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Original Article

Reliability Analysis of a Three-unit Pumping System

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Abstract *- Three-unit pumping system of a water pumping station is analyzed. The pumping station receives water from the desalination plant, which is then supplied to the consumers through three pumps. The pumps need to be functional to supply potable water continuously without interrupting the water network. To ensure the optimized performance of the pumps, maintenance practices need to be evaluated based on the performance of the pumps, and hence is a potential case study from a reliability outcomes perspective. Five years of real maintenance data of the three pumps are collected from the pumping station, and various failures and restoration rates are estimated from the data. The objective is to obtain the reliability indices of the system, such as mean time to system failure for two pumps, availability, and the expected busy period of the maintenance team, for gauging the operational capabilities of the pumps. Semi-Markov and regenerative processes have been used for this analysis.*

Keywords *- Reliability, Pumps, Failures, Repairs, Semi–Markov processes, Regenerative processes.*

1. Nomenclature

$\mathcal{S}_0$	Operative state of the pump station with all 3 pumps operational
$\mathcal{S}_1$	Pump 1 fails, and the other 2 pumps are in operation
$\mathcal{S}_2$	Pump 2 fails, and the other 2 pumps are in operation
$\mathcal{S}_3$	Pump 3 fails, and the other 2 pumps are in operation
$\mathcal{S}_4$	Pump 1 Repair continues, Pump 2 is waiting for repair, and Pump 3 is in operation
$\mathcal{S}_5$	Pump 2 Repair continues, Pump 3 is waiting for repair, and Pump 1 is in operation
$\mathcal{S}_6$	Pump 3 Repair continues, Pump 1 is waiting for repair, and Pump 2 is in operation
λ_i	Rate of failure of Pump $i,\ where\ i = 1,2,3$
μ_i	Repair rate for Pump $i,\ where\ i = 1,2,3$
©	The symbol for Laplace Convolution
(s)	Symbol for Stieltje's convolution
*	The symbol for Laplace Transforms
**	Symbol for Laplace Stieltje's transforms
Λ_0	Steady-state availability of the system
B_0	An expected busy period of the maintenance facility
$\zeta_i(t)$	c.d.f. of first passage time from a regenerative state i to a failed state j
$p_{ij}(t), Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state I to the regenerative state j or a failed state j in (0, t].

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$g_1(t)$, $G_1(t)$	p.d.f. and c.d.f. of repair rate for repairable failures for Pump 1
$g_2(t)$, $G_2(t)$	p.d.f. and c.d.f. of repair rate for repairable failures for Pump 2
$g_3(t)$, $G_3(t)$	p.d.f. and c.d.f. of repair rate for repairable failures for Pump 3

*p.d.f.: probability density function
c.d.f.: cumulative distribution function

2. Introduction

Many industrial systems under different operating conditions were analyzed in the past. The objective was to obtain the relevant indices reflecting the system's operational capabilities. Mostly, the semi-Markovian process was adopted for the system analysis.

Mahmoud [1] worked on a system reliability study of a two-unit cold standby redundant system with two types of failure and preventive maintenance. Gopalan & Muralitharan [2] analyzed a one-unit repairable system subject to online preventive maintenance and repair; Mokaddis et al. [3] worked on a two-unit warm standby system subject to degradation; Li and Chen [4] talked about the aging properties of the residual life length of k-out-of-n system with independent but non-identical components; Rizwan [5] wrote about modeling strategy of an industrial system. Zuhair & Rizwan [6] discussed the reliability analysis of a two-unit system and estimated indices of interest; Parashar & Taneja [7] evaluated the profit of a PLC hot standby system based on the master-slave concept and two types of repair facilities; Gupta & Tewari [8] worked on a thermal power plant for obtaining reliability indices of interest. Mathew et al. [9]-[11] extensively worked on a continuous casting plant and developed models for system analysis. Rizwan et al. [12] carried out a cost-benefit analysis of a desalination unit; Sharma & Taneja [13] analyzed a two standby oil delivery system with a provision of switching over to another system at need to increase the availability; Sharma & Kaur [14]-[15] worked on compressor systems with and without Provision of Priority to a failed compressor Unit. Kumar et al. [16] worked on a furnace draft air cycle in a thermal power plant. Bhatia et al. [17] considered a 3-unit induced draft fan cold standby system working at full/reduced capacity. Parashar et al. [18] analyzed a 3-unit induced draft fan system with one warm standby system. Ram & Singh [19], Bhardwaj & Singh [20], Gupta & Gupta [21], and Niwas et al. [22] worked on the system analysis of head of-Line repair approach, asymptotic performance analysis of a standby system with server failure, Stochastic analysis of a reliability model of the one-unit system with post-inspection, post-repair, preventive maintenance, and replacement, and MTSF and profit analysis of a single unit system with inspection for the feasibility of repair beyond warranty. Later, Rizwan et al. [23]-[25] analyzed desalination plants with shutdown during the winter Season and repair/maintenance on an FCFS basis and domestic wastewater treatment plants. Padmavathi et al. [26] compared the models portraying two operating conditions of a desalination plant. Al Rahbi et al. [27]-[34] extensively focused on the rodding anode plant in the aluminum industry and developed models for different operating conditions of the plant for reliability analysis. Barak et al. [35]-[36] discussed a two-unit cold standby system operating under different weather conditions and stochastic analysis of the two-unit redundant system with priority to inspection over repair. Wang et al. [37], Yusuf et al. [38], Goyal et al. [39], Gupta et al. [40], Saini & Kumar [41], Dahiya et al. [42], and Singh et al. [43] have covered reliability analysis of a two dissimilar-unit warm standby repairable system with priority in use, reliability assessment of a repairable system under online and offline preventive maintenance, sensitivity analysis of physical processing unit of the sewage treatment plant, operational availability analysis of generators in steam turbine power plants, stochastic modeling of a single-unit system operating under different environmental conditions subject to inspection and degradation, modeling and analysis of concrete mixture plant subject to coverage factor and robust reliability approach, and cost-benefit analysis of two non-identical units' cold standby system subject to heavy rain with partially operative after repair. Taj et al. [44]-[52] developed multiple models for various operating conditions of the sub-systems and the main systems of a cable plant for system analysis in terms of reliability outcomes and cost-benefit analysis with comparisons among the models. Kadyan et al. [53] recently conducted a stochastic analysis of a three-unit non-identical repairable system, prioritizing the main unit for operations and repair. The literature review shows that most models use maintenance data for system analysis as case studies. The conventional modeling and analysis approach has been used to obtain the system behavior's reliability indices.

Therefore, the novelty of the entire work lies in its application to the industrial systems as a potential case study from a reliability perspective and obtaining the relevant reliability indices which reflect the system behavior and the cost-benefit analysis. Hence, the objective of the present work is a case study for analyzing a pumping station operating with three pumps and obtaining the relevant reliability indices which reflect the system or the pumping station's performance over a period. One such pumping station system in Oman is identified and analyzed to understand the operational capabilities of the pumps by obtaining the reliability indices such as mean time to system failure for two pumps, steady-state availability, and the expected busy period for restoring the system to an operational state. Five years of maintenance data of the pumps have been collected from the station. Failure and restoration rates are estimated from the data for further analysis.

The water transmission system is one of the major systems for transporting the potable water produced at the desalination plants. This transmission system plays a major role in providing drinking water to consumers. Potable water from the Association of Clean Water Agencies (ACWA) power and desalination plant is supplied with a maximum capacity of 3800 m^3/hr. The two storage reservoirs, Main Reservoir 1 (MR1) and Main Reservoir 2 (MR2), each have a capacity of 91,200 m^3. Potable water from SOMHAN Reservoir (SMN) power and RO plant is supplied having a maximum capacity of 5000m3/hr to the other two storage reservoirs, MR3 and MR4, each having a capacity of 130,000 m3 and 230,000 m3, respectively. Water in these reservoirs is pumped to different service reservoirs in the governorate area. Pumps are located in the pump house at the main pumping station and connected to the two common discharge headers. In emergencies, there is a provision to supply the water from the main reservoirs by pumping from an alternative facility in place. The operation of the water transmission system is carried out from the control room with the help of Programmable Logic Controllers (PLCs) and Supervisory Control and Data Acquisition (SCADA) system.

The main function of the water pump under discussion is to pump water from the reservoirs. The pumps circulate water through the distribution network and maintain the pressure required in the water network. The failures or malfunctions usually occur in the pump in the form of no water delivered, insufficient flow, intermittent flow, insufficient pressure, or pump leaking. Many components in the pump, if not functioning properly, will lead to major failures in the pump to achieve its work smoothly. The failures as noticed in the data could be the fault in bearing due to Lubricant loss through seal, insufficient or improper lubrication, over greased bearing, or fatigue cracks. The faults in impeller I the form of chamber wear due to erosion, corrosion, cavitation damage due to small diameter suction pipe and loose on the shaft, and wear ring damage.

Moreover, the shaft failures are bending fatigue (surface cracks), shaft diameter too small for the application, overloaded during operation, and machining dimension error. Mechanical seals also sometimes have failures like water hammer pressure increased, major vibration moved to seal, and the pump shaft bent. The coupling failures include loosening the coupling fastened to the shaft, the shaft is not straight, and the loss of lubricant film. There are also Electrical failures noted like poor motor power conditioning, phase drop due to winding failure, interruption of power supplier, and high operating temperature. Therefore, the failures discussed above result in the pump's poor performance and efficiency getting reduced and, therefore, unable to meet the end-user requirements. It is important to evaluate or plan the proper maintenance strategies to avoid or reduce failures. Therefore its reliability analysis would form the basis for assessing the operational capabilities of the pumps.

Pumping stations with three pumps are analyzed probabilistically using semi-Markov and regenerative processes.

3. Summarization of the Data

Estimated Rates for Pumps:

The estimated rate of failure of Pump 1 (λ_1) = 0.00099 per hour
The estimated rate of failure of Pump 2 (λ_2) = 0.000126 per hour
The estimated rate of failure of Pump 3 (λ_3) = 0.0015 per hour
Estimated restoration rate for Pump 1 (μ_1) = 0.13469 per hour
Estimated restoration rate for Pump 2 (μ_2) = 0.10186 per hour
Estimated restoration rate for Pump 3 (μ_3) = 0.09697 per hour

4. State Transition Table

The description of the states for the pump station is depicted in the state transition table, Table 1:

Table 1. State Transition Table

	$\mathcal{S}_0$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{S}_4$	$\mathcal{S}_5$	$\mathcal{S}_6$
$\mathcal{S}_0$	0	λ_1	λ_2	λ_3	0	0	0
$\mathcal{S}_1$	$g_1(t)$	0	0	0	λ_2	0	0
$\mathcal{S}_2$	$g_2(t)$	0	0	0	0	λ_3	0
$\mathcal{S}_3$	$g_3(t)$	0	0	0	0	0	λ_1
$\mathcal{S}_4$	0	0	$g_1(t)$	0	0	0	0
$\mathcal{S}_5$	0	0	0	$g_2(t)$	0	0	0
$\mathcal{S}_6$	0	$g_3(t)$	0	0	0	0	0

4.1. States of the System

Regenerative states: $0, 1, 2$ and 3
Non-regenerative states: $4, 5$ and 6
$\mathcal{S}_0(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3)$: All pumps are in an operative state
$\mathcal{S}_1(\mathcal{P}_{1r}, \mathcal{P}_2, \mathcal{P}_3)$: Pump 1 is failed and is under repair; the pumping station is working with two pumps
$\mathcal{S}_2$ $(\mathcal{P}_1, \mathcal{P}_{2r}, \mathcal{P}_3)$: Pump 2 is failed and is under repair; the pumping station is working with two pumps
$\mathcal{S}_3(\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_{3r})$: Pump 3 is failed and is under repair; the pumping station is working with two pumps
$\mathcal{S}_4(\mathcal{P}_{1R}, \mathcal{P}_{2rw}, \mathcal{P}_3)$: Pump 2 is failed and waiting for repair once the repair team is free from the repair continued from state 1, and the pumping station is working with one pump
$\mathcal{S}_5(\mathcal{P}_1, \mathcal{P}_{2R}, \mathcal{P}_{3rw})$: Pump 3 is failed and waiting for repair

once the repair team is free from the repair continued from state 2, and the pumping station is working with one pump $\mathcal{S}_6(\mathcal{P}_{1rw}, \mathcal{P}_2, \mathcal{P}_{3R})$: Pump 1 is failed and waiting for repair once the repair team is free from the repair continued from state 3, and the pumping station is working with one pump.

4.2. Model Assumptions

- Initially, all pumps are in the operating state
- At least one pump is operative all the time with a reduced supply of water
- The maintenance team attends one failed pump at a time and starts working on the other failed pump once the pump being attended is brought into operation
- All failure times are exponentially distributed
- Repair time distributions are taken as arbitrary
- Pump failures are self-announcing

5. Transition Probabilities and Mean Sojourn Times

The regeneration points are the epochs of entry into states 0, 1, 2, and 3. The transition probabilities are given by:

$$dQ_{01}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2+\lambda_3)\,t}dt,$$
$$dQ_{02}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2+\lambda_3)\,t}dt,$$
$$dQ_{03}(t) = \lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3)\,t}dt,$$
$$dQ_{10}(t) = e^{-\lambda_2 t} g_1(t)dt,$$
$$dQ_{12}^{(4)}(t) = (\lambda_2 e^{-\lambda_2 t} © 1)\, g_1(t)dt,$$
$$dQ_{14}(t) = \lambda_2 e^{-\lambda_2\,t}\overline{G_1}(t)dt,$$
$$dQ_{20}(t) = e^{-\lambda_3 t} g_2(t)dt,$$
$$dQ_{23}^{(5)}(t) = (\lambda_3 e^{-\lambda_2 t} © 1)\, g_2(t)dt,$$
$$dQ_{25}(t) = \lambda_3 e^{-\lambda_3\,t}\overline{G_2}(t)dt,$$
$$dQ_{30}(t) = e^{-\lambda_1 t} g_3(t)dt,$$
$$dQ_{31}^{(6)}(t) = (\lambda_1 e^{-\lambda_1 t} © 1)\, g_3(t)dt,$$
$$dQ_{36}(t) = \lambda_1 e^{-\lambda_1\,t}\overline{G_3}(t)dt.$$

(1-12)

Further the non-zero p_{ij}'s can be evaluated as follows:

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3};$$
$$p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3};$$
$$p_{03} = \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3};$$
$$p_{10} = g_1^*(\lambda_2) = \frac{\mu_1}{\mu_1+\lambda_2};$$
$$p_{12}^{(4)} = 1 - g_1^*(\lambda_2) = \frac{\lambda_2}{\mu_1+\lambda_2};$$
$$p_{14} = \frac{\lambda_2}{\mu_1+\lambda_2};$$
$$p_{20} = g_2^*(\lambda_3) = \frac{\mu_2}{\mu_2+\lambda_3};$$
$$p_{23}^{(5)} = 1 - g_2^*(\lambda_3) = \frac{\lambda_3}{\mu_2+\lambda_3};$$
$$p_{25} = \frac{\lambda_3}{\mu_2+\lambda_3};\ p_{30} = g_3^*(\lambda_1) = \frac{\mu_3}{\mu_3+\lambda_1};$$
$$p_{31}^{(6)} = 1 - g_3^*(\lambda_1) = \frac{\lambda_1}{\mu_3+\lambda_1};$$
$$p_{36} = \frac{\lambda_1}{\mu_3+\lambda_1}.$$

(13-24)

By these transition probabilities, it can be verified that:

$$p_{10} + p_{12}^{(4)} = p_{20} + p_{23}^{(5)} = p_{30} + p_{31}^{(6)} = 1$$

The mean sojourn time, μ_i in the regenerative state, 'i' is defined as the time of stay in that state before transitioning to any other state. If T denotes the sojourn time in the regenerative state 'i', then:

$$\mu_i = E(T) = P(T > t);$$
$$\mu_0 = \int_0^\infty e^{-(\lambda_1+\lambda_2+\lambda_3)t}dt = \frac{1}{\lambda_1+\lambda_2+\lambda_3}$$
$$\mu_1 = \int_0^\infty \overline{G_1}(t)e^{-\lambda_2 t}dt = \frac{1}{\lambda_2}[1 - g_1^*(\lambda_2)]$$
$$\mu_2 = \int_0^\infty \overline{G_2}(t)e^{-\lambda_3 t}dt = \frac{1}{\lambda_3}[1 - g_2^*(\lambda_3)]$$
$$\mu_3 = \int_0^\infty \overline{G_3}(t)e^{-\lambda_1 t}dt = \frac{1}{\lambda_1}[1 - g_3^*(\lambda_1)]$$

(25-28)

When measured from the epoch of arrival into state $'i'$, the unconditional mean time taken by the system to transit for any regeneration state $'j'$ is mathematically defined as:

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -{q_{ij}}^{*'}(0)$$

(Unconditional time taken to transit),

and $\sum_j m_{ij} = \mu_i$

$$\mu_i = E(T) = P(T > t);$$

$$m_{01} = \frac{\lambda_1}{(\lambda_1+\lambda_2+\lambda_3)^2};$$
$$m_{02} = \frac{\lambda_2}{(\lambda_1+\lambda_2+\lambda_3)^2};$$
$$m_{03} = \frac{\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)^2};$$
$$m_{10} = \frac{\mu_1}{(\mu_1+\lambda_2)^2};$$
$$m_{12} = \frac{1}{\mu_1} - \frac{\mu_1}{(\mu_1+\lambda_2)^2};$$
$$m_{14} = \frac{\lambda_2}{(\mu_1+\lambda_2)^2};$$

$$m_{20} = \frac{\mu_2}{(\mu_2+\lambda_3)^2};$$
$$m_{23} = \frac{1}{\mu_2} - \frac{\mu_2}{(\mu_2+\lambda_3)^2};$$
$$m_{25} = \frac{\lambda_3}{(\mu_2+\lambda_3)^2};$$
$$m_{30} = \frac{\mu_3}{(\mu_3+\lambda_1)^2};$$
$$m_{31} = \frac{1}{\mu_3} - \frac{\mu_3}{(\mu_3+\lambda_1)^2};$$
$$m_{36} = \frac{\lambda_1}{(\mu_3+\lambda_1)^2}.$$

(29-40)

6. The Mathematical Analysis

6.1. Mean Time to System Failure (2 Pumps)

Regarding the failed states as absorbing states when the 2 pumps fail out of the total 3, the following recursive relation, $\zeta_i(t)$ for regenerative processes is derived by applying the arguments used for regenerative processes and using Stieltje's convolution:

$$\zeta_0(t) = Q_{01}(t)(s)\,\zeta_1(t) + Q_{02}(t)(s)\,\zeta_2(t) + Q_{03}(t)(s)\,\zeta_3(t)$$
$$\zeta_1(t) = Q_{10}(t)(s)\,\zeta_0(t) + Q_{14}(t)$$
$$\zeta_2(t) = Q_{20}(t)(s)\,\zeta_0(t) + Q_{25}(t)$$
$$\zeta_3(t) = Q_{30}(t)(s)\,\zeta_0(t) + Q_{36}(t)$$

(41-44)

Now using Laplace Stieltje's transforms $(\zeta^{**}(s))$, when the unit started at the beginning of the state $\mathcal{S}_0$MTSF is computed as

$$MTSF = \lim_{s\to 0}\frac{1-\zeta_0^{**}(s)}{s} = \lim_{s\to 0}\frac{1-\frac{\mathcal{N}(s)}{\mathcal{D}(s)}}{s}$$

(45)

$$= \lim_{s\to 0}\frac{\mathcal{D}(s)-\mathcal{N}(s)}{s\mathcal{D}(s)}$$
$$= \lim_{s\to 0}\frac{\mathcal{D}'(s)-\mathcal{N}'(s)}{s\mathcal{D}'(s)+\mathcal{D}(s)} = \frac{\mathcal{D}'(0)-\mathcal{N}'(0)}{\mathcal{D}(0)} = \frac{\mathcal{N}}{\mathcal{D}}$$

(45)

where $\mathcal{N} = m_{10}p_{01} + m_{14}p_{01} + m_{20}p_{02} + m_{25}p_{02} + m_{30}p_{03} + m_{36}p_{03}$ $\mathcal{D} = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}$.

6.2. Availability Analysis of the System

The following recursive relations for $\Lambda_i(t)$ are derived using the probabilistic arguments and defining $\Lambda_i(t)$ as the probability of the unit entering upstate at the instant time t, given that the system entered regenerative state i at $t = 0$:

$$\Lambda_0(t) = \mathcal{M}_0(t) + q_{01}(t)©\Lambda_1(t) + q_{02}(t)©\Lambda_2(t) + q_{03}(t)©\Lambda_3(t)$$
$$\Lambda_1(t) = \mathcal{M}_1(t) + q_{10}(t)©\Lambda_0(t) + q_{12}^{(4)}(t)©\Lambda_2(t)$$
$$\Lambda_2(t) = \mathcal{M}_2(t) + q_{20}(t)©\Lambda_0(t) + q_{23}^{(5)}(t)©\Lambda_3(t)$$
$$\Lambda_3(t) = \mathcal{M}_3(t) + q_{30}(t)©\Lambda_0(t) + q_{31}^{(6)}(t)©\Lambda_1(t)$$

(46-49)

where

$$\mathcal{M}_0(t) = e^{-(\lambda_1+\lambda_2+\lambda_3)t};$$
$$\mathcal{M}_1(t) = \overline{G_1}(t)e^{-\lambda_2 t};$$
$$\mathcal{M}_2(t) = \overline{G_2}(t)e^{-\lambda_3 t};$$
$$\mathcal{M}_3(t) = \overline{G_3}(t)e^{-\lambda_1 t}$$

(50-53)

Taking Laplace transforms of the above equations and solving for$\Lambda_0^*(s)$, the following is obtained:

$$\Lambda_0 = \lim_{s\to 0} s\Lambda_0^*(s) = \frac{\mathcal{N}_1}{\mathcal{D}_1}$$

(54)

where

$$\mathcal{N}_1 = \mathcal{M}_3p_{03} + \mathcal{M}_3p_{02}p_{23} + \mathcal{M}_3p_{01}p_{12}p_{23} + \mathcal{M}_1p_{03}p_{31} + \mathcal{M}_2p_{03}p_{12}p_{31} + \mathcal{M}_1p_{02}p_{23}p_{31} + \mathcal{M}_0p_{12}p_{23}p_{31}$$
$$\mathcal{D}_1 = m_{30}p_{03} + m_{31}p_{03}p_{10} + m_{31}p_{03}p_{12}p_{20} + m_{30}p_{02}p_{23} + m_{31}p_{02}p_{10}p_{23} + m_{31}p_{12}p_{23} + m_{30}p_{01}p_{12}p_{23} + m_{03}p_{30} + m_{23}p_{02}p_{30} + m_{23}p_{01}p_{12}p_{30} + m_{02}p_{23}p_{30} + m_{12}p_{01}p_{23}p_{30} + m_{01}p_{12}p_{23}p_{30} + m_{10}p_{03}p_{31} + m_{03}p_{10}p_{31} + m_{23}p_{02}p_{10}p_{31} + m_{23}p_{12}p_{31} + m_{20}p_{03}p_{12}p_{31} + m_{12}p_{03}p_{20}p_{31} + m_{03}p_{12}p_{20}p_{31} + m_{12}p_{23}p_{31} + m_{10}p_{02}p_{23}p_{31} + m_{02}p_{10}p_{23}p_{31}.$$

6.3. Busy Period Analysis of the System

Following recursive relations are established by defining $\mathcal{B}_i(t)$ as the probability that the repairman is occupied at instant t, given that the system reached regenerative state i at $t = 0$:

$$\mathcal{B}_0(t) = q_{01}(t)©\mathcal{B}_1(t) + q_{02}(t)©\mathcal{B}_2(t) + q_{03}(t)©\mathcal{B}_3(t)$$
$$\mathcal{B}_1(t) = W_1(t) + q_{10}(t)©\mathcal{B}_1(t) + q_{12}^{(4)}(t)©\mathcal{B}_2(t)$$
$$\mathcal{B}_2(t) = W_2(t) + q_{20}(t)©\mathcal{B}_0(t) + q_{23}^{(5)}(t)©\mathcal{B}_3(t)$$
$$\mathcal{B}_3(t) = W_3(t) + q_{30}(t)©\mathcal{B}_0(t) + q_{31}^{(6)}(t)©\mathcal{B}_1(t)$$

(55-58)

where,

$$W_1(t) = \overline{G_1}(t)e^{-\lambda_2 t};$$
$$W_2(t) = \overline{G_2}(t)e^{-\lambda_3 t};$$
$$W_3(t) = \overline{G_3}(t)e^{-\lambda_1 t}$$

(59-61)

Taking Laplace transforms of above equations and solving for $\mathcal{B}_0{}^*(s)$The expected busy period of the maintenance facility in steady-state is obtained as:

$$\mathcal{B}_0 = \lim_{s\to 0} s\,\mathcal{B}_0{}^*(s) = \frac{\mathcal{N}_2}{\mathcal{D}_2},$$

(62)

where,

$$\mathcal{N}_2 = W_3p_{03} + W_3p_{02}p_{23} + W_3p_{01}p_{12}p_{23} + W_1p_{03}p_{31} + W_2p_{03}p_{12}p_{31} + W_1p_{02}p_{23}p_{31}$$
$$\mathcal{D}_2 = m_{30}p_{03} + m_{31}p_{03}p_{10} + m_{31}p_{03}p_{12}p_{20} + m_{30}p_{02}p_{23} + m_{31}p_{02}p_{10}p_{23} + m_{31}p_{12}p_{23} + m_{30}p_{01}p_{12}p_{23} + m_{03}p_{30} + m_{23}p_{02}p_{30} + m_{23}p_{01}p_{12}p_{30} + m_{02}p_{23}p_{30} + m_{12}p_{01}p_{23}p_{30} +$$

$$m_{01}p_{12}p_{23}p_{30} + m_{10}p_{03}p_{31} + m_{03}p_{10}p_{31} + m_{23}p_{02}p_{10}p_{31} + m_{23}p_{12}p_{31} + m_{20}p_{03}p_{12}p_{31} + m_{12}p_{03}p_{20}p_{31} + m_{03}p_{12}p_{20}p_{31} + m_{12}p_{23}p_{31} + m_{10}p_{02}p_{23}p_{31} + m_{02}p_{10}p_{23}p_{31}.$$

Using the data as summarized in Section 3 and various expressions for reliability indicators obtained in sections 5 & 6, the following values of the system effectiveness are estimated:

Mean Time to System Failure (2 Pumps)
$= 24278.2\ hours.$

Availability for the system Λ_0 = 0.999606

Expected busy period Bo =0.0339684

6.4. Profit Analysis

The profit incurred by the system can be obtained by using the following equation:

$$P = C_0\Lambda_0 - C_1B_0 \tag{63}$$

where,

C_0 does the system generate the revenue per unit up-time

C_1 the maintenance cost per unit time for which the repairman is busy.

7. Conclusion

The result shows that the mean time to pump failure is 24278.2 hours, the probability of system operational capability is quite satisfactory, which is 0.999606, and the expected busy period of the maintenance team is 0.0339684. The overall profitability of the system is OMR 2498.336 per hour based on the revenue and maintenance costs. The maintenance practices adopted by the team look organized and prompt. However, the reasons for the failure of the pumps and the restoration time could be relooked by the team to improve upon the results further.

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