_k\left(\frac{\partial\theta_1}{\partial x_1}+v\frac{\partial\theta_2}{\partial x_2}\right)=\Gamma_k M_{11};$$
$$\bar{M}_{22}=D_1\Gamma_k\left(\frac{\partial\theta_2}{\partial x_2}+v\frac{\partial\theta_1}{\partial x_1}\right)=\Gamma_k M_{22};$$
$$\bar{M}_{12}=D_2\Gamma_k\left(\frac{\partial\theta_1}{\partial x_2}+\frac{\partial\theta_2}{\partial x_1}\right)=\Gamma_k M_{12}\ (6)$$

where

$$D_2=D_1/2,\quad M_{22}=-D_1\left(\frac{\partial\theta_2}{\partial x_2}+v\frac{\partial\theta_1}{\partial x_1}\right),$$
$$M_{11}=-D_1\left(\frac{\partial\theta_1}{\partial x_1}+v\frac{\partial\theta_2}{\partial x_2}\right),\quad M_{12}=D_2\left(\frac{\partial\theta_1}{\partial x_2}+\frac{\partial\theta_2}{\partial x_1}\right),$$

and integrating over x_3, we have

$$\delta A=-\int_s\left[-\frac{\partial}{\partial x_j}\left(\bar{M}_{ij}\delta\theta_i\right)+\frac{\partial}{\partial x_j}\left(h\delta_{3j}\delta W\right)\right]dS+$$
$$+\int_s\left(-\frac{\partial\bar{M}_{ij}}{\partial x_j}\delta\theta_i+\frac{\partial\left(h\bar{\sigma}_{3j}\right)}{\partial x_j}\delta W+h\bar{\sigma}_{3i}\delta\theta_i-\right.$$
$$\left.-\rho h\ddot{W}\delta W-\frac{\rho h^3}{12}\ddot{\theta}_i\delta\theta_i\right)dS=0. \quad (7)$$

Integrating (7) in parts and equating the coefficients for variations to zero δW and $\delta\theta_I$ inside the body and at its boundaries, we obtain the following system of differential equations

$$\begin{cases}-\dfrac{\partial M_{12}}{\partial x_2}-\dfrac{\partial M_{11}}{\partial x_1}+h\sigma_{31}-\dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_1=0;\\ -\dfrac{\partial M_{22}}{\partial x_2}-\dfrac{\partial M_{12}}{\partial x_1}+h\sigma_{32}-\dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_2=0;\\ \dfrac{\partial(h\sigma_{32})}{\partial x_2}+\dfrac{\partial(h\sigma_{31})}{\partial x_1}-\dfrac{\rho h\ddot{W}}{\Gamma_k}=0\end{cases}\quad (8)$$

with natural boundary conditions:

$$\begin{cases}M_{12}=0;\\ M_{11}=0;\\ h\sigma_{31}=0,x_1=0,l_1\end{cases}$$

The main alternatives to which will be the following:

$$\begin{cases}\theta_1=0;\\ \theta_2=0;\\ W=0,x_1=0,l_1\end{cases}$$

Equations (8) are differential equations with complex coefficients, and it can be written in the following form

$$\begin{pmatrix}-\dfrac{\partial M_{12}}{\partial x_2}-\dfrac{\partial M_{11}}{\partial x_1}+h\tau_{31}-\dfrac{sh^3}{12\Gamma_{KR}}\theta_1''\\ \dfrac{-\partial M_{22}}{\partial x_2}-\dfrac{\partial M_{12}}{\partial x_1}+h\tau_{32}-\dfrac{sh^3}{12\Gamma_{KR}}\theta''\\ \dfrac{\partial(h\tau_{32})}{\partial x_2}+\dfrac{\partial(h\tau_{31})}{\partial x_1}-\dfrac{sh^3}{\Gamma_{K12}}\ddot{W}\end{pmatrix}+$$
$$+i\Gamma_{KI}\begin{pmatrix}-\dfrac{\partial M_{12}}{\partial x_2}-\dfrac{\partial M_{11}}{\partial x_1}+h\tau_{31}\\ \dfrac{-\partial M_{22}}{\partial x_2}-\dfrac{\partial M_{12}}{\partial x_1}+h\tau_{32}\\ \dfrac{\partial(h\tau_{32})}{\partial x_2}+\dfrac{\partial(h\tau_{31})}{\partial x_1}\end{pmatrix}=0$$

The main variables in this system will be: W_1, θ_1, θ_2, M_{12}, M_{11}, $Q_1=h\,\sigma_{31}$. Exclude the variables M22 and Q from the last equations, where

$$M_{22}=-\frac{Eh^3}{12}\frac{\partial\theta_2}{\partial x_2}+vM_{11};\quad Q_2=h\sigma_{32}=\frac{\chi Eh}{2(1+v)}\left(\frac{\partial W}{\partial x_2}-\theta_2\right).$$

Then we arrive at the following system of equations:

$$\begin{cases}\dfrac{\partial W}{\partial x_1}=\theta_1+\dfrac{2(1+v)}{\chi Eh}Q_1;\\ \dfrac{\partial\theta_2}{\partial x_1}=-\dfrac{\partial\theta_1}{\partial x_2}-\dfrac{24(1+v)}{Eh^3}M_{12};\\ \dfrac{\partial\theta_1}{\partial x_1}=-v\dfrac{\partial\theta_2}{\partial x_2}-\dfrac{12(1-v^2)}{Eh^2}M_{12};\\ \dfrac{\partial M_{11}}{\partial x_1}=-\dfrac{\partial M_{12}}{\partial x_2}+Q_1-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_1;\\ \dfrac{\partial M_{22}}{\partial x_1}=-\dfrac{Eh^3}{12}\dfrac{\partial^2\theta_2}{\partial x_2^2}-v\dfrac{\partial M_{11}}{\partial x_2}+\dfrac{\chi Eh}{2(1+v)}\left(\dfrac{\partial W}{\partial x_2}-\theta_2\right)-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_2;\\ \dfrac{\partial Q_1}{\partial x_1}=-\dfrac{\chi Eh}{2(1+v)}\left(\dfrac{\partial^2 W}{\partial x_2^2}-\dfrac{\partial\theta_2}{\partial x_2}\right)+\dfrac{\rho h\ddot{W}}{\Gamma_k}.\end{cases}$$

(9)

or

$$\frac{\partial y_1}{\partial x_1}=y_2+\frac{y_4}{\chi h};\quad \frac{дy_2}{дx_1}=-v\frac{дy_3}{дx_2}-\frac{6(1-v)}{h^3}y_5;\ \frac{\partial y_3}{\partial x_1}=-\frac{\partial y_2}{\partial x_2}-\frac{12}{h^3}y_6;$$

$$\frac{\partial y_4}{\partial x_1}=\chi h\frac{\partial}{\partial x_2}\left(y_3-\frac{\partial y_1}{\partial x_2}\right)+\frac{h}{\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};$$
$$\frac{\partial y_5}{\partial x_1}=-\frac{\partial y_6}{\partial x_2}+y_4-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};$$
$$\frac{\partial y_6}{\partial x_1}=\frac{\partial}{\partial x_2}\left(\frac{(1+v)h^3}{6}\cdot\frac{\partial y_3}{\partial x_2}-vy_5\right)+$$
$$+\chi h\left(\frac{\partial y_1}{\partial x_2}-y_3\right)-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_3}{\partial\tilde{t}^2}\quad (10)$$

where

$$y_1 = W; \quad y_2 = \theta_2; \quad y_3 = \theta/\nu; \quad y_4 = \frac{2(1+\nu)}{E} Q_1;$$
$$y_5 = \frac{4(1+\nu)}{1-\nu} M_{12}; \quad y_6 = \frac{h(1-\nu^2)}{E\nu} M_{12}$$
$$M_{22} = -D\left(\frac{\partial\theta_2}{\partial x_2} + \nu\frac{\partial\theta_1}{\partial x_1}\right) + \nu M_{11} - \nu M_{11} =$$
$$= -D(1-\nu^2)\frac{\partial\theta_2}{\partial x_2} + \nu M_{11} =$$
$$= -\frac{Eh^3}{12(1-\nu^2)}(1-\nu^2)\frac{\partial\theta_2}{\partial x_2} + \nu M_{11} =$$
$$= -\frac{Eh^3}{12}\frac{\partial\theta_2}{\partial x_2} + \nu M_{11}$$

Finding, as before, solutions describing plane harmonic waves propagating along the axes x_1, we will look for a solution of system (10) in the form

$$\begin{cases} y_1 = z_1(x_1)\cos(kx_2 - \omega t); \\ y_2 = z_2(x_1)\cos(kx_2 - \omega t); \\ y_3 = z_3(x_1)\sin(kx_2 - \omega t); \\ y_4 = z_4(x_1)\cos(kx_2 - \omega t); \\ y_5 = z_5(x_1)\cos(kx_2 - \omega t); \\ y_6 = z_6(x_1)\sin(kx_2 - \omega t). \end{cases} \qquad (11)$$

Substituting relations (11) into the system of partial differential equations (10), we obtain a system of first-order ordinary differential equations resolved with respect to derivatives:

$$\begin{cases} z_1' = z_2 + \frac{z_n}{\chi h}; \\ z_2' = -\nu\kappa\kappa_3 - \frac{6(1-\nu)}{3} z_5; \\ z_3' = \kappa z_2 - \frac{12}{h^3} z_6; \\ z_4' = \chi h\kappa\kappa_3 + \kappa^2\left(\chi h - \frac{hc^2}{\Gamma_л}\right) z_1; \\ z_5 = -\kappa z_6 + z_4 + \frac{h^3}{12\Gamma_л}\omega^2 z_2; \\ z_6' = -\chi h\kappa\kappa_1 - \left[\chi h + \frac{\kappa^2 h^3}{12\Gamma_л}\left(2(1+\nu) - \frac{c^2}{\Gamma_л}\right)\right] z_3 + \nu\kappa\kappa_5. \end{cases} \qquad (12)$$

The boundary conditions for this system can be written as follows:

a) free left edge of the plate:

$z_4 = z_5 = z_6 = 0, \quad x_1 = 0;$ (13)

b) free right edge of the record:

$z_4 = z_5 = z_6 = 0, \quad x_1 = l_1$ (14)

c) pinched right edge of the plate:

$z_1 = z_2 = z_3 = 0, \quad x_1 = l_1;$ (15)

Thus, we formulate the spectral boundary value problem (12-15) with respect to the parameter ω, describing the propagation of bending plane edge waves in the Timoshenko plate.

3. Results and Analysis

For a Kirchhoff plate of variable thickness, the first five modes with the lowest complex phase propagation velocities $C = C_R + iC_I$ were studied, where C_R- is the phase velocity of wave propagation; C_I - damping speed. The following dimensionless shell parameters are used in all calculation variants:

$$E = 1, \rho = 1, \nu = 0{,}25, G = 1, A = 0{,}048;$$
$$\beta = 0{,}05; \quad \alpha = 0{,}10.$$

The boundary conditions for the resolving system of equations (12) of the Timoshenko plate have the form:

$x=0,1; \quad z_4=z_5=z_6=0$. (16)

Then the expressions for the waveforms are

$$z_1 = A_1\cos 2\pi n x_2; \quad z_4 = A_4\sin 2\pi n x_2;$$
$$z_2 = A_2\cos 2\pi n x_2; \quad z_5 = A_5\sin 2\pi n x_2; \qquad (17)$$
$$z_3 = A_3\sin 2\pi n x_2; \quad z_6 = A_6\cos 2\pi n x_2;$$

In relations (17), the constants A_i (i=1,…,6) are determined from the solution of a linear algebraic system of equations

$$\begin{cases} A_3 + \frac{A_4}{\chi h} = 0; \\ \kappa A_3 - \frac{12}{h^3} A_5 = 0; \\ -\nu\kappa A_2 - \frac{6(1-\nu)}{h^3} A_6 = 0; \\ \chi h\kappa A_2 + \kappa^2(\chi h - hc^2) A_1 = 0; \\ -\chi h\kappa A_1 - \left[\frac{(1-\nu)h^3}{6}\kappa^2 + \chi h - \frac{h^3}{12}\omega^2\right] A_2 + \nu\kappa A_6 = 0; \\ -\kappa A_5 + A_4 + \frac{h^3}{12}\omega^2 A_3 = 0. \end{cases} \qquad (18)$$

The system of equations (18) is obtained by substituting expressions (17) into the resolving system of differential equations (12). Conditions for the determinant of the matrix of system (18) being equal to zero is the dispersion equation of the boundary value problem (12) - (13).

Figure 2 shows the first three modes of the real part of the complex phase velocities of the Timoshenko plate (b) in comparison with the corresponding modes of the Kirchhoff plate (a) with a Poisson's ratio of 0.25. In the case of the Kirchhoff plate, the limiting phase velocities are higher at modes II and III, and this difference increases as the mode number increases.

The obtained discrepancies with the results of [17] indicate the need for a more detailed study within the framework of the general theory of elasticity. However, in general, the numerical analysis of edge waves in Kirchhoff-Love and Timoshenko plates allows us to conclude that the Kirchhoff-Love hypotheses are quite suitable for calculating wave processes in wedge-shaped plates, including at frequencies with a wavelength of the order of plate thickness. This discrepancy with the classical results of the theory of Kirchhoff-Love plates of constant thickness is explained by the phenomenon of localization of the waveform with increasing frequency, which occurs only in plates of variable thickness.

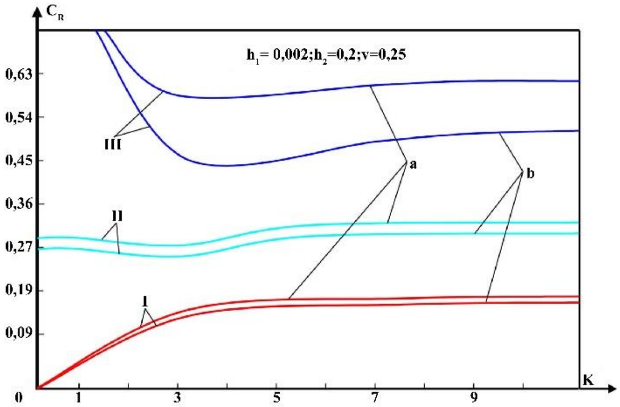

Fig. 2 Dispersion curves of phase velocities *a*-Kirchhoff-Love*;* *b*-Timoshenko

At the same time, the relative simplicity of the mathematical apparatus of the Kirchhoff plate theory makes it possible to study the dispersion characteristics of waveguides with a more complex cross-section configuration, which is very difficult to construct in the framework of a three-dimensional theory.

4. Conclusion

Based on the results obtained, the following conclusions are made:

- As the wave number increases, the propagation velocities of the real and imaginary parts of normal modes in the Timoshenko wedge-shaped plate tend to have constant values. In this case, the localization of motionis observed near the sharp edge of the waveguide;
- For small wedge angles, a comparison of the results obtained according to the Kirchhoff and Timoshenko theories shows a satisfactory match;
- The real and imaginary parts of the complex phase velocity of the first mode propagation in a wedge-shaped plate are practically independent of Poisson's ratio (variation within 0.5 %).

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